Users' manual for the `Sollya` tool

weekly-2026-10-26

Sylvain Chevillard
sylvain.chevillard@ens-lyon.org

Christoph Lauter
christoph.lauter@christoph-lauter.org

Mioara Joldeş
joldes@laas.fr

License

The `Sollya` tool is Copyright © 2006-2022 by

Laboratoire de l'Informatique du Parallélisme,
UMR CNRS - ENS Lyon - UCB Lyon 1 - INRIA 5668
Lyon, France,

LORIA (CNRS, INPL, INRIA, UHP, U-Nancy 2), Nancy, France,

Laboratoire d'Informatique de Paris 6, equipe PEQUAN,
UPMC Universite Paris 06 - CNRS - UMR 7606 - LIP6, Paris, France,

Laboratoire d'Informatique de Paris 6 - Équipe PEQUAN
Sorbonne Universités
UPMC Univ Paris 06
UMR 7606, LIP6
Boîte Courrier 169
4, place Jussieu
F-75252 Paris Cedex 05
France,

Sorbonne Université
CNRS, Laboratoire d'Informatique de Paris 6, LIP6
F - 75005 Paris
France,

CNRS, LIP6, UPMC
Sorbonne Universités, UPMC Univ Paris 06,
CNRS, LIP6 UMR 7606, 4 place Jussieu 75005 Paris,

University of Alaska Anchorage, College of Engineering

and by

Centre de recherche INRIA Sophia Antipolis Méditerranée,
Équipes APICS, FACTAS,
Sophia Antipolis, France.

All rights reserved.

This software is governed by the CeCILL-C license under French law and abiding by the rules of distribution of free software. You can use, modify and/ or redistribute the software under the terms of the CeCILL-C license as circulated by CEA, CNRS and INRIA at the following URL `http://www.cecill.info`.
As a counterpart to the access to the source code and rights to copy, modify and redistribute granted by the license, users are provided only with a limited warranty and the software's author, the holder of the economic rights, and the successive licensors have only limited liability.
In this respect, the user's attention is drawn to the risks associated with loading, using, modifying and/or developing or reproducing the software by the user in light of its specific status of free software, that may mean that it is complicated to manipulate, and that also therefore means that it is reserved for developers and experienced professionals having in-depth computer knowledge. Users are therefore encouraged to load and test the software's suitability as regards their requirements in conditions enabling the security of their systems and/or data to be ensured and, more generally, to use and operate it in the same conditions as regards security.
The fact that you are presently reading this means that you have had knowledge of the CeCILL-C license and that you accept its terms.
This program is distributed WITHOUT ANY WARRANTY; without even the implied warranty of MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.

Contents

1 Compilation and installation of `Sollya` 1
- 1.1 Compilation dependencies . . . 1
- 1.2 `Sollya` command line options . . . 1

2 Introduction 3

3 General principles 5

4 Variables 7

5 Data types 9
- 5.1 Booleans . . . 9
- 5.2 Numbers . . . 9
- 5.3 Rational numbers and rational arithmetic . . . 13
- 5.4 Intervals and interval arithmetic . . . 14
- 5.5 Functions . . . 19
- 5.6 Strings . . . 20
- 5.7 Particular values . . . 20
- 5.8 Lists . . . 20
- 5.9 Structures . . . 22

6 Iterative language elements: assignments, conditional statements and loops 24
- 6.1 Blocks . . . 24
- 6.2 Assignments . . . 24
- 6.3 Conditional statements . . . 26
- 6.4 Loops . . . 26

7 Functional language elements: procedures and pattern matching 27
- 7.1 Procedures . . . 27
- 7.2 Pattern matching . . . 29

8 Commands and functions 40
- 8.1 abs . . . 41
- 8.2 absolute . . . 41
- 8.3 accurateinfnorm . . . 42
- 8.4 acos . . . 43
- 8.5 acosh . . . 43
- 8.6 && . . . 43
- 8.7 annotatefunction . . . 44
- 8.8 :. . . . 46
- 8.9 ∼ . . . 46
- 8.10 asciiplot . . . 48
- 8.11 asin . . . 49
- 8.12 asinh . . . 50
- 8.13 atan . . . 50
- 8.14 atanh . . . 50
- 8.15 autodiff . . . 50
- 8.16 autosimplify . . . 52
- 8.17 bashevaluate . . . 53
- 8.18 bashexecute . . . 54
- 8.19 bezout . . . 54
- 8.20 binary . . . 56
- 8.21 bind . . . 56
- 8.22 boolean . . . 58
- 8.23 canonical . . . 59

8.24 ceil . . . 60
8.25 chebyshevform . . . 60
8.26 checkinfnorm . . . 61
8.27 coeff . . . 63
8.28 composepolynomials . . . 63
8.29 @ . . . 64
8.30 constant . . . 66
8.31 cos . . . 66
8.32 cosh . . . 67
8.33 D . . . 67
8.34 DD . . . 67
8.35 DE . . . 67
8.36 decimal . . . 67
8.37 default . . . 67
8.38 degree . . . 68
8.39 denominator . . . 69
8.40 diam . . . 69
8.41 dieonerrormode . . . 70
8.42 diff . . . 72
8.43 dirtyfindzeros . . . 72
8.44 dirtyinfnorm . . . 73
8.45 dirtyintegral . . . 74
8.46 dirtysimplify . . . 75
8.47 display . . . 76
8.48 div . . . 77
8.49 / . . . 78
8.50 double . . . 80
8.51 doubledouble . . . 80
8.52 doubleextended . . . 81
8.53 dyadic . . . 82
8.54 == . . . 82
8.55 erf . . . 85
8.56 erfc . . . 86
8.57 error . . . 86
8.58 evaluate . . . 87
8.59 execute . . . 88
8.60 exp . . . 89
8.61 expand . . . 89
8.62 expm1 . . . 90
8.63 exponent . . . 90
8.64 externaldata . . . 91
8.65 externalplot . . . 92
8.66 externalproc . . . 93
8.67 false . . . 96
8.68 file . . . 97
8.69 findzeros . . . 97
8.70 fixed . . . 98
8.71 floating . . . 98
8.72 floor . . . 99
8.73 fpminimax . . . 99
8.74 fullparentheses . . . 106
8.75 function . . . 107
8.76 gcd . . . 110
8.77 >= . . . 111
8.78 getbacktrace . . . 112
8.79 getsuppressedmessages . . . 114

8.80 > . . . 116
8.81 guessdegree . . . 116
8.82 halfprecision . . . 117
8.83 head . . . 118
8.84 hexadecimal . . . 118
8.85 honorcoeffprec . . . 119
8.86 hopitalrecursions . . . 119
8.87 horner . . . 120
8.88 HP . . . 121
8.89 implementconstant . . . 121
8.90 implementpoly . . . 126
8.91 in . . . 129
8.92 inf . . . 130
8.93 infnorm . . . 131
8.94 integer . . . 133
8.95 integral . . . 133
8.96 interpolate . . . 134
8.97 isbound . . . 138
8.98 isevaluable . . . 139
8.99 <= . . . 140
8.100 length . . . 141
8.101 library . . . 142
8.102 libraryconstant . . . 143
8.103 list of . . . 145
8.104 log . . . 145
8.105 log10 . . . 146
8.106 log1p . . . 146
8.107 log2 . . . 146
8.108 < . . . 146
8.109 mantissa . . . 147
8.110 max . . . 148
8.111 mid . . . 149
8.112 midpointmode . . . 149
8.113 min . . . 150
8.114 − . . . 151
8.115 mod . . . 153
8.116 ∗ . . . 154
8.117 nearestint . . . 155
8.118 != . . . 155
8.119 nop . . . 156
8.120 ! . . . 157
8.121 numberroots . . . 158
8.122 numerator . . . 159
8.123 object . . . 160
8.124 objectname . . . 160
8.125 off . . . 162
8.126 on . . . 163
8.127 || . . . 163
8.128 parse . . . 164
8.129 perturb . . . 165
8.130 pi . . . 165
8.131 plot . . . 166
8.132 + . . . 168
8.133 points . . . 169
8.134 postscript . . . 170
8.135 postscriptfile . . . 170

8.136 ^ . . . 170
8.137 powers . . . 172
8.138 prec . . . 172
8.139 precision . . . 173
8.140 .: . . . 174
8.141 print . . . 174
8.142 printdouble . . . 177
8.143 printexpansion . . . 177
8.144 printsingle . . . 178
8.145 printxml . . . 179
8.146 proc . . . 180
8.147 procedure . . . 185
8.148 QD . . . 186
8.149 quad . . . 186
8.150 quit . . . 187
8.151 range . . . 187
8.152 rationalapprox . . . 188
8.153 rationalmode . . . 189
8.154 RD . . . 190
8.155 readfile . . . 190
8.156 readxml . . . 191
8.157 relative . . . 192
8.158 remez . . . 192
8.159 rename . . . 195
8.160 restart . . . 197
8.161 return . . . 198
8.162 revert . . . 199
8.163 RN . . . 200
8.164 round . . . 200
8.165 roundcoefficients . . . 201
8.166 roundcorrectly . . . 203
8.167 roundingwarnings . . . 203
8.168 RU . . . 204
8.169 RZ . . . 204
8.170 searchgal . . . 205
8.171 SG . . . 206
8.172 showmessagenumbers . . . 206
8.173 simplify . . . 208
8.174 sin . . . 209
8.175 single . . . 209
8.176 sinh . . . 210
8.177 sort . . . 210
8.178 sqrt . . . 211
8.179 string . . . 211
8.180 subpoly . . . 211
8.181 substitute . . . 212
8.182 sup . . . 213
8.183 supnorm . . . 214
8.184 suppressmessage . . . 215
8.185 tail . . . 218
8.186 tan . . . 219
8.187 tanh . . . 219
8.188 taylor . . . 219
8.189 taylorform . . . 220
8.190 taylorrecursions . . . 223
8.191 TD . . . 224

8.192 time . . . 224
8.193 timing . . . 225
8.194 tripledouble . . . 226
8.195 true . . . 226
8.196 unsuppressmessage . . . 227
8.197 var . . . 229
8.198 verbosity . . . 230
8.199 void . . . 231
8.200 worstcase . . . 232
8.201 write . . . 233
8.202 _x_ . . . 234

9 Appendix: interval arithmetic philosophy in `Sollya` 236
9.1 Univariate functions . . . 236
9.2 Bivariate functions . . . 236

10 Appendix: the `Sollya` library 237
10.1 Introduction . . . 237
10.2 Sollya object data-type . . . 237
10.3 Conventions in use in the library . . . 239
10.4 Displaying `Sollya` objects and numerical values . . . 239
10.5 Creating `Sollya` objects . . . 240
10.5.1 Numerical constants . . . 240
10.5.2 Functional expressions . . . 241
10.5.3 Other simple objects . . . 241
10.5.4 Lists . . . 243
10.5.5 Structures . . . 244
10.5.6 Library functions, library constants and procedure functions . . . 245
10.5.7 External procedures and external data symbols . . . 246
10.6 Getting the type of an object . . . 248
10.7 Recovering the value of a range . . . 250
10.8 Recovering the value of a numerical constant or a constant expression . . . 250
10.9 Converting a string from `Sollya` to C . . . 252
10.10 Recovering the contents of a `Sollya` list . . . 252
10.11 Recovering the contents of a `Sollya` structure . . . 253
10.12 Decomposing a functional expression . . . 253
10.13 Faithfully evaluate a functional expression . . . 258
10.14 Comparing objects structurally and computing hashes on `Sollya` objects . . . 259
10.15 Executing `Sollya` procedures . . . 261
10.16 Name of the free variable . . . 261
10.17 Commands and functions . . . 261
10.18 Warning messages in library mode . . . 262
10.18.1 Catching warning messages . . . 262
10.18.2 Emitting warning messages . . . 265
10.19 Using `Sollya` in a program that has its own allocation functions . . . 265

1 Compilation and installation of Sollya

Sollya comes in two flavors:

- Either as an interactive tool. This is achieved by running the Sollya executable file.
- Or as a C library that provides all the features of the tool within the C programming language.

The installation of the tool and the library follow the same steps, described below. The present documentation focuses more on the interactive tool. As a matter of fact, the library works exactly the same way as the tool, so it is necessary to know a little about the tool in order to correctly use the library. The reader who is only interested in the library should at least read the following Sections 2, 3 and 5. A documentation specifically describing the library usage is available in Appendix 10 at the end of the present documentation.

1.1 Compilation dependencies

The Sollya distribution can be compiled and installed using the usual ./configure, make, make install procedure. Besides a C and a C++ compiler, Sollya needs the following software libraries and tools to be installed:

- GMP
- MPFR
- MPFI
- fplll
- libxml2
- gnuplot (external tool).

The ./configure script checks for the installation of the libraries. However Sollya will build without error if gnuplot is not installed. In this case an error will be displayed at runtime.

The use of the external tool rlwrap is highly recommended but not required to use the Sollya interactive tool. Use the -A option of rlwrap for correctly displayed ANSI X3.64/ ISO/IEC 6429 colored prompts (see below).

1.2 Sollya command line options

Sollya can read input on standard input or in a file whose name is given as an argument when Sollya is invoked. The tool will always produce its output on standard output, unless specifically instructed by a particular Sollya command that writes to a file. The following lines are valid invocations of Sollya, assuming that bash is used as a shell:

```
~/% sollya
...
~/% sollya myfile.sollya
...
~/% sollya < myfile.sollya
```

If a file given as an input does not exist, an error message is displayed.

All configurations of the internal state of the tool are done by commands given on the Sollya prompt or in Sollya scripts. Nevertheless, some command line options are supported; they work at a very basic I/O-level and can therefore not be implemented as commands.

The following options are supported when calling Sollya:

- `--args`: This special argument indicates to `Sollya` that subsequent command line arguments are no longer to be interpreted but are to be passed as-is to the predefined `Sollya` variable `__argv`. The `--args` argument is implicitly assumed if a `Sollya` script filename has already been specified with a preceding command line argument and none of the subsequent command line arguments is one of the special options given in this list.
- `--donotmodifystacksize`: When invoked, `Sollya` tries to increase the stack size that is available to a user process to the maximum size supported by the kernel. On some systems, the correspondent `ioctl` does not work properly. Use the option to prevent `Sollya` from changing the stack size.
- `--flush`: When this option is given, `Sollya` will flush all its input and output buffers after parsing and executing each command resp. sequence of commands. This option is needed when pipes are used to communicate with `Sollya` from another program.
- `--help`: Prints help on the usage of the tool and quits.
- `--nocolor`: `Sollya` supports coloring of the output using ANSI X3.64/ ISO/IEC 6429 escape sequences. Coloring is deactivated when `Sollya` is connected on standard input to a file that is not a terminal. This option forces the deactivation of ANSI coloring. This might be necessary on very old gray-scale terminals or when encountering problems with old versions of `rlwrap`.
- `--noprompt`: `Sollya` prints a prompt symbol when connected on standard input to a pseudo-file that is a terminal. The option deactivates the prompt.
- `--oldautoprint`: The behavior of an undocumented feature for displaying values has changed in `Sollya` from version 1.1 to version 2.0. The old feature is deprecated. If you wish to use it nevertheless, use this deprecated option.
- `--oldexternalprocprint`: The behavior of an undocumented feature for displaying `Sollya` objects representing external procedures upon automatic printing at the `Sollya` prompt has been changed in `Sollya` from version 4.1 to version 5.0. The old feature is deprecated. If you wish to use it nevertheless, use this deprecated option.
- `--oldrlwrapcompatible`: This option is deprecated. It makes `Sollya` emit a non ANSI X3.64 compliant coloring escape sequence for making it compatible with versions of `rlwrap` that do not support the `-A` option. The option is considered a hack since it is known to garble the output of the tool under some particular circumstances.
- `--warninfile[append] <file>`: Normally, `Sollya` emits warning and information messages together with all other displayed information on either standard output or standard error. This option allows all warning and information messages to get redirected to a file. The filename to be used must be given after the option. When `--warninfile` is used, the existing content (if any) of the file is first removed before writing to the file. With `--warninfileappend`, the messages are appended to an existing file. Even if coloring is used for the displaying all other `Sollya` output, no coloring sequences are ever written to the file. Let us emphasize on the fact that any file of a unixoid system can be used for output, for instance also a named pipe. This allows for error messaging to be performed on a separate terminal. The use of this option is mutually exclusive with the `--warnonstderr` option.
- `--warnonstderr`: Normally, `Sollya` prints warning and information messages on standard output, using a warning color when coloring is activated. When this option is given, `Sollya` will output all warning and information messages on standard error. Coloring will be used even on standard error, when activated. The use of this option is mutually exclusive with the `--warninfile[append]` option.

The `Sollya` interactive tool process returns the following exit status values, depending on the various reasons the tool exits:

- The exit status `0` is returned when `Sollya` is quit using the `quit` command. This is the standard way of terminating the `Sollya` process. The same exit status is returned for cases when `Sollya` is run with one of the `--help` or `--version` options (see above).

- The exit status `1` is returned when the `Sollya` is terminated due to an internal error. This case should never happen. This exit status `1` is also returned when two or more command line options (as documented above) are inconsistent, syntactically incorrect or provoke some other low-level error (such as a file in input not being readable).
- The exit status `2` is returned when the `Sollya` tool is terminated upon a `Sollya` language level error and `dieonerrormode` is set to `on` (see the documentation of that keyword for details).
- The exit status `3` is returned when the last `Sollya` command gets parsed and executed correctly but input reaches an end-of-file condition without the `quit` command being executed beforehand.
- The exit status `4` is returned when `Sollya` reaches an end-of-file condition upon incomplete input, *i.e.* when it started parsing a new `Sollya` command (or expression), which is incomplete. Remark that the empty input (end-of-file immediately upon `Sollya` process launch) is an incomplete input for reasons to be found in the `Sollya` grammar.

2 Introduction

`Sollya` is an interactive tool for handling numerical functions and working with arbitrary precision. It can evaluate functions accurately, compute polynomial approximations of functions, automatically implement polynomials for use in math libraries, plot functions, compute infinity norms, etc. `Sollya` is also a full-featured script programming language with support for procedures etc.

Let us begin this manual with an example. `Sollya` does not allow command line edition; since this may quickly become uncomfortable, we highly suggest to use the `rlwrap` tool with `Sollya`:

```
~/% rlwrap -A sollya
>
```

`Sollya` manipulates only functions in one variable. The first time that an unbound variable is used, this name is fixed. It will be used to refer to the free variable. For instance, try

```
> f = sin(x)/x;
> g = cos(y)-1;
Warning: the identifier "y" is neither assigned to, nor bound to a library funct
ion nor external procedure, nor equal to the current free variable.
Will interpret "y" as "x".
> g;
cos(x) - 1
```

Now, the name x can only be used to refer to the free variable:

```
> x = 3;
Warning: the identifier "x" is already bound to the free variable, to a library
function, library constant, external procedure or an external data symbol.
The command will have no effect.
Warning: the last assignment will have no effect.
```

If you really want to unbind x, you can use the `rename` command and change the name of the free variable:

```
> rename(x,y);
Information: the free variable has been renamed from "x" to "y".
> g;
cos(y) - 1
> x=3;
> x;
3
```

Sollya has a reserved keyword that can always be used to refer to the free variable. This keyword is _x_. This is particularly useful in contexts when the name of the variable is not known: typically when referring to the free variable in a pattern matching or inside a procedure.

```
> f == sin(_x_)/_x_;
true
```

As you have seen, you can name functions and easily work with them. The basic thing to do with a function is to evaluate it at some point:

```
> f(-2);
Warning: rounding has happened. The value displayed is a faithful rounding to 16
5 bits of the true result.
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
0.45464871341284084769800993295587242135112748572394
> evaluate(f,-2);
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
0.45464871341284084769800993295587242135112748572394
```

The printed value is generally a faithful rounding of the exact value at the working precision (*i.e.*, one of the two floating-point numbers enclosing the exact value). Internally Sollya represents numbers as floating-point numbers in arbitrary precision with radix 2: the fact that a faithful rounding is performed in binary does not imply much on the exactness of the digits displayed in decimal. The working precision is controlled by the global variable prec:

```
> prec;
165
> prec=200;
The precision has been set to 200 bits.
> prec;
200
> f(-2);
Warning: rounding has happened. The value displayed is a faithful rounding to 20
0 bits of the true result.
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
0.45464871341284084769800993295587242135112748572394513418948 65
```

Sometimes a faithful rounding cannot easily be computed. In such a case, a value is printed that was obtained using floating-point approximations without control on the final accuracy:

```
> log2(5)/log2(17) - log(5)/log(17);
Warning: rounding may have happened.
If there is rounding, the displayed value is *NOT* guaranteed to be a faithful r
ounding of the true result.
0
```

The philosophy of Sollya is: *Whenever something is not exact, print a warning.* This explains the warnings in the previous examples. If the result can be shown to be exact, there is no warning:

```
> sin(0);
0
```

Let us finish this Section with a small complete example that shows a bit of what can be done with Sollya:

```
> restart;
The tool has been restarted.
> prec=50;
The precision has been set to 50 bits.
> f=cos(2*exp(x));
> d=[-1/8;1/8];
> p=remez(f,2,d);
> derivativeZeros = dirtyfindzeros(diff(p-f),d);
> derivativeZeros = inf(d).:derivativeZeros:.sup(d);
> maximum=0;
> for t in derivativeZeros do {
    r = evaluate(abs(p-f), t);
    if r > maximum then { maximum=r; argmaximum=t; };
  };
> print("The infinity norm of", p-f, "is", maximum, "and is reached at", argmaxi
mum);
The infinity norm of -3.89710727796949e-2 * x^2 + -1.79806720921853 * x + (-0.41
62655728752966) - cos(2 * exp(x)) is 8.6306594443227e-4 and is reached at 6.6635
5088071379e-2
```

In this example, we define a function f, an interval d and we compute the best degree-2 polynomial approximation of f on d with respect to the infinity norm. In other words, $\max_{x \in d}\{|p(x) - f(x)|\}$ is minimal among polynomials with degree not greater than 2. Then, we compute the list of the zeros of the derivative of $p - f$ and add the bounds of d to this list. Finally, we evaluate $|p - f|$ for each point in the list and store the maximum and the point where it is reached. We conclude by printing the result in a formatted way.

Let us mention as a side-note that you do not really need to use such a script for computing an infinity norm; as we will see, the command `dirtyinfnorm` does this for you.

3 General principles

The first purpose of `Sollya` is to help people using numerical functions and numerical algorithms in a safe way. It is first designed to be used interactively but it can also be used in scripts[1].

One of the particularities of `Sollya` is to work with multi-precision arithmetic (it uses the `MPFR` library). For safety purposes, `Sollya` knows how to use interval arithmetic. It uses interval arithmetic to produce tight and safe results with the precision required by the user.

The general philosophy of `Sollya` is: *When you can perform a computation exactly and sufficiently quickly, do it; when you cannot, do not, unless you have been explicitly asked for.*

The precision of the tool is set by the global variable `prec`. In general, the variable `prec` determines the precision of the outputs of commands: more precisely, the command will internally determine how much precision should be used during the computations in order to ensure that the output is a faithfully rounded result with `prec` bits.

For decidability and efficiency reasons, this general principle cannot be applied every time, so be careful. Moreover certain commands are known to be unsafe: they give in general excellent results and give almost `prec` correct bits in output for everyday examples. However they are merely based on heuristics and should not be used when the result must be safe. See the documentation of each command to know precisely how confident you can be with their result.

A second principle (that comes together with the first one) is the following one: *When a computation leads to inexact results, inform the user with a warning.* This can be quite irritating in some circumstances: in particular if you are using `Sollya` within other scripts. The global variable `verbosity`

[1] Remark: some of the behaviors of `Sollya` slightly change when it is used in scripts. For example, no prompt is printed.

lets you change the level of verbosity of Sollya. When the variable is set to 0, Sollya becomes completely silent on standard output and prints only very important messages on standard error. Increase verbosity if you want more information about what Sollya is doing. Please keep in mind that when you affect a value to a global variable, a message is always printed even if verbosity is set to 0. In order to silently affect a global variable, use !:

```
> prec=30;
The precision has been set to 30 bits.
> prec=30!;
>
```

For conviviality reasons, values are displayed in decimal by default. This lets a normal human being understand the numbers they manipulate. But since constants are internally represented in binary, this causes permanent conversions that are sources of roundings. Thus you are loosing in accuracy and Sollya is always complaining about inexact results. If you just want to store or communicate your results (to another tools for instance) you can use bit-exact representations available in Sollya. The global variable display defines the way constants are displayed. Here is an example of the five available modes:

```
> prec=30!;
> a = 17.25;
> display=decimal;
Display mode is decimal numbers.
> a;
17.25
> display=binary;
Display mode is binary numbers.
> a;
1.000101_2 * 2^(4)
> display=powers;
Display mode is dyadic numbers in integer-power-of-2 notation.
> a;
69 * 2^(-2)
> display=dyadic;
Display mode is dyadic numbers.
> a;
69b-2
> display=hexadecimal;
Display mode is hexadecimal numbers.
> a;
0x1.14p4
```

Please keep in mind that it is possible to maintain the general verbosity level at some higher setting while deactivating all warnings on roundings. This feature is controlled using the roundingwarnings global variable. It may be set to on or off. By default, the warnings are activated (roundingwarnings = on) when Sollya is connected on standard input to a pseudo-file that represents a terminal. They are deactivated when Sollya is connected on standard input to a real file. See 8.167 for further details; the behavior is illustrated with examples there.

As always, the symbol e means $\times 10^{\square}$. The same way the symbol b means $\times 2^{\square}$. The symbol p means $\times 2^{\square}$ and is used only with the 0x prefix. The prefix 0x indicates that the digits of the following number until a symbol p or white-space are hexadecimal. For instance 0x1a.e2p3 represents the number $(1\times 16^1+10\times 16^0+14\times 16^{-1}+2\times 16^{-2})\times 2^3$. The suffix _2 indicates to Sollya that the previous number has been written in binary. Sollya can parse these notations even if you are not in the corresponding display mode, so you can always use them.

You can also use memory-dump hexadecimal notation frequently used to represent IEEE 754 double and single precision numbers. Since this notation does not allow for exactly representing numbers with arbitrary precision, there is no corresponding display mode. However, the commands printdouble

respectively `printsingle` round the value to the nearest `double` respectively `single`. The number is then printed in hexadecimal as the integer number corresponding to the memory representation of the IEEE 754 `double` or `single` number:

```
> printdouble(a);
0x4031400000000000
> printsingle(a);
0x418a0000
```

`Sollya` can parse these memory-dump hexadecimal notation back in any `display` mode. The difference of this memory-dump notation with the hexadecimal notation (as defined above) is made by the presence or absence of a `p` indicator.

4 Variables

As already explained, `Sollya` can manipulate variate functional expressions in one variable. These expressions contain a unique free variable the name of which is fixed by its first usage in an expression that is not a left-hand-side of an assignment. This global and unique free variable is a variable in the mathematical sense of the term.

`Sollya` also provides variables in the sense programming languages give to the term. These variables, which must be different in their name from the global free variable, may be global or declared and attached to a block of statements, *i.e.*, a begin-end-block. These programming language variables may hold any object of the `Sollya` language, as for example functional expressions, strings, intervals, constant values, procedures, external functions and procedures, etc.

Notice that the variable `__program_name` is filled when `Sollya` is started (either as an interactive tool or as a library). It contains the executable's 0th argument (in the library case, we mean the executable that loaded the library). It is however a normal variable (not protected) and it can be subsequently assigned any value. Please take care that there is no way to recover its original value after it has been replaced by something else.

Global variables need not to be declared. They start existing, *i.e.*, can be correctly used in expressions that are not left-hand-sides of assignments, when they are assigned a value in an assignment. Since they are global, this kind of variables is recommended only for small `Sollya` scripts. Larger scripts with code reuse should use declared variables in order to avoid name clashes for example in loop variables.

Declared variables are attached to a begin-end-block. The block structure builds scopes for declared variables. Declared variables in inner scopes shadow (global and declared) variables of outer scopes. The global free variable, *i.e.*, the mathematical variable for variate functional expressions in one variable, cannot be shadowed. Variables are declared using the `var` keyword. See Section 8.197 for details on its usage and semantic.

The following code examples illustrate the use of variables.

```
> f = exp(x);
> f;
exp(x)
> a = "Hello world";
> a;
Hello world
> b = 5;
> f(b);
Warning: rounding has happened. The value displayed is a faithful rounding to 16
5 bits of the true result.
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
148.413159102576603421115580040552279623487667593 88
> {var b; b = 4; f(b); };
Warning: rounding has happened. The value displayed is a faithful rounding to 16
5 bits of the true result.
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
54.598150033144239078110261202860878402790737038614
> {var x; x = 3; };
Warning: the identifier "x" is already bound to the current free variable.
It cannot be declared as a local variable. The declaration of "x" will have no e
ffect.
Warning: the identifier "x" is already bound to the free variable, to a library
function, library constant, external procedure or an external data symbol.
The command will have no effect.
Warning: the last assignment will have no effect.
> {var a, b; a=5; b=3; {var a; var b; b = true; a = 1; a; b;}; a; b; };
1
true
5
3
> a;
Hello world
```

Let us state that a variable identifier, just as every identifier in `Sollya`, contains at least one character, starts with a ASCII letter and continues with ASCII letters or numerical digits.

Two predefined variables exist when `Sollya` is started:

- `__argv` : This variable contains, on `Sollya` startup and after the execution of the `restart` command, a list of character strings that correspond to the command line options given to `Sollya` after the (implicit) command line argument `--args` (see above).
- `__unique_id` : This variable contains, on `Sollya` startup and after the execution of the `restart` command, a character string that uniquely identifies the given `Sollya` session on a system. It hence allows for concurrent execution of `Sollya` scripts that use temporary files for communication with other tools. The character string is made of alphanumeric characters ([0-9a-zA-Z]) and dashes (-) and underscores (_). In particular, it does not contain any space. After the execution of the `restart` command, the `__unique_id` variable is refreshed with a new, unique value.

Even though these variables exist upon `Sollya` startup with predefined values, they behave like any other variable: the predefined value can be overwritten by assigning any new value to the variables, the variables can be shadowed by declared, local variables of the same name and so on.

5 Data types

Sollya has a (very) basic system of types. If you try to perform an illicit operation (such as adding a number and a string, for instance), you will get a typing error. Let us see the available data types.

5.1 Booleans

There are two special values true and false. Boolean expressions can be constructed using the boolean connectors && (and), || (or), ! (not), and comparisons.

The comparison operators <, <=, > and >= can only be used between two numbers or constant expressions.

The comparison operators == and != are polymorphic. You can use them to compare any two objects, like two strings, two intervals, etc. As a matter of fact, polymorphism is allowed on both sides: it is possible to compare objects of different type. Such objects of different type, as they can never be syntactically equal, will always compare unequal (see exception for error, Section 8.57) and never equal. It is important to remember that testing the equality between two functions will return true if and only if the expression trees representing the two functions are exactly the same or automatic simplification is activated and both functions are polynomials that are equal. See 8.57 for an exception concerning the special object error. Example:

```
> 1+x==1+x;
true
```

5.2 Numbers

Sollya represents numbers as binary multi-precision floating-point values. For integer values and values in dyadic, binary, hexadecimal or memory dump notation, it automatically uses a precision needed for representing the value exactly (unless this behavior is overridden using the syntax given below). Additionally, automatic precision adaption takes place for all integer values (even in decimal notation) written without the exponent sign e or with the exponent sign e and an exponent sufficiently small that they are less than 10^{999}. Otherwise the values are represented with the current precision prec. When a number must be rounded, it is rounded to the precision prec before the expression get evaluated:

```
> prec=12!;
> 4097.1;
Warning: Rounding occurred when converting the constant "4097.1" to floating-poi
nt with 12 bits.
If safe computation is needed, try to increase the precision.
4098
> 4098.1;
Warning: Rounding occurred when converting the constant "4098.1" to floating-poi
nt with 12 bits.
If safe computation is needed, try to increase the precision.
4098
> 4097.1+1;
Warning: Rounding occurred when converting the constant "4097.1" to floating-poi
nt with 12 bits.
If safe computation is needed, try to increase the precision.
4099
```

As a matter of fact, each variable has its own precision that corresponds to its intrinsic precision or, if it cannot be represented, to the value of prec when the variable was set. Thus you can work with variables having a precision higher than the current precision.

The same way, if you define a function that refers to some constant, this constant is stored in the function with the current precision and will keep this value in the future, even if prec becomes smaller.

If you define a function that refers to some variable, the precision of the variable is kept, independently of the current precision:

```
> prec = 50!;
> a = 4097.1;
Warning: Rounding occurred when converting the constant "4097.1" to floating-poi
nt with 50 bits.
If safe computation is needed, try to increase the precision.
> prec = 12!;
> f = x + a;
> g = x + 4097.1;
Warning: Rounding occurred when converting the constant "4097.1" to floating-poi
nt with 12 bits.
If safe computation is needed, try to increase the precision.
> prec = 120;
The precision has been set to 120 bits.
> f;
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
4097.09999999999854480847716331481933 6 + x
> g;
4098 + x
```

In some rare cases, it is necessary to read in decimal constants with a particular precision being used in the conversion to the binary floating-point format, which `Sollya` uses. Setting `prec` to that precision may prove to be an insufficient means for doing so, for example when several different precisions have to be used in one expression. For these rare cases, `Sollya` provides the following syntax: decimal constants may be written `%`*precision*`%`*constant*, where *precision* is a constant integer, written in decimal, and *constant* is the decimal constant. `Sollya` will convert the constant *constant* with precision *precision*, regardless of the global variable `prec` and regardless if *constant* is an integer or would otherwise be representable.

```
> prec = 24;
The precision has been set to 24 bits.
> a = 0.1;
Warning: Rounding occurred when converting the constant "0.1" to floating-point
with 24 bits.
If safe computation is needed, try to increase the precision.
> b = 33554433;
> prec = 64;
The precision has been set to 64 bits.
> display = binary;
Display mode is binary numbers.
> a;
1.10011001100110011001101_2 * 2^(-4)
> 0.1;
Warning: Rounding occurred when converting the constant "0.1" to floating-point
with 64 bits.
If safe computation is needed, try to increase the precision.
1.1001100110011001100110011001100110011001100110011001100110011001101_2 * 2^(-4)
> %24%0.1;
Warning: Rounding occurred when converting the constant "0.1" to floating-point
with 24 bits.
If safe computation is needed, try to increase the precision.
1.10011001100110011001101_2 * 2^(-4)
> c = 33554433;
> b;
1.00000000000000000000001_2 * 2^(25)
> c;
1.00000000000000000000001_2 * 2^(25)
> %24%33554433;
Warning: Rounding occurred when converting the constant "33554433" to floating-p
oint with 24 bits.
If safe computation is needed, try to increase the precision.
1_2 * 2^(25)
>
>
```

`Sollya` is an environment that uses floating-point arithmetic. The IEEE 754-2008 standard on floating-point arithmetic does not only define floating-point numbers that represent real numbers but also floating-point data representing infinities and Not-a-Numbers (NaNs). `Sollya` also supports infinities and NaNs in the spirit of the IEEE 754-2008 standard without taking the standard's choices literally.

- Signed infinities are available through the `Sollya` objects `infty, -infty, @Inf@` and `-@Inf@`.
- Not-a-Numbers are supported through the `Sollya` objects `NaN` and `@NaN@`. `Sollya` does not have support for NaN payloads, signaling or quiet NaNs or signs of NaNs. Signaling NaNs are supported on input for single and double precision memory notation (see Section 3). However, they immediately get converted to plain `Sollya` NaNs.

The evaluation of an expression involving a NaN or the evaluation of a function at a point being NaN always results in a NaN.

Infinities are considered to be the limits of expressions tending to infinity. They are supported as bounds of intervals in some cases. However, particular commands might prohibit their use even though there might be a mathematical meaning attached to such expressions. For example, while `Sollya` will evaluate expressions such as $\lim_{x \to -\infty} e^x$, expressed *e.g.*, through `evaluate(exp(x),[-infty;0])`, it will

not accept to compute the (finite) value of

$$\int_{-\infty}^{0} e^x \, \mathrm{d}x.$$

The following examples give an idea of what can be done with `Sollya` infinities and NaNs. Here is what can be done with infinities:

```
> f = exp(x) + 5;
> f(-infty);
5
> evaluate(f,[-infty;infty]);
[5;infty]
> f(infty);
infty
> [-infty;5] * [3;4];
[-infty;20]
> -infty < 5;
true
> log(0);
-infty
> [log(0);17];
Warning: inclusion property is satisfied but the diameter may be greater than th
e least possible.
Warning: at least one of the given expressions is not a constant but requires ev
aluation.
Evaluation is guaranteed to ensure the inclusion property. The approximate resul
t is at least 165 bit accurate.
[-infty;17]
>
```

And the following example illustrates NaN behavior.

```
> 3/0;
Warning: the given expression is undefined or numerically unstable.
NaN
> (-3)/0;
Warning: the given expression is undefined or numerically unstable.
NaN
> infty/infty;
Warning: the given expression is undefined or numerically unstable.
NaN
> infty + infty;
infty
> infty - infty;
Warning: the given expression is undefined or numerically unstable.
NaN
> f = exp(x) + 5;
> f(NaN);
NaN
> NaN == 5;
false
> NaN == NaN;
false
> NaN != NaN;
true
> X = "Vive la Republique!";
> !(X == X);
false
> X = 5;
> !(X == X);
false
> X = NaN;
> !(X == X);
true
>
```

5.3 Rational numbers and rational arithmetic

The `Sollya` tool is mainly based on floating-point arithmetic: wherever possible, floating-point algorithms, including algorithms using interval arithmetic, are used to produce approximate but safe results. For some particular cases, floating-point arithmetic is not sufficient: some algorithms just require natural and rational numbers to be handled exactly. More importantly, for these applications, it is required that rational numbers be displayed as such.

`Sollya` implements a particular mode that offers a lightweight support for rational arithmetic. When needed, it can be enabled by assigning `on` to the global variable `rationalmode`. It is disabled by assigning `off`; the default is `off`.

When the mode for rational arithmetic is enabled, `Sollya`'s behavior will change as follows:

- When a constant expression is given at the `Sollya` prompt, `Sollya` will first try to simplify the expression to a rational number. If such an evaluation to a rational number is possible, `Sollya` will display that number as an integer or a fraction of two integers. Only if `Sollya` is not able to simplify the constant expression to a rational number, it will launch the default behavior of evaluating constant expressions to floating-point numbers that are generally faithful roundings of the expressions.

- When the global mode `autosimplify` is `on`, which is the default, `Sollya` will additionally use rational arithmetic while trying to simplify expressions given in argument of commands.

Even when `rationalmode` is `on`, `Sollya` will not be able to exhibit integer ratios between transcendental quantities. For example, `Sollya` will not display $\frac{1}{6}$ for $\arcsin\left(\frac{1}{2}\right)/\pi$ but $0.16666\ldots$. `Sollya`'s evaluator for rational arithmetic is only able to simplify rational expressions based on addition, subtraction, multiplication, division, negation, perfect squares (for square root) and integer powers.

The following example illustrates what can and what cannot be done with `Sollya`'s mode for rational arithmetic:

```
> 1/3 - 1/7;
Warning: rounding has happened. The value displayed is a faithful rounding to 16
5 bits of the true result.
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
0.19047619047619047619047619047619047619047619047619
> rationalmode = on;
Rational mode has been activated.
> 1/3 - 1/7;
4 / 21
> (2 + 1/7)^2 + (6/7)^2 + 2 * (2 + 1/7) * 6/7;
9
> rationalmode = off;
Rational mode has been deactivated.
> (2 + 1/7)^2 + (6/7)^2 + 2 * (2 + 1/7) * 6/7;
Warning: rounding has happened. The value displayed is a faithful rounding to 16
5 bits of the true result.
9
> rationalmode = on;
Rational mode has been activated.
> asin(1)/pi;
Warning: rounding has happened. The value displayed is a faithful rounding to 16
5 bits of the true result.
0.5
> sin(1/6 * pi);
Warning: rounding has happened. The value displayed is a faithful rounding to 16
5 bits of the true result.
0.5
> exp(1/7 - 3/21) / 7;
1 / 7
> rationalmode = off;
Rational mode has been deactivated.
> exp(1/7 - 3/21) / 7;
Warning: rounding has happened. The value displayed is a faithful rounding to 16
5 bits of the true result.
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
0.142857142857142857142857142857142857142857142857145
> print(1/7 - 3/21);
1 / 7 - 3 / 21
> rationalmode = on;
Rational mode has been activated.
> print(1/7 - 3/21);
0
```

5.4 Intervals and interval arithmetic

`Sollya` can manipulate intervals that are closed subsets of the real numbers. Several ways of defining intervals exist in `Sollya`. There is the most common way where intervals are composed of two numbers

or constant expressions representing the lower and the upper bound. These values are separated either by commas or semi-colons. Interval bound evaluation is performed in a way that ensures the inclusion property: all points in the original, unevaluated interval will be contained in the interval with its bounds evaluated to floating-point numbers.

```
> d=[1;2];
> d2=[1,1+1];
> d==d2;
true
> prec=12!;
> 8095.1;
Warning: Rounding occurred when converting the constant "8095.1" to floating-poi
nt with 12 bits.
If safe computation is needed, try to increase the precision.
8096
> [8095.1; 8096.1];
Warning: Rounding occurred when converting the constant "8095.1" to floating-poi
nt with 12 bits.
The inclusion property is nevertheless satisfied.
Warning: Rounding occurred when converting the constant "8096.1" to floating-poi
nt with 12 bits.
The inclusion property is nevertheless satisfied.
[8094;8098]
```

`Sollya` has a mode for printing intervals that are that thin that their bounds have a number of decimal digits in common when printed. That mode is called `midpointmode`; see below for an introduction and Section 8.112 for details. As `Sollya` must be able to parse back its own output, a syntax is provided to input intervals in midpoint mode. However, please pay attention to the fact that the notation used in midpoint mode generally increases the width of intervals: hence when an interval is displayed in midpoint mode and read again, the resulting interval may be wider than the original interval.

```
> midpointmode = on!;
> [1.725e4;1.75e4];
0.17~2/5~e5
> 0.17~2/5~e5;
0.17~2/5~e5
> midpointmode = off!;
> 0.17~2/5~e5;
[17200;17500]
```

In some cases, intervals become infinitely thin in theory, in which case one tends to think of point intervals even if their floating-point representation is not infinitely thin. `Sollya` provides a very convenient way for input of such point intervals. Instead of writing `[a;a]`, it is possible to just write `[a]`. `Sollya` will expand the notation while making sure that the inclusion property is satisfied:

```
> [3];
[3;3]
> [1/7];
Warning: at least one of the given expressions is not a constant but requires ev
aluation.
Evaluation is guaranteed to ensure the inclusion property. The approximate resul
t is at least 24 bit accurate.
Warning: For at least 2 of the constants displayed in decimal, rounding has happ
ened.
[0.14285713;0.14285715]
> [exp(8)];
Warning: at least one of the given expressions is not a constant but requires ev
aluation.
Evaluation is guaranteed to ensure the inclusion property. The approximate resul
t is at least 24 bit accurate.
Warning: For at least 2 of the constants displayed in decimal, rounding has happ
ened.
[2980.9578;2980.958]
```

When the mode `midpointmode` is set to `on` (see 8.112), `Sollya` will display intervals that are provably reduced to one point in this extended interval syntax. It will use `midpointmode` syntax for intervals that are sufficiently thin but not reduced to one point (see Section 8.112 for details):

```
> midpointmode = off;
Midpoint mode has been deactivated.
> [17;17];
[17;17]
> [exp(pi);exp(pi)];
Warning: at least one of the given expressions is not a constant but requires ev
aluation.
Evaluation is guaranteed to ensure the inclusion property. The approximate resul
t is at least 165 bit accurate.
Warning: For at least 2 of the constants displayed in decimal, rounding has happ
ened.
[23.140692632779269005729086367948547380266106242 6;23.140692632779269005729086367948547380266106242601]
> midpointmode = on;
Midpoint mode has been activated.
> [17;17];
[17]
> [exp(pi);exp(pi)];
Warning: at least one of the given expressions is not a constant but requires ev
aluation.
Evaluation is guaranteed to ensure the inclusion property. The approximate resul
t is at least 165 bit accurate.
0.2314069263277926900572908636794854738026610624 26~0/1~e2
>
```

`Sollya` intervals are internally represented with floating-point numbers as bounds; rational numbers are not supported here. If bounds are defined by constant expressions, these are evaluated to floating-point numbers using the current precision. Numbers or variables containing numbers keep their precision for the interval bounds.

Constant expressions get evaluated to floating-point values immediately; this includes π and rational numbers, even when `rationalmode` is `on` (see Section 5.3 for this mode).

```
> prec = 300!;
> a = 4097.1;
Warning: Rounding occurred when converting the constant "4097.1" to floating-poi
nt with 300 bits.
If safe computation is needed, try to increase the precision.
> prec = 12!;
> d = [4097.1; a];
Warning: Rounding occurred when converting the constant "4097.1" to floating-poi
nt with 12 bits.
The inclusion property is nevertheless satisfied.
> prec = 300!;
> d;
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
[4096;4097.1]
> prec = 30!;
> [-pi;pi];
Warning: at least one of the given expressions is not a constant but requires ev
aluation.
Evaluation is guaranteed to ensure the inclusion property. The approximate resul
t is at least 30 bit accurate.
Warning: For at least 2 of the constants displayed in decimal, rounding has happ
ened.
[-3.141592655;3.141592655]
```

You can get the upper-bound (respectively the lower-bound) of an interval with the command `sup` (respectively `inf`). The middle of the interval can be computed with the command `mid`. Let us also mention that these commands can also be used on numbers (in that case, the number is interpreted as an interval containing only one single point. In that case the commands `inf`, `mid` and `sup` are just the identity):

```
> d=[1;3];
> inf(d);
1
> mid(d);
2
> sup(4);
4
```

Let us mention that the `mid` operator never provokes a rounding. It is rewritten as an unevaluated expression in terms of `inf` and `sup`.

`Sollya` permits intervals to also have non-real bounds, such as infinities or NaNs. When evaluating certain expressions, in particular given as interval bounds, `Sollya` will itself generate intervals containing infinities or NaNs. When evaluation yields an interval with a NaN bound, the given expression is most likely undefined or numerically unstable. Such results should not be trusted; a warning is displayed.

While computations on intervals with bounds being NaN will always fail, `Sollya` will try to interpret infinities in the common way as limits. However, this is not guaranteed to work, even if it is guaranteed that no unsafe results will be produced. See also section 5.2 for more detail on infinities in `Sollya`. The behavior of interval arithmetic on intervals containing infinities or NaNs is subject to debate; moreover, there is no complete consensus on what should be the result of the evaluation of a function f over an interval I containing points where f is not defined. `Sollya` has its own philosophy regarding these questions. This philosophy is explained in Appendix 9 at the end of this document.

```
> evaluate(exp(x),[-infty;0]);
[0;1]
> dirtyinfnorm(exp(x),[-infty;0]);
Warning: a bound of the interval is infinite or NaN.
This command cannot handle such intervals.
NaN
>
> f = log(x);
> [f(0); f(1)];
Warning: inclusion property is satisfied but the diameter may be greater than th
e least possible.
Warning: at least one of the given expressions is not a constant but requires ev
aluation.
Evaluation is guaranteed to ensure the inclusion property. The approximate resul
t is at least 165 bit accurate.
[-infty;0]
>
```

`Sollya` internally uses interval arithmetic extensively to provide safe answers. In order to provide for algorithms written in the `Sollya` language being able to use interval arithmetic, `Sollya` offers native support of interval arithmetic. Intervals can be added, subtracted, multiplied, divided, raised to powers, for short, given in argument to any `Sollya` function. The tool will apply the rules of interval arithmetic in order to compute output intervals that safely encompass the hull of the image of the function on the given interval:

```
> [1;2] + [3;4];
[4;6]
> [1;2] * [3;4];
[3;8]
> sqrt([9;25]);
[3;5]
> exp(sin([10;100]));
Warning: For at least 2 of the constants displayed in decimal, rounding has happ
ened.
[0.36787942;2.718282]
```

When such expressions involving intervals are given, `Sollya` will follow the rules of interval arithmetic in precision `prec` for immediately evaluating them to interval enclosures. While `Sollya`'s evaluator always guarantees the inclusion property, it also applies some optimizations in some cases in order to make the image interval as thin as possible. For example, `Sollya` will use a Taylor expansion based evaluation if a composed function, call it f, is applied to an interval. In other words, in this case `Sollya` will behave as if the `evaluate` command (see Section 8.58) were implicitly used. In most cases, the result will be different from the one obtained by replacing all occurrences of the free variable of a function by the interval the function is to be evaluated on:

```
> f = x - sin(x);
> [-1b-10;1b-10] - sin([-1b-10;1b-10]);
Warning: For at least 2 of the constants displayed in decimal, rounding has happ
ened.
[-1.95312484477957829894e-3;1.95312484477957829894e-3]
> f([-1b-10;1b-10]);
Warning: For at least 2 of the constants displayed in decimal, rounding has happ
ened.
[-1.552204217011176269e-10;1.552204217011176269e-10]
> evaluate(f,[-1b-10;1b-10]);
Warning: For at least 2 of the constants displayed in decimal, rounding has happ
ened.
[-1.552204217011176269e-10;1.552204217011176269e-10]
```

5.5 Functions

`Sollya` knows only about functions with one single variable. The first time in a session that an unbound name is used (without being assigned) it determines the name used to refer to the free variable.

The basic functions available in `Sollya` are the following:

- `+`, `-`, `*`, `/`, `^`
- `sqrt`
- `abs`
- `sin`, `cos`, `tan`, `sinh`, `cosh`, `tanh`
- `asin`, `acos`, `atan`, `asinh`, `acosh`, `atanh`
- `exp`, `expm1` (defined as $\mathrm{expm1}(x) = \exp(x) - 1$)
- `log` (natural logarithm), `log2` (binary logarithm), `log10` (decimal logarithm), `log1p` (defined as $\mathrm{log1p}(x) = \log(1 + x)$)
- `erf`, `erfc`
- `halfprecision`, `single`, `double`, `doubleextended`, `doubledouble`, `quad`, `tripledouble` (see sections 8.82, 8.175, 8.50, 8.52, 8.51, 8.149 and 8.194)
- `HP`, `SG`, `D`, `DE`, `DD`, `QD`, `TD` (see sections 8.82, 8.175, 8.50, 8.52, 8.51, 8.149 and 8.194)
- `floor`, `ceil`, `nearestint`.

The constant π is available through the keyword `pi` as a 0-ary function:

```
> display=binary!;
> prec=12!;
> a=pi;
> a;
Warning: rounding has happened. The value displayed is a faithful rounding to 12
 bits of the true result.
1.10010010001_2 * 2^(1)
> prec=30!;
> a;
Warning: rounding has happened. The value displayed is a faithful rounding to 30
 bits of the true result.
1.10010010000111111011010101001_2 * 2^(1)
```

The reader may wish to see Sections 8.101 and 8.75 for ways of dynamically adding other base functions to `Sollya`.

5.6 Strings

Anything written between quotes is interpreted as a string. The infix operator `@` concatenates two strings. To get the length of a string, use the `length` function. You can access the i-th character of a string using brackets (see the example below). There is no character type in `Sollya`: the i-th character of a string is returned as a string itself.

```
> s1 = "Hello "; s2 = "World!";
> s = s1@s2;
> length(s);
12
> s[0];
H
> s[11];
!
```

Strings may contain the following escape sequences: \\, \¨, \?, \´, \n, \t, \a, \b, \f, \r, \v, \x[hexadecimal number] and \[octal number]. Refer to the C99 standard for their meaning.

5.7 Particular values

`Sollya` knows about some particular values. These values do not really have a type. They can be stored in variables and in lists. A (possibly not exhaustive) list of such values is the following one:

- `on`, `off` (see sections 8.126 and 8.125)
- `dyadic`, `powers`, `binary`, `decimal`, `hexadecimal` (see sections 8.53, 8.137, 8.20, 8.36 and 8.84)
- `file`, `postscript`, `postscriptfile` (see sections 8.68, 8.134 and 8.135)
- `RU`, `RD`, `RN`, `RZ` (see sections 8.168, 8.154, 8.163 and 8.169)
- `absolute`, `relative` (see sections 8.2 and 8.157)
- `floating`, `fixed` (see sections 8.71 and 8.70)
- `halfprecision`, `single`, `double`, `doubleextended`, `doubledouble`, `quad`, `tripledouble` (see sections 8.82, 8.175, 8.50, 8.52, 8.51, 8.149 and 8.194)
- `HP`, `SG`, `D`, `DE`, `DD`, `QD`, `TD` (see sections 8.82, 8.175, 8.50, 8.52, 8.51, 8.149 and 8.194)
- `perturb` (see Section 8.129)
- `honorcoeffprec` (see Section 8.85)
- `default` (see Section 8.37)
- `error` (see Section 8.57)
- `void` (see Section 8.199)

5.8 Lists

Objects can be grouped into lists. A list can contain elements with different types. As for strings, you can concatenate two lists with `@`. The function `length` also gives the length of a list.

You can prepend an element to a list using `.:` and you can append an element to a list using `:.` The following example illustrates some features:

```
> L = [| "foo" |];
> L = L:.1;
> L = "bar".:L;
> L;
[|"bar", "foo", 1|]
> L[1];
foo
> L@L;
[|"bar", "foo", 1, "bar", "foo", 1|]
```

Lists can be considered arrays and elements of lists can be referenced using brackets. Possible indices start at 0. The following example illustrates this point:

```
> L = [|1,2,3,4,5|];
> L;
[|1, 2, 3, 4, 5|]
> L[3];
4
```

Lists may contain ellipses indicated by `,...,` between elements that are constant and evaluate to integers that are incrementally ordered. `Sollya` translates such ellipses to the full list upon evaluation. The use of ellipses between elements that are not constants is not allowed. This feature is provided for ease of programming; remark that the complexity for expanding such lists is high. For illustration, see the following example:

```
> [|1,...,5|];
[|1, 2, 3, 4, 5|]
> [|-5,...,5|];
[|-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5|]
> [|3,...,1|];
Warning: at least one of the given expressions or a subexpression is not correct
ly typed
or its evaluation has failed because of some error on a side-effect.
error
> [|true,...,false|];
Warning: at least one of the given expressions or a subexpression is not correct
ly typed
or its evaluation has failed because of some error on a side-effect.
error
```

Lists may be continued to infinity by means of the `...` indicator after the last element given. At least one element must explicitly be given. If the last element given is a constant expression that evaluates to an integer, the list is considered as continued to infinity by all integers greater than that last element. If the last element is another object, the list is considered as continued to infinity by re-duplicating this last element. Let us remark that bracket notation is supported for such end-elliptic lists even for implicitly given elements. However, evaluation complexity is high. Combinations of ellipses inside a list and in its end are possible. The usage of lists described here is best illustrated by the following examples:

```
> L = [|1,2,true,3...|];
> L;
[|1, 2, true, 3...|]
> L[2];
true
> L[3];
3
> L[4];
4
> L[1200];
1200
> L = [|1,...,5,true...|];
> L;
[|1, 2, 3, 4, 5, true...|]
> L[1200];
true
```

5.9 Structures

In a similar way as in lists, `Sollya` allows data to be grouped in – untyped – structures. A structure forms an object to which other objects can be added as elements and identified by their names. The elements of a structure can be retrieved under their name and used as usual. The following sequence shows that point:

```
> s.a = 17;
> s.b = exp(x);
> s.a;
17
> s.b;
exp(x)
> s.b(1);
Warning: rounding has happened. The value displayed is a faithful rounding to 16
5 bits of the true result.
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
2.71828182845904523536028747135266249775724709 37
> s.d.a = [-1;1];
> s.d.b = sin(x);
> inf(s.d.a);
-1
> diff(s.d.b);
cos(x)
```

Structures can also be defined literally using the syntax illustrated in the next example. They will also be printed in that syntax.

```
> a = { .f = exp(x), .dom = [-1;1] };
> a;
{ .f = exp(x), .dom = [-1;1] }
> a.f;
exp(x)
> a.dom;
[-1;1]
> b.f = sin(x);
> b.dom = [-1b-5;1b-5];
> b;
{ .dom = [-3.125e-2;3.125e-2], .f = sin(x) }
> { .f = asin(x), .dom = [-1;1] }.f(1);
Warning: rounding has happened. The value displayed is a faithful rounding to 16
5 bits of the true result.
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
1.5707963267948966192313216916397514420985846996875 4
```

If the variable `a` is bound to an existing structure, it is possible to use the "dot notation" `a.b` to assign the value of the field `b` of the structure `a`. This works even if `b` is not yet a field of `a`: in this case a new field is created inside the structure `a`.

Besides, the dot notation can be used even when `a` is unassigned. In this case a new structure is created with a field `b`, and this structure is bound to `a`. However, the dot notation cannot be used if `a` is already bound to something that is not a structure.

These principles apply recursively: for instance, if `a` is a structure that contains only one field `d`, the command `a.b.c = 3` creates a new field named `b` inside the structure `a`; this field itself is a structure containing the field `c`. The command `a.d.c = 3` is allowed if `a.d` is already a structure, but forbidden otherwise (*e.g.*, if `a.d` was equal to `sin(x)`). This is summed up in the following example.

```
> restart;
The tool has been restarted.
> a.f = exp(x);
> a.dom = [-1;1];
> a.info.text = "My akrnoximation problem";
> a;
{ .info = { .text = "My akrnoximation problem" }, .dom = [-1;1], .f = exp(x) }
>
> a.info.text = "My approximation problem";
> a;
{ .info = { .text = "My approximation problem" }, .dom = [-1;1], .f = exp(x) }
>
> b = exp(x);
> b.a = 5;
Warning: cannot modify an element of something that is not a structure.
Warning: the last assignment will have no effect.
> b;
exp(x)
>
> a.dom.a = -1;
Warning: cannot modify an element of something that is not a structure.
Warning: the last assignment will have no effect.
> a;
{ .info = { .text = "My approximation problem" }, .dom = [-1;1], .f = exp(x) }
```

When printed, the elements of a structure are not sorted in any manner. They get printed in an arbitrary order that just maintains the order given in the definition of literate structures. That said,

when compared, two structures compare equal iff they contain the same number of identifiers, with the same names and iff the elements of corresponding names all compare equal. This means the order does not matter in comparisons and otherwise does only for printing.

The following example illustrates this matter:

```
> a = { .f = exp(x), .a = -1, .b = 1 };
> a;
{ .f = exp(x), .a = -1, .b = 1 }
> a.info = "My function";
> a;
{ .info = "My function", .f = exp(x), .a = -1, .b = 1 }
>
> b = { .a = -1, .f = exp(x), .info = "My function", .b = 1 };
> b;
{ .a = -1, .f = exp(x), .info = "My function", .b = 1 }
>
> a == b;
true
>
> b.info = "My other function";
> a == b;
false
>
> b.info = "My function";
> a == b;
true
> b.something = true;
> a == b;
false
```

6 Iterative language elements: assignments, conditional statements and loops

6.1 Blocks

Statements in `Sollya` can be grouped in blocks, so-called begin-end-blocks. This can be done using the key tokens { and }. Blocks declared this way are considered to be one single statement. As already explained in Section 4, using begin-end-blocks also opens the possibility of declaring variables through the keyword `var`.

6.2 Assignments

`Sollya` has two different assignment operators, `=` and `:=`. The assignment operator `=` assigns its right-hand-object "as is", *i.e.*, without evaluating functional expressions. For instance, `i = i + 1;` will dereferentiate the identifier `i` with some content, notate it y, build up the expression (function) $y + 1$ and assign this expression back to `i`. In the example, if `i` stood for the value 1000, the statement `i = i + 1;` would assign "1000 + 1" – and not "1001" – to `i`. The assignment operator `:=` evaluates constant functional expressions before assigning them. On other expressions it behaves like `=`. Still in the example, the statement `i := i + 1;` really assigns 1001 to `i`.

Both `Sollya` assignment operators support indexing of lists or strings elements using brackets on the left-hand-side of the assignment operator. The indexed element of the list or string gets replaced by the right-hand-side of the assignment operator. When indexing strings this way, that right-hand side must evaluate to a string of length 1. End-elliptic lists are supported with their usual semantic for this kind of assignment. When referencing and assigning a value in the implicit part of the end-elliptic list, the list gets expanded to the corresponding length.

The following examples well illustrate the behavior of assignment statements:

```
> autosimplify = off;
Automatic pure tree simplification has been deactivated.
> i = 1000;
> i = i + 1;
> print(i);
1000 + 1
> i := i + 1;
> print(i);
1002
> L = [|1,...,5|];
> print(L);
[|1, 2, 3, 4, 5|]
> L[3] = L[3] + 1;
> L[4] := L[4] + 1;
> print(L);
[|1, 2, 3, 4 + 1, 6|]
> L[5] = true;
> L;
[|1, 2, 3, 5, 6, true|]
> s = "Hello world";
> s;
Hello world
> s[1] = "a";
> s;
Hallo world
> s[2] = "foo";
Warning: the string to be assigned is not of length 1.
This command will have no effect.
> L = [|true,1,...,5,9...|];
> L;
[|true, 1, 2, 3, 4, 5, 9...|]
> L[13] = "Hello";
> L;
[|true, 1, 2, 3, 4, 5, 9, 10, 11, 12, 13, 14, 15, "Hello", 17...|]
```

The indexing of lists on left-hand sides of assignments is reduced to the first order. Multiple indexing of lists of lists on assignment is not supported for complexity reasons. Multiple indexing is possible in right-hand sides.

```
> L = [| 1, 2, [|"a", "b", [|true, false|] |] |];
> L[2][2][1];
false
> L[2][2][1] = true;
Warning: the first element of the left-hand side is not an identifier.
This command will have no effect.
> L[2][2] = "c";
Warning: the first element of the left-hand side is not an identifier.
This command will have no effect.
> L[2] = 3;
> L;
[|1, 2, 3|]
```

6.3 Conditional statements

`Sollya` supports conditional statements expressed with the keywords `if`, `then` and optionally `else`. Let us mention that only conditional statements are supported and not conditional expressions.

The following examples illustrate both syntax and semantic of conditional statements in `Sollya`. Concerning syntax, be aware that there must not be any semicolon before the `else` keyword.

```
> a = 3;
> b = 4;
> if (a == b) then print("Hello world");
> b = 3;
> if (a == b) then print("Hello world");
Hello world
> if (a == b) then print("You are telling the truth") else print("Liar!");
You are telling the truth
```

6.4 Loops

`Sollya` supports three kinds of loops. General *while-condition* loops can be expressed using the keywords `while` and `do`. One has to be aware of the fact that the condition test is executed always before the loop, there is no *do-until-condition* loop. Consider the following examples for both syntax and semantic:

```
> verbosity = 0!;
> prec = 30!;
> i = 5;
> while (expm1(i) > 0) do { expm1(i); i := i - 1; };
147.4131591
53.59815
19.08553693
6.3890561
1.718281828
> print(i);
0
```

The second kind of loops are loops on a variable ranging from a numerical start value and a end value. These kind of loops can be expressed using the keywords `for`, `from`, `to`, `do` and optionally `by`. The `by` statement indicates the width of the steps on the variable from the start value to the end value. Once again, syntax and semantic are best explained with an example:

```
> for i from 1 to 5 do print ("Hello world",i);
Hello world 1
Hello world 2
Hello world 3
Hello world 4
Hello world 5
> for i from 2 to 1 by -0.5 do print("Hello world",i);
Hello world 2
Hello world 1.5
Hello world 1
```

The third kind of loops are loops on a variable ranging on values contained in a list. In order to ensure the termination of the loop, that list must not be end-elliptic. The loop is expressed using the keywords `for`, `in` and `do` as in the following examples:

```
> L = [|true, false, 1,...,4, "Hello", exp(x)|];
> for i in L do i;
true
false
1
2
3
4
Hello
exp(x)
```

For both types of `for` loops, assigning the loop variable is allowed and possible. When the loop terminates, the loop variable will contain the value that made the loop condition fail. Consider the following examples:

```
> for i from 1 to 5 do { if (i == 3) then i = 4 else i; };
1
2
5
> i;
6
```

7 Functional language elements: procedures and pattern matching

7.1 Procedures

`Sollya` has some elements of functional languages. In order to avoid confusion with mathematical functions, the associated programming objects are called *procedures* in `Sollya`.

`Sollya` procedures are common objects that can be, for example, assigned to variables or stored in lists. Procedures are declared by the `proc` keyword; see Section 8.146 for details. The returned procedure object must then be assigned to a variable. It can hence be applied to arguments with common application syntax. The `procedure` keyword provides an abbreviation for declaring and assigning a procedure; see Section 8.147 for details.

`Sollya` procedures can return objects using the `return` keyword at the end of the begin-end-block of the procedure. Section 8.161 gives details on the usage of `return`. Procedures further can take any type of object in argument, in particular also other procedures that are then applied to arguments. Procedures can be declared inside other procedures.

Common `Sollya` procedures are declared with a certain number of formal parameters. When the procedure is applied to actual parameters, a check is performed if the right number of actual parameters is given. Then the actual parameters are applied to the formal parameters. In some cases, it is required that the number of parameters of a procedure be variable. `Sollya` provides support for the case with procedures with an arbitrary number of actual arguments. When the procedure is called, those actual arguments are gathered in a list which is applied to the only formal list parameter of a procedure with an arbitrary number of arguments. See Section 8.147 for the exact syntax and details; an example is given just below.

Let us remark that declaring a procedure does not involve any evaluation or other interpretation of the procedure body. In particular, this means that constants are evaluated to floating-point values inside `Sollya` when the procedure is applied to actual parameters and the global precision valid at this moment.

`Sollya` procedures are well illustrated with the following examples:

```
> succ = proc(n) { return n + 1; };
> succ(5);
6
> 3 + succ(0);
4
> succ;
proc(n)
{
nop;
return (n) + (1);
}
```

```
> add = proc(m,n) { var res; res := m + n; return res; };
> add(5,6);
11
> hey = proc() { print("Hello world."); };
> hey();
Hello world.
> print(hey());
Hello world.
void
> hey;
proc()
{
print("Hello world.");
return void;
}
```

```
> fac = proc(n) { var res; if (n == 0) then res := 1 else res := n * fac(n - 1);
 return res; };
> fac(5);
120
> fac(11);
39916800
> fac;
proc(n)
{
var res;
if (n) == (0) then
res := 1
else
res := (n) * (fac((n) - (1)));
return res;
}
```

```
> sumall = proc(args = ...) { var i, acc; acc = 0; for i in args do acc = acc +
i; return acc; };
> sumall;
proc(args = ...)
{
var i, acc;
acc = 0;
for i in args do
acc = (acc) + (i);
return acc;
}
> sumall();
0
> sumall(1);
1
> sumall(1,5);
6
> sumall(1,5,9);
15
> sumall @ [|1,5,9,4,8|];
27
>
```

Let us note that, when writing a procedure, one does not know what will be the name of the free variable at run-time. This is typically the context when one should use the special keyword `_x_`:

```
> ChebPolynomials = proc(n) {
    var i, res;
    if (n<0) then res = [||]
    else if (n==0) then res = [|1|]
    else {
       res = [|1, _x_|];
       for i from 2 to n do res[i] = horner(2*_x_*res[i-1]-res[i-2]);
    };
    return res;
  };
>
> f = sin(x);
> T = ChebPolynomials(4);
> canonical = on!;
> for i from 0 to 4 do T[i];
1
x
-1 + 2 * x^2
-3 * x + 4 * x^3
1 - 8 * x^2 + 8 * x^4
```

`Sollya` also supports external procedures, *i.e.*, procedures written in `C` (or some other language) and dynamically bound to `Sollya` identifiers. See 8.66 for details.

7.2 Pattern matching

Starting with version 3.0, `Sollya` supports matching expressions with expression patterns. This feature is important for an extended functional programming style. Further, and most importantly, it allows expression trees to be recursively decomposed using native constructs of the `Sollya` language. This means no help from external procedures or other compiled-language mechanisms is needed here anymore.

Basically, pattern matching supports relies on one `Sollya` construct:

```
match expr with
pattern1 :  (return-expr1)
pattern2 :  (return-expr2)
...
patternN : (return-exprN)
```

That construct has the following semantic: try to match the expression *expr* with the patterns *pattern1* through *patternN*, proceeding in natural order. If a pattern *patternI* is found that matches, evaluate the whole `match ... with` construct to the return expression *return-exprI* associated with the matching pattern *patternI*. If no matching pattern is found, display an error warning and return `error`. Note that the parentheses around the expressions *return-exprI* are mandatory.

Matching a pattern means the following:

- If a pattern does not contain any programming-language-level variables (different from the free mathematical variable), it matches expressions that are syntactically equal to itself. For instance, the pattern `exp(sin(3 * x))` will match the expression `exp(sin(3 * x))`, but it does not match `exp(sin(x * 3))` because the expressions are not syntactically equal.
- If a pattern does contain variables, it matches an expression *expr* if these variables can be bound to subexpressions of *expr* such that once the pattern is evaluated with that variable binding, it becomes syntactically equal to the expression *expr*. For instance, the pattern `exp(sin(a * x))` will match the expression `exp(sin(3 * x))` as it is possible to bind `a` to `3` such that `exp(sin(a * x))` evaluates to `exp(sin(3 * x))`.

If a pattern *patternI* with variables is matched in a `match ... with` construct, the variables in the pattern stay bound during the evaluation of the corresponding return expression *return-exprI*. This allows subexpressions to be extracted from expressions and/or recursively handled as needed.

The following examples illustrate the basic principles of pattern matching in `Sollya`. One can remark that it is useful to use the keyword `_x_` when one wants to be sure to refer to the free variable in a pattern matching:

```
> match exp(x) with
      exp(x)      : (1)
      sin(x)      : (2)
      default     : (3);
1
>
> match sin(x) with
      exp(x)      : (1)
      sin(x)      : (2)
      default     : (3);
2
>
> match exp(sin(x)) with
      exp(x)      : ("Exponential of x")
      exp(sin(x)) : ("Exponential of sine of x")
      default     : ("Something else");
Exponential of sine of x
>
> match exp(sin(x)) with
      exp(x)      : ("Exponential of x")
      exp(a)      : ("Exponential of " @ a)
      default     : ("Something else");
Exponential of sin(x)
>
>
> procedure differentiate(f) {
      return match f with
          g + h   : (differentiate(g) + differentiate(h))
          g * h   : (differentiate(g) * h + differentiate(h) * g)
          g / h   : ((differentiate(g) * h - differentiate(h) * g) / (h^2))
          exp(_x_)  : (exp(_x_))
          sin(_x_)  : (cos(_x_))
          cos(_x_)  : (-sin(_x_))
          g(h)    : ((differentiate(g))(h) * differentiate(h))
          _x_       : (1)
          h(_x_)    : (NaN)
          c       : (0);
  };
>
> rename(x,y);
Information: the free variable has been renamed from "x" to "y".
> differentiate(exp(sin(y + y)));
exp(sin(y * 2)) * cos(y * 2) * 2
> diff(exp(sin(y + y)));
exp(sin(y * 2)) * cos(y * 2) * 2
>
```

As `Sollya` is not a purely functional language, the `match ... with` construct can also be used in a more imperative style, which makes it become closer to constructs like `switch` in `C` or `Perl`. In lieu of a simple return expression, a whole block of imperative statements can be given. The expression to be returned by that block is indicated in the end of the block, using the `return` keyword. That syntax is illustrated in the next example:

```
> match exp(sin(x)) with
      exp(a)  : {
                   write("Exponential of ", a, "\n");
                   return a;
                }
      sin(x)  : {
                   var foo;
                   foo = 17;
                   write("Sine of x\n");
                   return foo;
                }
      default : {
                   write("Something else\n");
                   bashexecute("LANG=C date");
                   return true;
                };
Exponential of sin(x)
sin(x)
>
> match sin(x) with
      exp(a)  : {
                   write("Exponential of ", a, "\n");
                   return a;
                }
      sin(x)  : {
                   var foo;
                   foo = 17;
                   write("Sine of x\n");
                   return foo;
                }
      default : {
                   write("Something else\n");
                   bashexecute("LANG=C date");
                   return true;
                };
Sine of x
17
>
> match acos(17 * pi * x) with
      exp(a)  : {
                   write("Exponential of ", a, "\n");
                   return a;
                }
      sin(x)  : {
                   var foo;
                   foo = 17;
                   write("Sine of x\n");
                   return foo;
                }
      default : {
                   write("Something else\n");
                   bashexecute("LANG=C date");
                   return true;
                };
Something else
Fri Feb 18 13:12:15 AKST 2022
true
```

In the case when no return statement is indicated for a statement-block in a `match ... with` construct, the construct evaluates to the special value `void` if that pattern matches.

In order to well understand pattern matching in `Sollya`, it is important to realize the meaning of variables in patterns. This meaning is different from the one usually found for variables. In a pattern, variables are never evaluated to whatever they might have set before the pattern is executed. In contrast, all variables in patterns are new, free variables that will freshly be bound to subexpressions of the matching expression. If a variable of the same name already exists, it will be shadowed during the evaluation of the statement block and the return expression corresponding to the matching expression. This type of semantic implies that patterns can never be computed at run-time, they must always be hard-coded beforehand. However this is necessary to make pattern matching context-free.

As a matter of course, all variables figuring in the expression *expr* to be matched are evaluated before pattern matching is attempted. In fact, *expr* is a usual `Sollya` expression, not a pattern.

In `Sollya`, the use of variables in patterns does not need to be linear. This means the same variable might appear twice or more in a pattern. Such a pattern will only match an expression if it contains the same subexpression, associated with the variable, in all places indicated by the variable in the pattern.

The following examples illustrate the use of variables in patterns in detail:

```
> a = 5;
> b = 6;
> match exp(x + 3) with
        exp(a + b) : {
                           print("Exponential");
                           print("a = ", a);
                           print("b = ", b);
                     }
        sin(x)     : {
                           print("Sine of x");
                     };
Exponential
a =  x
b =  3
> print("a = ", a, ", b = ", b);
a =  5 , b =  6
>
> a = 5;
> b = 6;
> match exp(x + 3) with
        exp(a + b) : {
                           var a, c;
                           a = 17;
                           c = "Hallo";
                           print("Exponential");
                           print("a = ", a);
                           print("b = ", b);
                           print("c = ", c);
                     }
        sin(x)     : {
                           print("Sine of x");
                     };
Exponential
a =  17
b =  3
c =  Hallo
> print("a = ", a, ", b = ", b);
a =  5 , b =  6
```

```
> match exp(sin(x)) + sin(x) with
        exp(a) + a : {
                        print("Winner");
                        print("a = ", a);
                     }
        default    : {
                        print("Loser");
                     };
Winner
a =  sin(x)
>
> match exp(sin(x)) + sin(3 * x) with
        exp(a) + a : {
                        print("Winner");
                        print("a = ", a);
                     }
        default    : {
                        print("Loser");
                     };
Loser
>
> f = exp(x);
> match f with
        sin(x) : (1)
        cos(x) : (2)
        exp(x) : (3)
        default : (4);
3
```

Pattern matching is meant to be a means to decompose expressions structurally. For this reason and in an analogous way to variables, no evaluation is performed at all on (sub-)expressions that form constant functions. As a consequence, patterns match constant expressions only if they are structurally identical. For example $5 + 1$ only matches $5 + 1$ and not $1 + 5$, $3 + 3$ nor 6.

This general rule on constant expressions admits one exception. Intervals in `Sollya` can be defined using constant expressions as bounds. These bounds are immediately evaluated to floating-point constants, though. In order to permit pattern matching on intervals, constant expressions given as bounds of intervals that form patterns are evaluated before pattern matching. However, in order not conflict with the rules of no evaluation of variables, these constant expressions as bounds of intervals in patterns must not contain free variables.

```
> autosimplify = off!;
> match 5 + 1 with
      1 + 5 : ("One plus five")
      6     : ("Six")
      5 + 1 : ("Five plus one");
Five plus one
>
> match 6 with
      1 + 5 : ("One plus five")
      6     : ("Six")
      5 + 1 : ("Five plus one");
Six
>
> match 1 + 5 with
      1 + 5 : ("One plus five")
      6     : ("Six")
      5 + 1 : ("Five plus one");
One plus five
>
> match [1; 5 + 1] with
      [1; 1 + 5] : ("Interval from one to one plus five")
      [1; 6]     : ("Interval from one to six")
      [1; 5 + 1] : ("Interval from one to five plus one");
Interval from one to one plus five
>
> match [1; 6] with
      [1; 1 + 5] : ("Interval from one to one plus five")
      [1; 6]     : ("Interval from one to six")
      [1; 5 + 1] : ("Interval from one to five plus one");
Interval from one to one plus five
>
```

The `Sollya` keyword `default` has a special meaning in patterns. It acts like a wild-card, matching any (sub-)expression, as long as the whole expression stays correctly typed. Upon matching with `default`, no variable gets bound. This feature is illustrated in the next example. Remark that `default` does match `error`, as well. If `error` is the result of an evaluation error on that very pattern matching construct, a warning message is displayed. Here are illustrations for `default` and `error`:

```
> match exp(x) with
      sin(x)    : ("Sine of x")
      atan(x^2) : ("Arctangent of square of x")
      default   : ("Something else")
      exp(x)    : ("Exponential of x");
Something else
>
> match atan(x^2) with
      sin(x)          : ("Sine of x")
      atan(default^2) : ("Arctangent of the square of something")
      default         : ("Something else");
Arctangent of the square of something
>
> match atan(exp(x)^2) with
      sin(x)          : ("Sine of x")
      atan(default^2) : ("Arctangent of the square of something")
      default         : ("Something else");
Arctangent of the square of something
>
> match exp("Hello world") with
      exp(default)    : ("A miracle has happened")
      default         : ("Something else");
Warning: at least one of the given expressions or a subexpression is not correct
ly typed
or its evaluation has failed because of some error on a side-effect.
Warning: the evaluation of the expression to be matched in a match... with const
ruct yields error due to a syntax error or an error on a side-effect.
The effect or return value of the match... with construct may be meaningless.
Something else
>
> match error with
      @NaN@           : ("Not A Number")
      [@NaN@, @NaN@]  : ("Interval of Not A Numbers")
      error           : ("Error")
      default         : ("Something else");
Error
>
> match [@NaN@, @NaN@] with
      @NaN@           : ("Not A Number")
      [@NaN@, @NaN@]  : ("Interval of Not A Numbers")
      error           : ("Error")
      default         : ("Something else");
Interval of Not A Numbers
>
> match @NaN@ with
      @NaN@           : ("Not A Number")
      [@NaN@, @NaN@]  : ("Interval of Not A Numbers")
      error           : ("Error")
      default         : ("Something else");
Not A Number
>
> match exp(17) with
      @NaN@           : ("Not A Number")
      [@NaN@, @NaN@]  : ("Interval of Not A Numbers")
      error           : ("Error")
      default         : ("Something else");
Something else
>
> match exp("This is wrong") with
      @NaN@           : ("Not A Number")
      [@NaN@, @NaN@]  : ("Interval of Not A Numbers")
      error           : ("Error")
      default         : ("Something else");
Warning: at least one of the given expressions or a subexpression is not correct
```

In Sollya, pattern matching is possible on the following Sollya types and operations defined on them:

- Expressions that define univariate functions, as explained above,
- Intervals with one, two or no bound defined in the pattern by a variable,
- Character sequences, literate or defined using the @ operator, possibly with a variable on one of the sides of the @ operator,
- Lists, literate, literate with variables or defined using the .:, :. and @ operators, possibly with a variable on one of the sides of the @ operator or one or two variables for .: and :.,
- Structures, literate or literate with variables, and
- All other Sollya objects, matchable with themselves (DE matches DE, on matches on, perturb matches perturb etc.) Remark that this includes error, which may be literally given or may be the result of a syntax error or an evaluation error. See the example above for an illustration.

```
> procedure detector(obj) {
     match obj with
         exp(a * x)            : { "Exponential of ", a, " times x"; }
         [ a; 17 ]             : { "An interval from ", a, " to 17"; }
         [| |]                 : { "Empty list"; }
         [| a, b, 2, exp(c) |] : { "A list of ", a, ", ", b, ", 2 and ",
                                   "exponential of ", c; }
         a @ [| 2, 3 |]        : { "Concatenation of the list ", a, " and ",
                                   "the list of 2 and 3"; }
         a .: [| 9 ... |]      : { a, " prepended to all integers >= 9"; }
         "Hello" @ w           : { "Hello concatenated with ", w; }
         { .a = sin(b);
           .b = [c;d] }        : { "A structure containing as .a the ",
                                   "sine of ", b,
                                   " and as .b the range from ", c,
                                   " to ", d; }
         perturb               : { "The special object perturb"; }
         default               : { "Something else"; };
  };
>
> detector(exp(5 * x));
Exponential of 5 times x
> detector([3.25;17]);
An interval from 3.25 to 17
> detector([||]);
Empty list
> detector([| sin(x), nearestint(x), 2, exp(5 * atan(x)) |]);
A list of sin(x), nearestint(x), 2 and exponential of 5 * atan(x)
> detector([| sin(x), cos(5 * x), "foo", 2, 3 |]);
Concatenation of the list [|sin(x), cos(x * 5), "foo"|] and the list of 2 and 3
> detector([| DE, 9... |]);
doubleextended prepended to all integers >= 9
> detector("Hello world");
Hello concatenated with  world
> detector({ .a = sin(x); .c = "Hello"; .b = [9;10] });
A structure containing as .a the sine of x and as .b the range from 9 to 10
> detector(perturb);
The special object perturb
> detector([13;19]);
Something else
```

Concerning intervals, please pay attention to the fact that expressions involving intervals are immediately evaluated and that structural pattern matching on functions on intervals is not possible. This point is illustrated in the next example:

```
> match exp([1;2]) with
        [a;b]              : {
                                a,", ",b;
                             }
        default            : {
                                "Something else";
                             };
Warning: For at least 2 of the constants displayed in decimal, rounding has happ
ened.
2.71828182845904523536028747135266249775724709369999, 7.389056098930650227230427460575007813180315570551 9
>
> match exp([1;2]) with
        exp([a;b])         : {
                                a,", ", b;
                             }
        default            : {
                                "Something else";
                             };
Warning: at least one of the given expressions or a subexpression is not correct
ly typed
or its evaluation has failed because of some error on a side-effect.
error
>
> match exp([1;2]) with
    exp(a)  : {
                "Exponential of ", a;
              }
    default : {
                "Something else";
              };
Something else
```

With respect to pattern matching on lists or character sequences defined using the `@` operator, the following is to be mentioned:

- Patterns like `a @ b` are not allowed as they would need to perform an ambiguous cut of the list or character sequence to be matched. This restriction is maintained even if the variables (here `a` and `b`) are constrained by other occurrences in the pattern (for example in a list) which would make the cut unambiguous.
- Recursive use of the `@` operator (even mixed with the operators `.:` and `:.`) is possible under the condition that there must not exist any other parenthesizing of the term in concatenations (`@`) such that the rule of one single variable for `@` above gets violated. For instance, `( [| 1 |] @ a) @ (b @ [| 4 |])` is not possible as it can be re-parenthesized `[| 1 |] @ (a @ b) @ [| 4 |]`, which exhibits the ambiguous case.

These points are illustrated in this example:

```
> match [| exp(sin(x)), sin(x), 4, DE(x), 9... |] with
        exp(a) .: (a .: (([||] :. 4) @ (b @ [| 13... |]))) :
                              { "a = ", a, ", b = ", b; };
a = sin(x), b = [|doubleextended(x), 9, 10, 11, 12|]
>
> match [| 1, 2, 3, 4, D... |] with
        a @ [| 4, D...|] : (a);
[|1, 2, 3|]
>
> match [| 1, 2, 3, 4, D... |] with
        a @ [| D...|] : (a);
[|1, 2, 3, 4|]
>
> match [| 1, 2, 3, 4... |] with
        a @ [| 3...|] : (a);
[|1, 2|]
>
> match [| 1, 2, 3, 4... |] with
        a @ [| 4...|] : (a);
[|1, 2, 3|]
>
> match [| 1, 2, 3, 4... |] with
        a @ [| 17...|] : (a);
[|1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16|]
>
> match [| 1, 2, 3, 4... |] with
        a @ [| 17, 18, 19 |] : (a)
        default              : ("Something else");
Something else
```

As mentioned above, pattern matching on `Sollya` structures is possible. Patterns for such a match are given in a literately, *i.e.*, using the syntax `{ .a =` *exprA*`, .b =` *exprB*`, ... }`. A structure pattern *sp* will be matched by a structure *s* iff that structure *s* contains at least all the elements (like `.a`, `.b` etc.) of the structure pattern *sp* and iff each of the elements of the structure *s* matches the pattern in the corresponding element of the structure pattern *sp*. The user should be aware of the fact that the structure to be matched is only supposed to have at least the elements of the pattern but that it may contain more elements is a particular `Sollya` feature. For instance with pattern matching, it is hence possible to ensure that access to particular elements will be possible in a particular code segment. The following example is meant to clarify this point:

```
> structure.f = exp(x);
> structure.dom = [1;2];
> structure.formats = [| DD, D, D, D |];
> match structure with
       { .f = sin(x);
         .dom = [a;b]
       }                    : { "Sine, ",a,", ",b; }
       { .f = exp(c);
         .dom = [a;b];
         .point = default
       }                    : { "Exponential, ",a, ", ", b, ", ", c; }
       { .f = exp(x);
         .dom = [a;b]
       }                    : { "Exponential, ",a, ", ", b; }
       default              : { "Something else"; };
Exponential, 1, 2
>
> structure.f = sin(x);
> match structure with
       { .f = sin(x);
         .dom = [a;b]
       }                    : { "Sine, ",a,", ",b; }
       { .f = exp(c);
         .dom = [a;b];
         .point = default
       }                    : { "Exponential, ",a, ", ", b, ", ", c; }
       { .f = exp(x);
         .dom = [a;b]
       }                    : { "Exponential, ",a, ", ", b; }
       default              : { "Something else"; };
Sine, 1, 2
>
> structure.f = exp(x + 2);
> structure.point = 23;
> match structure with
       { .f = sin(x);
         .dom = [a;b]
       }                    : { "Sine, ",a,", ",b; }
       { .f = exp(c);
         .dom = [a;b];
         .point = default
       }                    : { "Exponential, ",a, ", ", b, ", ", c; }
       { .f = exp(x);
         .dom = [a;b]
       }                    : { "Exponential, ",a, ", ", b; }
       default              : { "Something else"; };
Exponential, 1, 2, 2 + x
```

8 Commands and functions

The documentation of any command can be obtained from within the interactive tool by typing

```
help name_of_command;
```

at prompt. Similarly, it can be obtained from within the library thanks to the function

```
const char *sollya_lib_get_help_text(const char *)
```

that returns a string containing the documentation of the command whose name is stored in the input string argument. More precisely, that function returns `NULL` in the case when there is no command with such a name, and a valid pointer containing the help text otherwise. That pointer does not need to be freed by the user.

8.1 abs

Name: **abs**
the absolute value.

Library names:
```
sollya_obj_t sollya_lib_abs(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_abs(sollya_obj_t)
#define SOLLYA_ABS(x) sollya_lib_build_function_abs(x)
```

Description:

- **abs** is the absolute value function. **abs**(x)=$\begin{cases} x & x > 0 \\ -x & x \leq 0 \end{cases}$.

8.2 absolute

Name: **absolute**
indicates an absolute error for **externalplot**, **fpminimax** or **supnorm**

Library names:
```
sollya_obj_t sollya_lib_absolute()
int sollya_lib_is_absolute(sollya_obj_t)
```

Usage:

absolute : absolute|relative

Description:

- The use of **absolute** in the command **externalplot** indicates that during plotting in **externalplot** an absolute error is to be considered.

  See **externalplot** for details.

- Used with **fpminimax**, **absolute** indicates that **fpminimax** must try to minimize the absolute error.

  See **fpminimax** for details.

- When given in argument to **supnorm**, **absolute** indicates that an absolute error is to be considered for supremum norm computation.

  See **supnorm** for details.

Example 1:

```
> bashexecute("gcc -fPIC -c externalplotexample.c");
> bashexecute("gcc -shared -o externalplotexample externalplotexample.o -lgmp -l
mpfr");
> externalplot("./externalplotexample",absolute,exp(x),[-1/2;1/2],12,perturb);
```

See also: **externalplot** (8.65), **fpminimax** (8.73), **relative** (8.157), **bashexecute** (8.18), **supnorm** (8.183)

8.3 accurateinfnorm

Name: **accurateinfnorm**
computes a faithful rounding of the infinity norm of a function

Usage:

accurateinfnorm(*function*,*range*,*constant*) : (function, range, constant) → constant
accurateinfnorm(*function*,*range*,*constant*,*exclusion range 1*,...,*exclusion range n*) : (function, range, constant, range, ..., range) → constant

Parameters:

- *function* represents the function whose infinity norm is to be computed
- *range* represents the infinity norm is to be considered on
- *constant* represents the number of bits in the significant of the result
- *exclusion range 1* through *exclusion range n* represent ranges to be excluded

Description:

- The command **accurateinfnorm** computes an upper bound to the infinity norm of function *function* in *range*. This upper bound is the least floating-point number greater than the value of the infinity norm that lies in the set of dyadic floating point numbers having *constant* significant mantissa bits. This means the value **accurateinfnorm** evaluates to is at the time an upper bound and a faithful rounding to *constant* bits of the infinity norm of function *function* on range *range*.

  If given, the fourth and further arguments of the command **accurateinfnorm**, *exclusion range 1* through *exclusion range n* the infinity norm of the function *function* is not to be considered on.

- The command **accurateinfnorm** is now considered DEPRECATED in `Sollya`. Users should be aware about the fact that the algorithm behind **accurateinfnorm** is highly inefficient and that other, better suited algorithms, such as **supnorm**, are available inside `Sollya`. As a matter of fact, while **accurateinfnorm** is maintained for compatibility reasons with legacy `Sollya` codes, users are advised to avoid using **accurateinfnorm** in new `Sollya` scripts and to replace it, where possible, by the **supnorm** command.

Example 1:

```
> p = remez(exp(x), 5, [-1;1]);
> accurateinfnorm(p - exp(x), [-1;1], 20);
4.52055246569216251373291015625e-5
> accurateinfnorm(p - exp(x), [-1;1], 30);
4.52055139703588793054223060791015625e-5
> accurateinfnorm(p - exp(x), [-1;1], 40);
4.52055139671392325340093520935624837875366210937 5e-5
```

Example 2:

```
> p = remez(exp(x), 5, [-1;1]);
> midpointmode = on!;
> infnorm(p - exp(x), [-1;1]);
0.45205~5/7~e-4
> accurateinfnorm(p - exp(x), [-1;1], 40);
4.52055139671392325340093520935624837875366210937 5e-5
```

See also: **infnorm** (8.93), **dirtyinfnorm** (8.44), **supnorm** (8.183), **checkinfnorm** (8.26), **remez** (8.158), **diam** (8.40)

8.4 acos

Name: **acos**
the arccosine function.

Library names:
```
sollya_obj_t sollya_lib_acos(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_acos(sollya_obj_t)
#define SOLLYA_ACOS(x) sollya_lib_build_function_acos(x)
```

Description:

- **acos** is the inverse of the function **cos**: **acos**(y) is the unique number $x \in [0; \pi]$ such that **cos**(x)=y.
- It is defined only for $y \in [-1; 1]$.

See also: **cos** (8.31)

8.5 acosh

Name: **acosh**
the arg-hyperbolic cosine function.

Library names:
```
sollya_obj_t sollya_lib_acosh(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_acosh(sollya_obj_t)
#define SOLLYA_ACOSH(x) sollya_lib_build_function_acosh(x)
```

Description:

- **acosh** is the inverse of the function **cosh**: **acosh**(y) is the unique number $x \in [0; +\infty]$ such that **cosh**(x)=y.
- It is defined only for $y \in [0; +\infty]$.

See also: **cosh** (8.32)

8.6 &&

Name: &&
boolean AND operator

Library name:
```
sollya_obj_t sollya_lib_and(sollya_obj_t, sollya_obj_t)
```

Usage:

expr1 && *expr2* : (boolean, boolean) → boolean

Parameters:

- *expr1* and *expr2* represent boolean expressions

Description:

- && evaluates to the boolean AND of the two boolean expressions *expr1* and *expr2*. && evaluates to true iff both *expr1* and *expr2* evaluate to true.

Example 1:
```
> true && false;
false
```

Example 2:
```
> (1 == exp(0)) && (0 == log(1));
true
```

See also: || (8.127), **!** (8.120)

8.7 annotatefunction

Name: **annotatefunction**

Annotates a `Sollya` function object with an approximation that is faster to evaluate

Library names:

```
sollya_obj_t sollya_lib_annotatefunction(sollya_obj_t, sollya_obj_t,
                                         sollya_obj_t, sollya_obj_t, ...);
sollya_obj_t sollya_lib_v_annotatefunction(sollya_obj_t, sollya_obj_t,
                                           sollya_obj_t, sollya_obj_t,
                                           va_list);
```

Usage:

annotatefunction(*f*, *g*, *I*, *d*) : (function, function, range, range) → function
annotatefunction(*f*, *g*, *I*, *d*, x_0) : (function, function, range, range, constant) → function

Parameters:

- f is a function.
- g is a function, in most cases a polynomial.
- I is an interval.
- d is an interval.
- x_0 is a constant (default value is 0 when not provided).

Description:

- When a given function f is to be evaluated at several points of a given interval I to a given precision, it might be useful to precompute a good approximant g of f and further evaluate it instead of f when the approximation is good enough to provide the desire precision. If f is a complicated expression, whereas g is, *e.g.*, a polynomial of low degree, the cost of precomputing g can be well compensated by the gain of time in each subsequent evaluation. The purpose of **annotatefunction** is to provide such a mechanism to the user.
- When using **annotatefunction**(*f*, *g*, *I*, *d*, x_0), resp. **annotatefunction**(*f*, *g*, *I*, *d*) (where x_0 is assumed to be zero), it is assumed that

  $$\forall x \in I, f(x) - g(x - x_0) \in d.$$

  It is the user responsibility to ensure this property. Otherwise, any subsequent use of f on points of I might lead to incorrect values.
- A call to **annotatefunction**(*f*, *g*, *I*, *d*, x_0) annotates the given `Sollya` function object f with the approximation g. In further use, when asked to evaluate f on a point x of I, `Sollya` will first evaluate g on $x - x_0$ and check if the result is accurate enough in the given context (accounting for the fact that the error of approximation between the true value and $g(x - x_0)$ belongs to d). If not (and only in this case), an evaluation of the expression of f on x is performed.
- The approximation g can be any `Sollya` function but particular performance is expected when g is a polynomial. Upon annotation with a polynomial, precomputations are performed to analyze certain properties of the given approximation polynomial.
- **annotatefunction** updates the internal representation of f so as to persistently keep this information attached with the `Sollya` object representing f. In particular, the annotation is persistent through copy or use of f as a subexpression to build up bigger expressions. Notice however, that there is no way of deducing an annotation for the derivative of f from an annotation of f. So, in general, it should not be expected that **diff**(f) will be automatically annotated (notice, however that f might be a subexpression of its derivative, *e.g.*, for f=**exp** or f=**tan**, in which case the corresponding subexpressions of the derivative could inherit the annotations from f. It is currently not specified whether `Sollya` does this automatically or not).

- **annotatefunction** really is an imperative statement that modifies the internal representation of *f*. However, for convenience **annotatefunction** returns *f* itself.
- `Sollya` function objects can be annotated more than once with different approximations on different domains, that do not need to be disjoint. Upon evaluation of the annotated function object, `Sollya` chooses an approximation annotation (if any) that provides for sufficient accuracy at the evaluation point. It is not specified in which order `Sollya` tries different possible annotations when several are available for a given point x.

Example 1:

```
> verbosity=1!;
> procedure EXP(X,n,p) {
            var res, oldPrec;
            oldPrec = prec;
            prec = p!;
            "Using procedure function exponential with X=" @ X @ ", n=" @ n @ ",
 and p=" @ p;
            res = exp(X);
            prec = oldPrec!;
            return res;
       };
> g = function(EXP);
> p = 467680523945888933825168701613328646980445149548996-165 + x * (23384026197
294446691258465802074096632225783601255b-164 + x * (584600654932361167294842666
130356538218192258774236-163 + x * (389733769954907444862769649080681513731982
1946501b-164 + x * (779467539909814871742274462137143483104884881741 7b-167 + x
 * (249429612771140759211229411741788494258098560367376-171 + x * (83143204257
048761156138389001050974564563711794716-172 + x * (190041609730397013715793569
916459322894226704029956-176 + x * (190041609726693241489121222544499121560039
26801563b-179 + x * (337851750625425975267386794938572294567023960422556-183 +
 x * (675703511364367437839362598826492688619186066989 1b-184 + x * (9828414707
511252769908089206114262766633532289937b-188 + x * (262088611080038133147245 15
233584738706961162212965b-193 + x * (322570642533259543159537423969994565772 23
350602741b-197 + x * (578429089657689569703509185903214676926704485495b-195 +
x * 24678885421766756585236271055409967789849594719 57b-201)))))))))))));
> h = annotatefunction(g, p, [-1/2;1/2], [-475294848522543b-124;475294848522543b
-124]);
> h == g;
true
> prec = 24;
The precision has been set to 24 bits.
> h(0.25);
Warning: rounding has happened. The value displayed is a faithful rounding to 24
 bits of the true result.
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
1.2840254
> prec = 165;
The precision has been set to 165 bits.
> h(0.25);
Using procedure function exponential with X=[0.25;0.25], n=0, and p=185
Warning: rounding has happened. The value displayed is a faithful rounding to 16
5 bits of the true result.
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
1.284025416687741484073420568062436458336280865281 47
```

See also: **chebyshevform** (8.25), **taylorform** (8.189), **remez** (8.158), **supnorm** (8.183), **infnorm** (8.93)

8.8 :.

Name: **:.**
add an element at the end of a list.

Library name:
`sollya_obj_t sollya_lib_append(sollya_obj_t, sollya_obj_t)`

Usage:

$$L\mathbf{:.}x : (\text{list, any type}) \to \text{list}$$

Parameters:

- L is a list (possibly empty).
- x is an object of any type.

Description:

- **:.** adds the element x at the end of the list L.
- Note that since x may be of any type, it can in particular be a list.

Example 1:

```
> [|2,3,4|]:.5;
[|2, 3, 4, 5|]
```

Example 2:

```
> [|1,2,3|]:.[|4,5,6|];
[|1, 2, 3, [|4, 5, 6|]|]
```

Example 3:

```
> [||]:.1;
[|1|]
```

See also: **.:** (8.140), **@** (8.29)

8.9 ∼

Name: ∼
floating-point evaluation of a constant expression

Library name:
`sollya_obj_t sollya_lib_approx(sollya_obj_t)`

Usage:

$$\sim \textit{expression} : \text{function} \to \text{constant}$$
$$\sim \textit{something} : \text{any type} \to \text{any type}$$

Parameters:

- *expression* stands for an expression that is a constant
- *something* stands for some language element that is not a constant expression

Description:

- ∼ *expression* evaluates the *expression* that is a constant term to a floating-point constant. The evaluation may involve a rounding. If *expression* is not a constant, the evaluated constant is a faithful rounding of *expression* with **precision** bits, unless the *expression* is exactly 0 as a result of cancellation. In the latter case, a floating-point approximation of some (unknown) accuracy is returned.
- ∼ does not do anything on all language elements that are not a constant expression. In other words, it behaves like the identity function on any type that is not a constant expression. It can hence be used in any place where one wants to be sure that expressions are simplified using floating-point computations to constants of a known precision, regardless of the type of actual language elements.
- ∼ **error** evaluates to error and provokes a warning.
- ∼ is a prefix operator not requiring parentheses. Its precedence is the same as for the unary + and − operators. It cannot be repeatedly used without brackets.

Example 1:

```
> print(exp(5));
exp(5)
> print(~ exp(5));
148.413159102576603421115580040552279623487667593 88
```

Example 2:

```
> autosimplify = off!;
```

Example 3:

```
> print(~sin(5 * pi));
0
```

Example 4:

```
> print(~exp(x));
exp(x)
> print(~ "Hello");
Hello
```

Example 5:

```
> print(~exp(x*5*Pi));
exp((pi) * 5 * x)
> print(exp(x* ~(5*Pi)));
exp(x * 15.707963267948966192313216916397514420985846996875 7)
```

Example 6:

```
> print(~exp(5)*x);
148.413159102576603421115580040552279623487667593 88 * x
> print( (~exp(5))*x);
148.413159102576603421115580040552279623487667593 88 * x
> print(~(exp(5)*x));
exp(5) * x
```

See also: **evaluate** (8.58), **prec** (8.138), **error** (8.57)

8.10 asciiplot

Name: **asciiplot**
plots a function in a range using ASCII characters

Library name:
`void sollya_lib_asciiplot(sollya_obj_t, sollya_obj_t)`

Usage:

asciiplot(*function*, *range*) : (function, range) → void

Parameters:

- *function* represents a function to be plotted
- *range* represents a range the function is to be plotted in

Description:

- **asciiplot** plots the function *function* in range *range* using ASCII characters. On systems that provide the necessary `TIOCGWINSZ ioctl`, `Sollya` determines the size of the terminal for the plot size if connected to a terminal. If it is not connected to a terminal or if the test is not possible, the plot is of fixed size 77×25 characters. The function is evaluated on a number of points equal to the number of columns available. Its value is rounded to the next integer in the range of lines available. A letter `x` is written at this place. If zero is in the hull of the image domain of the function, an x-axis is displayed. If zero is in range, a y-axis is displayed. If the function is constant or if the range is reduced to one point, the function is evaluated to a constant and the constant is displayed instead of a plot.

Example 1:

```
> asciiplot(exp(x),[1;2]);
                                                                      xx
                                                                     xx
                                                                    xx
                                                                   xx
                                                                  xx
                                                                xxx
                                                               xx
                                                             xxx
                                                            xx
                                                          xxx
                                                         xxx
                                                        xxx
                                                       xxx
                                                      xxx
                                                     xxxx
                                                    xxx
                                                   xxxx
                                                  xxxx
                                                 xxxx
                                               xxxx
                                             xxxxx
                                           xxxxx
                                        xxxxx
                                     xxxxx
                                  xxx
```

Example 2:

```
> asciiplot(expm1(x),[-1;2]);
                          |                                                    x
                          |                                                   x
                          |                                                  x
                          |                                                 x
                          |                                               xx
                          |                                              x
                          |                                             x
                          |                                           xx
                          |                                          x
                          |                                        xx
                          |                                      xx
                          |                                    xx
                          |                                  xx
                          |                                xx
                          |                              xx
                          |                            xxx
                          |                         xxx
                          |                     xxxx
                          |                xxxx
                          |           xxxx
                          |      xxxxxx
---------------------xxxxxxxx-------------------------------------------------
         xxxxxxxxxxxx     |
xxxxxxxxx                 |
```

Example 3:

```
> asciiplot(5,[-1;1]);
5
```

Example 4:

```
> asciiplot(exp(x),[1;1]);
2.71828182845904523536028747135266249775724709 37
```

See also: **plot** (8.131), **externalplot** (8.65)

8.11 asin

Name: **asin**
the arcsine function.

Library names:

```
sollya_obj_t sollya_lib_asin(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_asin(sollya_obj_t)
#define SOLLYA_ASIN(x) sollya_lib_build_function_asin(x)
```

Description:

- **asin** is the inverse of the function **sin**: **asin**(y) is the unique number $x \in [-\pi/2; \pi/2]$ such that **sin**(x)=y.
- It is defined only for $y \in [-1; 1]$.

See also: **sin** (8.174)

8.12 asinh

Name: **asinh**
the arg-hyperbolic sine function.

Library names:
```
sollya_obj_t sollya_lib_asinh(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_asinh(sollya_obj_t)
#define SOLLYA_ASINH(x) sollya_lib_build_function_asinh(x)
```

Description:

- **asinh** is the inverse of the function **sinh**: **asinh**(y) is the unique number $x \in [-\infty; +\infty]$ such that **sinh**(x)=y.
- It is defined for every real number y.

See also: **sinh** (8.176)

8.13 atan

Name: **atan**
the arctangent function.

Library names:
```
sollya_obj_t sollya_lib_atan(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_atan(sollya_obj_t)
#define SOLLYA_ATAN(x) sollya_lib_build_function_atan(x)
```

Description:

- **atan** is the inverse of the function **tan**: **atan**(y) is the unique number $x \in [-\pi/2; +\pi/2]$ such that **tan**(x)=y.
- It is defined for every real number y.

See also: **tan** (8.186)

8.14 atanh

Name: **atanh**
the hyperbolic arctangent function.

Library names:
```
sollya_obj_t sollya_lib_atanh(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_atanh(sollya_obj_t)
#define SOLLYA_ATANH(x) sollya_lib_build_function_atanh(x)
```

Description:

- **atanh** is the inverse of the function **tanh**: **atanh**(y) is the unique number $x \in [-\infty; +\infty]$ such that **tanh**(x)=y.
- It is defined only for $y \in [-1; 1]$.

See also: **tanh** (8.187)

8.15 autodiff

Name: **autodiff**
Computes the first n derivatives of a function at a point or over an interval.

Library name:
```
sollya_obj_t sollya_lib_autodiff(sollya_obj_t, sollya_obj_t, sollya_obj_t)
```

Usage:

autodiff(f, n, x_0) : (function, integer, constant) → list
autodiff(f, n, I) : (function, integer, range) → list

Parameters:

- f is the function to be differentiated.
- n is the order of differentiation.
- x_0 is the point at which the function is differentiated.
- I is the interval over which the function is differentiated.

Description:

- **autodiff** computes the first n derivatives of f at point x_0. The computation is performed numerically, without symbolically differentiating the expression of f. Yet, the computation is safe since small interval enclosures are produced. More precisely, **autodiff** returns a list $[f_0, \ldots, f_n]$ such that, for each i, f_i is a small interval enclosing the exact value of $f^{(i)}(x_0)$.
- Since it does not perform any symbolic differentiation, **autodiff** is much more efficient than **diff** and should be preferred when only numerical values are necessary.
- If an interval I is provided instead of a point x_0, the list returned by **autodiff** satisfies: $\forall i,\ f^{(i)}(I) \subseteq f_i$. A particular use is when one wants to know the successive derivatives of a function at a non representable point such as π. In this case, it suffices to call **autodiff** with the (almost) point interval $I = [\mathbf{pi}]$.
- When I is almost a point interval, the returned enclosures f_i are also almost point intervals. However, when the interval I begins to be fairly large, the enclosures can be deeply overestimated due to the dependency phenomenon present with interval arithmetic.
- As a particular case, f_0 is an enclosure of the image of f over I. However, since the algorithm is not specially designed for this purpose it is not very efficient for this particular task. In particular, it is not able to return a finite enclosure for functions with removable singularities (e.g. $\sin(x)/x$ at 0). The command **evaluate** is much more efficient for computing an accurate enclosure of the image of a function over an interval.

Example 1:

```
> L = autodiff(exp(cos(x))+sin(exp(x)), 5, 0);
> midpointmode = on!;
> for f_i in L do f_i;
0.3559752813266941742012789792982961497379810154498~2/4~e1
0.5403023058681397174009366074429766037323104206179~0/3~
-0.3019450507398802024611853185539984893647499733880~6/2~e1
-0.2524412954423689519957506964890896998867689182 39~6/4~e1
0.3122789875648103314521452918413972974632057906 9~1/3~e1
-0.1663430795900669603348405357933995688395595497 8~3/1~e2
```

Example 2:

```
> f = log(cos(x)+x);
> L = autodiff(log(cos(x)+x), 5, [2,4]);
> L[0];
[0;1.27643852425465597132446653114905059102580436018893]
> evaluate(f, [2,4]);
[0.45986058925497069206106494332976097408234056912429;1.20787210589964169595901037621103012113048821362855]
> fprime = diff(f);
> L[1];
[2.53086745013099407167484456656211083053393118778677e-2;1.756802495307928251372639094511829094135912887 3365]
> evaluate(fprime,[2,4]);
[2.71048755415961996452136364304380881763456815673085e-2;1.109195306639432908373972257886235314055584312 7995]
```

Example 3:

```
> L = autodiff(sin(x)/x, 0, [-1,1]);
> L[0];
[NaN;NaN]
> evaluate(sin(x)/x, [-1,1]);
[0.54030230586813971740093660744297660373231042061 79;1]
```

See also: **diff** (8.42), **evaluate** (8.58)

8.16 autosimplify

Name: **autosimplify**
activates, deactivates or inspects the value of the automatic simplification state variable

Library names:
`void sollya_lib_set_autosimplify_and_print(sollya_obj_t)`
`void sollya_lib_set_autosimplify(sollya_obj_t)`
`sollya_obj_t sollya_lib_get_autosimplify()`

Usage:

autosimplify = *activation value* : on|off → void
autosimplify = *activation value* ! : on|off → void
autosimplify : on|off

Parameters:

- *activation value* represents **on** or **off**, i.e. activation or deactivation

Description:

- An assignment **autosimplify** = *activation value*, where *activation value* is one of **on** or **off**, activates respectively deactivates the automatic safe simplification of expressions of functions generated by the evaluation of commands or in argument of other commands.

  `Sollya` commands like **remez**, **taylor** or **rationalapprox** sometimes produce expressions that can be simplified. Constant subexpressions can be evaluated to dyadic floating-point numbers, monomials with coefficients 0 can be eliminated. Further, expressions indicated by the user perform better in many commands when simplified before being passed in argument to a command. When the automatic simplification of expressions is activated, `Sollya` automatically performs a safe (not value changing) simplification process on such expressions.

  The automatic generation of subexpressions can be annoying, in particular if it takes too much time for not enough benefit. Further the user might want to inspect the structure of the expression tree returned by a command. In this case, the automatic simplification should be deactivated.

If the assignment **autosimplify** = *activation value* is followed by an exclamation mark, no message indicating the new state is displayed. Otherwise the user is informed of the new state of the global mode by an indication.

Example 1:

```
> autosimplify = on !;
> print(x - x);
0
> autosimplify = off ;
Automatic pure tree simplification has been deactivated.
> print(x - x);
x - x
```

Example 2:

```
> autosimplify = on !;
> print(rationalapprox(sin(pi/5.9),7));
33 / 65
> autosimplify = off !;
> print(rationalapprox(sin(pi/5.9),7));
33 / 65
```

See also: **print** (8.141), == (8.54), **!**= (8.118), **prec** (8.138), **points** (8.133), **diam** (8.40), **display** (8.47), **verbosity** (8.198), **canonical** (8.23), **taylorrecursions** (8.190), **timing** (8.193), **fullparentheses** (8.74), **midpointmode** (8.112), **hopitalrecursions** (8.86), **remez** (8.158), **rationalapprox** (8.152), **taylor** (8.188)

8.17 bashevaluate

Name: **bashevaluate**
executes a shell command and returns its output as a string

Library names:
`sollya_obj_t sollya_lib_bashevaluate(sollya_obj_t, ...)`
`sollya_obj_t sollya_lib_v_bashevaluate(sollya_obj_t, va_list)`

Usage:

bashevaluate(*command*) : string → string
bashevaluate(*command*,*input*) : (string, string) → string

Parameters:

- *command* is a command to be interpreted by the shell.
- *input* is an optional character sequence to be fed to the command.

Description:

- **bashevaluate**(*command*) will execute the shell command *command* in a shell. All output on the command's standard output is collected and returned as a character sequence.
- If an additional argument *input* is given in a call to **bashevaluate**(*command*,*input*), this character sequence is written to the standard input of the command *command* that gets executed.
- All characters output by *command* are included in the character sequence to which **bashevaluate** evaluates but two exceptions. Every NULL character ('\0') in the output is replaced with '?' as `Sollya` is unable to handle character sequences containing that character. Additionally, if the output ends in a newline character ('\n'), this character is stripped off. Other newline characters which are not at the end of the output are left as such.

Example 1:

```
> bashevaluate("LANG=C date");
Mon Feb 21 12:19:56 CET 2022
```

Example 2:

```
> [| bashevaluate("echo Hello") |];
[|"Hello"|]
```

Example 3:

```
> a = bashevaluate("sed -e 's/a/e/g;'", "Hallo");
> a;
Hello
```

See also: **bashexecute** (8.18)

8.18 bashexecute

Name: **bashexecute**
executes a shell command.

Library name:
`void sollya_lib_bashexecute(sollya_obj_t)`

Usage:

bashexecute(*command*) : string → void

Parameters:

- *command* is a command to be interpreted by the shell.

Description:

- **bashexecute**(*command*) lets the shell interpret *command*. It is useful to execute some external code within `Sollya`.
- **bashexecute** does not return anything. It just executes its argument. However, if *command* produces an output in a file, this result can be imported in `Sollya` with help of commands like **execute**, **readfile** and **parse**.

Example 1:

```
> bashexecute("LANG=C date");
Mon Feb 21 12:20:06 CET 2022
```

See also: **execute** (8.59), **readfile** (8.155), **parse** (8.128), **bashevaluate** (8.17)

8.19 bezout

Name: **bezout**
Computes the gcd as well as the co-factors (Bezout coefficients)

Library name:
`sollya_obj_t sollya_lib_bezout(sollya_obj_t, sollya_obj_t)`

Usage:

bezout(p, q) : (function, function) → structure

Parameters:

- p is a constant or a polynomial.
- q is a constant or a polynomial.

Description:

- When both p and q are integers, **bezout**(p,q) computes the greatest common divisor g of these two integers, *i.e.*, the greatest non-negative integer g dividing both p and q, as well as co-factors (Bezout coefficients), *i.e.*, a and b such that $a\,p + b\,q = g$.
- **bezout** always returns a structure containing four elements, `g`, `r`, `a` and `b`. These four elements always satisfy the following two properties: $\gcd(g,r) = \gcd(p,q)$ and $a\,p + b\,q = g$. In addition, the tool tries to ensure $r = 0$. This latter property cannot always be ensured, in particular, it cannot be ensured for constant expression such that r becomes zero but the tool is unable to prove this fact. **bezout** ensures that g is the greatest common divisor of p and q only if r ends up being zero.
- When both p and q are rational numbers, say s/t and u/v, **bezout**(p,q) computes the greatest common divisor of $s \cdot v$ and $u \cdot t$, divided by the product of the denominators, $t \cdot v$, as well as co-factors, still satisfying $a\,p + b\,q = g$.
- When both p and q are constants but at least one of them is no rational number, **bezout** does not run the extended Euclidian algorithm to reduce $\gcd(p,q)$, *i.e.*, the two elements `g`, `r` of the returned structure contain copies of the inputs p and q. The elements `a` and `b` are set to 0 and 1 accordingly. **bezout** differs from the **gcd** command in this point.
- When both p and q are polynomials with at least one being non-constant, **bezout**(p,q) returns the polynomial of greatest degree dividing both p and q, and whose leading coefficient is the greatest common divisor of the leading coefficients of p and q. Polynomial co-factors (Bezout coefficients), *i.e.*, a and b such that $a\,p + b\,q = g$, are returned as well.
- Similarly to the cases documented for **div** and **mod**, **bezout** may fail to return the unique polynomial of largest degree dividing both p and q in cases when certain coefficients of either p or q are constant expressions for which the tool is unable to determine whether they are zero or not. These cases typically involve polynomials whose leading coefficient is zero but the tool is unable to detect this fact. In contrast to the **gcd** command, the **bezout** command is completely safe in this respect, as it returns not only an alleged greatest common divisor `g` but also the final remainder `r` obtained in the extended Euclidian algorithm. The two properties $\gcd(g,r) = \gcd(p,q)$ and $a\,p + b\,q = g$ are always satisfied for the four elements of the returned structure.
- When at least one of p or q is a function that is no polynomial, **bezout** does not run the extended Euclidian algorithm to reduce $\gcd(p,q)$, *i.e.*, the two elements `g`, `r` of the returned structure contain copies of the inputs p and q. The elements `a` and `b` are set to 0 and 1 accordingly. **bezout** differs from the **gcd** command in this point.

Example 1:

```
> bezout(1001, 231);
{ .b = -4, .a = 1, .r = 0, .g = 77 }
> bezout(13, 17);
{ .b = -3, .a = 4, .r = 0, .g = 1 }
> bezout(-210, 462);
{ .b = 1, .a = 2, .r = 0, .g = 42 }
```

Example 2:

```
> rationalmode = on!;
> bezout(6/7, 33/13);
{ .b = -1, .a = 3, .r = 0, .g = 3 / 91 }
```

Example 3:

```
> bezout(exp(13),sin(17));
{ .b = 0, .a = 1, .r = -0.96139749187955685726163694486915609849206725405894, .g
 = 4.42413392008920503326102775949088281784391306060 59e5 }
```

Example 4:

```
> bezout(24 + 68 * x + 74 * x^2 + 39 * x^3 + 10 * x^4 + x^5, 480 + 776 * x + 476
 * x^2 + 138 * x^3 + 19 * x^4 + x^5);
{ .b = 9.16666666666666666666666666666666666666666666666665e-2 + x * (0.125 + x *
 5e-2), .a = -1.66666666666666666666666666666666666666666666666667 + x * (-0.575
+ x * (-5e-2)), .r = 0, .g = 4 + x * (4 + x) }
> bezout(1001 * x^2, 231 * x);
{ .b = 0.33333333333333333333333333333333333333333333333333, .a = 0, .r = 0, .g
= x * 77 }
```

Example 5:

```
> bezout(exp(x), x^2);
{ .b = 0, .a = 1, .r = x^2, .g = exp(x) }
```

See also: **gcd** (8.76), **div** (8.48), **mod** (8.115), **numberroots** (8.121)

8.20 binary

Name: **binary**
special value for global state **display**

Library names:
`sollya_obj_t sollya_lib_binary()`
`int sollya_lib_is_binary(sollya_obj_t)`

Description:

- **binary** is a special value used for the global state **display**. If the global state **display** is equal to **binary**, all data will be output in binary notation.

  As any value it can be affected to a variable and stored in lists.

See also: **decimal** (8.36), **dyadic** (8.53), **powers** (8.137), **hexadecimal** (8.84), **display** (8.47)

8.21 bind

Name: **bind**
partially applies a procedure to an argument, returning a procedure with one argument less

Usage:

bind(*proc*, *ident*, *obj*) : (procedure, identifier type, any type) → procedure

Parameters:

- *proc* is a procedure to be partially applied to an argument
- *ident* is one of the formal arguments of *proc*
- *obj* is any `Sollya` object *ident* is to be bound to

Description:

- **bind** allows a formal parameter *ident* of a procedure *proc* to be bound to an object *obj*, hence *proc* to be partially applied. The result of this curryfied application, returned by **bind**, is a procedure with one argument less. This way, **bind** permits specialization of a generic procedure, parameterized e.g. by a function or range.

- In the case when *proc* does not have a formal parameter named *ident*, **bind** prints a warning and returns the procedure *proc* unmodified.
- **bind** always returns a procedure, even if *proc* only has one argument, which gets bound to *ident*. In this case, **bind** returns a procedure which does not take any argument. Hence evaluation, which might provoke side effects, is only performed once the procedure gets used.
- If **bind** is used on a procedure with an arbitrary number of arguments, no check is performed whether the procedure actually has a parameter of name *ident*. **bind** returns a procedure with an arbitrary number of arguments whose argument has the same name as the original procedure *proc*. That returned procedure binds *ident* to *obj* and then runs *proc* within this context, applying it to its original argument. The execution of **bind** fails for procedures with an arbitrary number of arguments if the original procedure's argument has the same name as *ident*. In this case, the procedure is returned unmodified.

Example 1:

```
> procedure add(X,Y) { return X + Y; };
> succ = bind(add,X,1);
> succ(5);
6
> succ;
proc(Y)
{
nop;
return (proc(X, Y)
{
nop;
return (X) + (Y);
})(1, Y);
}
```

Example 2:

```
> procedure add(X,Y) { return X + Y; };
> succ = bind(add,X,1);
> five = bind(succ,Y,4);
> five();
5
> five;
proc()
{
nop;
return (proc(Y)
{
nop;
return (proc(X, Y)
{
nop;
return (X) + (Y);
})(1, Y);
})(4);
}
```

Example 3:

```
> verbosity = 1!;
> procedure add(X,Y) { return X + Y; };
> foo = bind(add,R,1);
Warning: the given procedure has no argument named "R". The procedure is returne
d unchanged.
> foo;
proc(X, Y)
{
nop;
return (X) + (Y);
}
```

Example 4:

```
> procedure sum_times(l=...) { var s, i; s = 0; for i in l do s = s + i; return
c * s; };
> foo = bind(sum_times, c, 17);
> foo;
proc(l = ...)
{
var c;
c = 17;
return (proc(l = ...)
{
var s, i;
s = 0;
for i in l do
s = (s) + (i);
return (c) * (s);
}) @ (l);
}
> foo(1, 2, 3);
102
```

See also: **procedure** (8.147), **proc** (8.146), **function** (8.75), **@** (8.29)

8.22 boolean

Name: **boolean**
keyword representing a boolean type

Library name:
SOLLYA_EXTERNALPROC_TYPE_BOOLEAN

Usage:

boolean : type type

Description:

- **boolean** represents the boolean type for declarations of external procedures by means of **externalproc**.

  Remark that in contrast to other indicators, type indicators like **boolean** cannot be handled outside the **externalproc** context. In particular, they cannot be assigned to variables.

See also: **externalproc** (8.66), **constant** (8.30), **function** (8.75), **integer** (8.94), **list of** (8.103), **range** (8.151), **string** (8.179), **object** (8.123)

8.23 canonical

Name: **canonical**

brings all polynomial subexpressions of an expression to canonical form or activates, deactivates or checks canonical form printing

Library names:

```
void sollya_lib_set_canonical_and_print(sollya_obj_t)
void sollya_lib_set_canonical(sollya_obj_t)
sollya_obj_t sollya_lib_canonical(sollya_obj_t)
sollya_obj_t sollya_lib_get_canonical()
```

Usage:

canonical(*function*) : function → function
canonical = *activation value* : on|off → void
canonical = *activation value* ! : on|off → void

Parameters:

- *function* represents the expression to be rewritten in canonical form
- *activation value* represents **on** or **off**, i.e. activation or deactivation

Description:

- The command **canonical** rewrites the expression representing the function *function* in a way such that all polynomial subexpressions (or the whole expression itself, if it is a polynomial) are written in canonical form, i.e. as a sum of monomials in the canonical base. The canonical base is the base of the integer powers of the global free variable. The command **canonical** does not endanger the safety of computations even in `Sollya`'s floating-point environment: the function returned is mathematically equal to the function *function*.
- An assignment **canonical** = *activation value*, where *activation value* is one of **on** or **off**, activates respectively deactivates the automatic printing of polynomial expressions in canonical form, i.e. as a sum of monomials in the canonical base. If automatic printing in canonical form is deactivated, automatic printing yields to displaying polynomial subexpressions in Horner form.

  If the assignment **canonical** = *activation value* is followed by an exclamation mark, no message indicating the new state is displayed. Otherwise the user is informed of the new state of the global mode by an indication.

Example 1:

```
> print(canonical(1 + x * (x + 3 * x^2)));
1 + x^2 + 3 * x^3
> print(canonical((x + 1)^7));
1 + 7 * x + 21 * x^2 + 35 * x^3 + 35 * x^4 + 21 * x^5 + 7 * x^6 + x^7
```

Example 2:

```
> print(canonical(exp((x + 1)^5) - log(asin(((x + 2) + x)^4 * (x + 1)) + x)));
exp(1 + 5 * x + 10 * x^2 + 10 * x^3 + 5 * x^4 + x^5) - log(asin(16 + 80 * x + 16
0 * x^2 + 160 * x^3 + 80 * x^4 + 16 * x^5) + x)
```

Example 3:

```
> canonical;
off
> (x + 2)^9;
512 + x * (2304 + x * (4608 + x * (5376 + x * (4032 + x * (2016 + x * (672 + x *
 (144 + x * (18 + x)))))))
> canonical = on;
Canonical automatic printing output has been activated.
> (x + 2)^9;
512 + 2304 * x + 4608 * x^2 + 5376 * x^3 + 4032 * x^4 + 2016 * x^5 + 672 * x^6 +
 144 * x^7 + 18 * x^8 + x^9
> canonical;
on
> canonical = off!;
> (x + 2)^9;
512 + x * (2304 + x * (4608 + x * (5376 + x * (4032 + x * (2016 + x * (672 + x *
 (144 + x * (18 + x)))))))
```

See also: **horner** (8.87), **print** (8.141), **autosimplify** (8.16)

8.24 ceil

Name: **ceil**
the usual function ceil.

Library names:
```
sollya_obj_t sollya_lib_ceil(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_ceil(sollya_obj_t)
#define SOLLYA_CEIL(x) sollya_lib_build_function_ceil(x)
```

Description:

- **ceil** is defined as usual: **ceil**(x) is the smallest integer y such that $y \geq x$.
- It is defined for every real number x.

See also: **floor** (8.72), **nearestint** (8.117), **round** (8.164), **RU** (8.168)

8.25 chebyshevform

Name: **chebyshevform**
computes a rigorous polynomial approximation

Library name:
```
sollya_obj_t sollya_lib_chebyshevform(sollya_obj_t, sollya_obj_t,
                                      sollya_obj_t);
```

Usage:

chebyshevform(f, n, I) : (function, integer, range) $\rightarrow$ list

Parameters:

- f is the function to be approximated.
- n is the degree of the polynomial that must approximate f.
- I is the interval over which the function is to be approximated. This interval cannot be a point interval, i.e. its endpoints have to be different.

Description:

- **WARNING: chebyshevform** is a certified command, not difficult to use but not completely straightforward to use either. In order to be sure to use it correctly, the reader is invited to carefully read this documentation entirely.
- **chebyshevform** computes an approximation polynomial and an interval error bound for a given function f.
  More precisely, it returns a list $L = [p, \text{coeffErrors}, \Delta, \text{chebCoeffs}]$ where:
  - p is an approximation polynomial of degree n which is roughly speaking a numerical Chebyshev interpolation polynomial of f over I represented in monomial basis.
  - coeffsErrors is a list of $n+1$ intervals. Each interval coeffsErrors$[i]$ contains an enclosure of all the errors accumulated when computing the i-th coefficient of p.
  - Δ is an interval that provides a bound for the approximation error between p and f. See details below.
  - chebCoeffs is a list of $n+1$ intervals. These are the interval coefficients obtained for the representation of interpolation polynomial considered in Chebyshev basis. See details below.
- The polynomial p and the bound Δ are obtained using Chebyshev Models principles.
- More formally, the mathematical property ensured by the algorithm may be stated as follows:
  Using monomial basis representation given by p: there exist (small) values $\varepsilon_i \in$ coeffsErrors$[i]$ such that: $\forall x \in I, \exists \delta \in \Delta,\ f(x) - p(x) = \sum_{i=0}^{n} \varepsilon_i\, x^i + \delta$.
  Using Chebyshev basis representation given by chebCoeffs: there exist values $\alpha_i \in$ chebCoeffs$[i]$ such that: $\forall x \in I, \exists \delta \in \Delta,\ f(x) - \sum_{i=0}^{n} \alpha_i\, T_i(y) = \delta$, where $[a, b] = I, y = (2x - b - a)/(b - a)$ and $T_i(y)$ is the i-th Chebyshev polynomial over $[-1, 1]$.
- The algorithm does not guarantee that by increasing the degree of the approximation, the remainder bound will become smaller. Moreover, it may even become larger due to the dependency phenomenon present with interval arithmetic. In order to reduce this phenomenon, a possible solution is to split the definition domain I into several smaller intervals.

Example 1:

```
> TL=chebyshevform(exp(x), 10, [-1,1]);
> p=TL[0];
> Delta=TL[2];
> p; Delta;
1.00000000000000000000000000000000000000000000017 + x * (1.000000000273898191
977395347145362676477711516641 7 + x * (0.5000000000227675436802332688254209736541
1870042312 + x * (0.16666666119045065616898899342137151053369328144400 3 + x * (
4.16666662114404339731887995095915661615696989046 44e-2 + x * (8.3333639749825520
034240517636254576844423981913793e-3 + x * (1.388891436361480907068621055372167 7
7944219976325265e-3 + x * (1.983427760664721422847273297002273894035581118659 06e
-4 + x * (2.479577275982778882822180679653242679557681464890 14e-5 + x * (2.82540
334409824772663163700125420195118047008360 66e-6 + x * 2.813698333371884313582809
116304019749063304559236 2e-7)))))))))
[-2.714064128271745057750850104614499265724608243203 73e-11;2.714064128271745057 7
508501046144992657246082432037 3e-11]
```

See also: **taylorform** (8.189), **diff** (8.42), **autodiff** (8.15), **taylor** (8.188), **remez** (8.158)

8.26 checkinfnorm

Name: **checkinfnorm**
checks whether the infinity norm of a function is bounded by a value

Library name:

```
sollya_obj_t sollya_lib_checkinfnorm(sollya_obj_t, sollya_obj_t,
                                     sollya_obj_t)
```

Usage:

checkinfnorm(*function*,*range*,*constant*) : (function, range, constant) → boolean

Parameters:

- *function* represents the function whose infinity norm is to be checked
- *range* represents the infinity norm is to be considered on
- *constant* represents the upper bound the infinity norm is to be checked to

Description:

- The command **checkinfnorm** checks whether the infinity norm of the given function *function* in the range *range* can be proven (by `Sollya`) to be less than the given bound *bound*. This means, if **checkinfnorm** evaluates to **true**, the infinity norm has been proven (by `Sollya`'s interval arithmetic) to be less than the bound. If **checkinfnorm** evaluates to **false**, there are two possibilities: either the bound is less than or equal to the infinity norm of the function or the bound is greater than the infinity norm but `Sollya` could not conclude using its internal interval arithmetic.

  checkinfnorm is sensitive to the global variable **diam**. The smaller **diam**, the more time `Sollya` will spend on the evaluation of **checkinfnorm** in order to prove the bound before returning **false** although the infinity norm is bounded by the bound. If **diam** is equal to 0, `Sollya` will eventually spend infinite time on instances where the given bound *bound* is less or equal to the infinity norm of the function *function* in range *range*. In contrast, with **diam** being zero, **checkinfnorm** evaluates to **true** iff the infinity norm of the function in the range is bounded by the given bound.

Example 1:

```
> checkinfnorm(sin(x),[0;1.75], 1);
true
> checkinfnorm(sin(x),[0;1.75], 1/2); checkinfnorm(sin(x),[0;20/39],1/2);
false
true
```

Example 2:

```
> p = remez(exp(x), 5, [-1;1]);
> b = dirtyinfnorm(p - exp(x), [-1;1]);
> checkinfnorm(p - exp(x), [-1;1], b);
false
> b1 = round(b, 15, RU);
> checkinfnorm(p - exp(x), [-1;1], b1);
true
> b2 = round(b, 25, RU);
> checkinfnorm(p - exp(x), [-1;1], b2);
false
> diam = 1b-20!;
> checkinfnorm(p - exp(x), [-1;1], b2);
true
```

See also: **infnorm** (8.93), **dirtyinfnorm** (8.44), **supnorm** (8.183), **accurateinfnorm** (8.3), **remez** (8.158), **diam** (8.40)

8.27 coeff

Name: **coeff**
gives the coefficient of degree n of a polynomial

Library name:
`sollya_obj_t sollya_lib_coeff(sollya_obj_t, sollya_obj_t)`

Usage:

coeff(f,n) : (function, integer) → constant

Parameters:

- f is a function (usually a polynomial).
- n is an integer

Description:

- If f is a polynomial, **coeff**(f, n) returns the coefficient of degree n in f.
- If f is a function that is not a polynomial, **coeff**(f, n) returns 0.

Example 1:

```
> coeff((1+x)^5,3);
10
```

Example 2:

```
> coeff(sin(x),0);
0
```

See also: **degree** (8.38), **roundcoefficients** (8.165), **subpoly** (8.180)

8.28 composepolynomials

Name: **composepolynomials**
computes an approximation to the composition of two polynomials and bounds the error

Library name:
`sollya_obj_t sollya_lib_composepolynomials(sollya_obj_t, sollya_obj_t)`

Usage:

composepolynomials(p,q) : (function, function) → structure

Parameters:

- p and q are polynomials

Description:

- Given two polynomials p and q, **composepolynomials**(p, q) computes an approximation r to the polynomial $(p \circ q)$ and bounds the error polynomial $r - (p \circ q)$ using interval arithmetic.
- **composepolynomials** always returns a structure containing two elements, `poly` and `radii`. The element `poly` is contains the approximate composed polynomial r. The element `radii` contains a list of $n + 1$ intervals a_i bounding the coefficients of the error polynomial, which is of the same degree n as is the composed polynomial $(p \circ q)$. This is, there exist $\alpha_i \in a_i$ such that

$$\sum_{i=0}^{n} \alpha_i \, x^i = r(x) - (p \circ q)(x).$$

- In the case when either of p or q is not a polynomial, **composepolynomials** behaves like **substitute** used in a literate structure. The list of intervals bounding the coefficients of the error polynomial is returned empty.

Example 1:

```
> composepolynomials(1 + 2 * x + 3 * x^2 + 4 * x^3, 5 + 6 * x + 7 * x^2);
{ .radii = [|[0;0], [0;0], [0;0], [0;0], [0;0], [0;0], [0;0]|], .poly = 586 + x
* (1992 + x * (4592 + x * (6156 + x * (6111 + x * (3528 + x * 1372))))) }
```

Example 2:

```
> print(composepolynomials(1/5 * x + exp(17) + log(2) * x^2, x^4 + 1/3 * x^2));
{ .radii = [|[-3.587324068671531701564747733222185296077470571203 9e-43;3.5873240
686715317015647477332221852960774705712039e-43], [0;0], [-2.6727647100921956461405364671514818788151968 80
105e-51;2.67276471009219564614053646715148187881519688 0105e-51], [0;0], [-1.0691058840368782584562145868605927515260787520 42e-50;1.0691
0588403687825845621458686059275152607875204 2e-50], [0;0], [-2.138211768073756516
912429173721185503052157504084e-50;2.13821176807375651691242917372118550305215 75
04084e-50], [0;0], [-1.0691058840368782584562145868605927515260787520 42e-50;1.06
9105884036878258456214586860592751526078752042e-50]|], .poly = 2.415495275357529
821477543518038582387986756735272 28e7 + x^2 * (6.6666666666666666666666666666666
666666666666666666e-2 + x^2 * (0.2770163533955494788241369023842418408972777927 0
67 + x^2 * (0.46209812037329687294482141430545104538366675624017 + x^2 * 0.69314
718055994530941723212145817656807550013436026))) }
```

Example 3:

```
> composepolynomials(sin(x),x + x^2);
{ .radii = [| |], .poly = sin(x * (1 + x)) }
```

See also: **substitute** (8.181)

8.29 @

Name: **@**
concatenates two lists or strings or applies a list as arguments to a procedure

Library name:
`sollya_obj_t sollya_lib_concat(sollya_obj_t, sollya_obj_t)`

Usage:

$$L1@L2 : (\text{list}, \text{list}) \to \text{list}$$
$$string1@string2 : (\text{string}, \text{string}) \to \text{string}$$
$$proc@L1 : (\text{procedure}, \text{list}) \to \text{any type}$$

Parameters:

- *L1* and *L2* are two lists.
- *string1* and *string2* are two strings.
- *proc* is a procedure or an external procedure.

Description:

- In its first usage form, @ concatenates two lists or strings.

- In its second usage form, @ applies the elements of a list as arguments to a procedure or an external procedure. In the case when *proc* is a procedure or external procedure with a fixed number of arguments, a check is done if the number of elements in the list corresponds to the number of formal parameters of *proc*. An empty list can therefore be applied only to a procedure that does not take any argument. In the case when *proc* accepts an arbitrary number of arguments, no such check is performed.

Example 1:

```
> [|1,...,3|]@[|7,8,9|];
[|1, 2, 3, 7, 8, 9|]
```

Example 2:

```
> "Hello "@"World!";
Hello World!
```

Example 3:

```
> procedure cool(a,b,c) {
  write(a,", ", b," and ",c," are cool guys.\n");
  };
> cool @ [| "Christoph", "Mioara", "Sylvain" |];
Christoph, Mioara and Sylvain are cool guys.
```

Example 4:

```
> procedure sayhello() {
  "Hello! how are you?";
  };
> sayhello();
Hello! how are you?
> sayhello @ [||];
Hello! how are you?
```

Example 5:

```
> bashexecute("gcc -fPIC -Wall -c externalprocexample.c");
> bashexecute("gcc -fPIC -shared -o externalprocexample externalprocexample.o");

> externalproc(foo, "./externalprocexample", (integer, integer) -> integer);
> foo;
foo
> foo @ [|5, 6|];
11
```

Example 6:

```
> procedure add(L = ...) {
  var acc, i;
  acc = 0;
  for i in L do acc = i + acc;
  return acc;
  };
> add(1,2);
3
> add(1,2,3);
6
> add @ [|1, 2|];
3
> add @ [|1, 2, 3|];
6
> add @ [||];
0
```

See also: **.:** (8.140), **:.** (8.8), **procedure** (8.147), **externalproc** (8.66), **proc** (8.146), **bind** (8.21), **getbacktrace** (8.78)

8.30 constant

Name: **constant**
keyword representing a constant type

Library name:
`SOLLYA_EXTERNALPROC_TYPE_CONSTANT`

Usage:

constant : type type

Description:

- **constant** represents the constant type for declarations of external procedures **externalproc**.

  Remark that in contrast to other indicators, type indicators like **constant** cannot be handled outside the **externalproc** context. In particular, they cannot be assigned to variables.

See also: **externalproc** (8.66), **boolean** (8.22), **function** (8.75), **integer** (8.94), **list of** (8.103), **range** (8.151), **string** (8.179), **object** (8.123)

8.31 cos

Name: **cos**
the cosine function.

Library names:
```
sollya_obj_t sollya_lib_cos(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_cos(sollya_obj_t)
#define SOLLYA_COS(x) sollya_lib_build_function_cos(x)
```

Description:

- **cos** is the usual cosine function.
- It is defined for every real number x.

See also: **acos** (8.4), **sin** (8.174), **tan** (8.186)

8.32 cosh

Name: **cosh**
the hyperbolic cosine function.

Library names:
```
sollya_obj_t sollya_lib_cosh(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_cosh(sollya_obj_t)
#define SOLLYA_COSH(x) sollya_lib_build_function_cosh(x)
```

Description:

- **cosh** is the usual hyperbolic function: $\cosh(x) = \frac{e^x+e^{-x}}{2}$.
- It is defined for every real number x.

See also: **acosh** (8.5), **sinh** (8.176), **tanh** (8.187), **exp** (8.60)

8.33 D

Name: **D**
short form for **double**

See also: **double** (8.50)

8.34 DD

Name: **DD**
short form for **doubledouble**

See also: **doubledouble** (8.51)

8.35 DE

Name: **DE**
short form for **doubleextended**

See also: **doubleextended** (8.52)

8.36 decimal

Name: **decimal**
special value for global state **display**

Library names:
```
sollya_obj_t sollya_lib_decimal()
int sollya_lib_is_decimal(sollya_obj_t)
```

Description:

- **decimal** is a special value used for the global state **display**. If the global state **display** is equal to **decimal**, all data will be output in decimal notation.

  As any value it can be affected to a variable and stored in lists.

See also: **dyadic** (8.53), **powers** (8.137), **hexadecimal** (8.84), **binary** (8.20), **display** (8.47)

8.37 default

Name: **default**
default value for some commands.

Library names:

```
sollya_obj_t sollya_lib_default()
int sollya_lib_is_default(sollya_obj_t)
```

Description:

- **default** is a special value and is replaced by something depending on the context where it is used. It can often be used as a joker, when you want to specify one of the optional parameters of a command and not the others: set the value of uninteresting parameters to **default**.
- Global variables can be reset by affecting them the special value **default**.

Example 1:

```
> p = remez(exp(x),5,[0;1],default,1e-5);
> q = remez(exp(x),5,[0;1],1,1e-5);
> p==q;
true
```

Example 2:

```
> prec;
165
> prec=200;
The precision has been set to 200 bits.
```

8.38 degree

Name: **degree**
gives the degree of a polynomial.

Library name:
`sollya_obj_t sollya_lib_degree(sollya_obj_t)`

Usage:

$$\textbf{degree}(f) : \mathsf{function} \to \mathsf{integer}$$

Parameters:

- f is a function (usually a polynomial).

Description:

- If f is a polynomial, **degree**(f) returns the degree of f.
- Contrary to the usage, `Sollya` considers that the degree of the null polynomial is 0.
- If f is a function that is not a polynomial, **degree**(f) returns -1.

Example 1:

```
> degree((1+x)*(2+5*x^2));
3
> degree(0);
0
```

Example 2:

```
> degree(sin(x));
-1
```

See also: **coeff** (8.27), **subpoly** (8.180), **roundcoefficients** (8.165)

8.39 denominator

Name: **denominator**
gives the denominator of an expression

Library name:
`sollya_obj_t sollya_lib_denominator(sollya_obj_t)`

Usage:

denominator(*expr*) : function → function

Parameters:

- *expr* represents an expression

Description:

- If *expr* represents a fraction *expr1*/*expr2*, **denominator**(*expr*) returns the denominator of this fraction, i.e. *expr2*.

  If *expr* represents something else, **denominator**(*expr*) returns 1.

  Note that for all expressions *expr*, **numerator**(*expr*) **/ denominator**(*expr*) is equal to *expr*.

Example 1:

```
> denominator(5/3);
3
```

Example 2:

```
> denominator(exp(x));
1
```

Example 3:

```
> a = 5/3;
> b = numerator(a)/denominator(a);
> print(a);
5 / 3
> print(b);
5 / 3
```

Example 4:

```
> a = exp(x/3);
> b = numerator(a)/denominator(a);
> print(a);
exp(x / 3)
> print(b);
exp(x / 3)
```

See also: **numerator** (8.122), **rationalmode** (8.153)

8.40 diam

Name: **diam**
parameter used in safe algorithms of `Sollya` and controlling the maximal length of the involved intervals.

Library names:
`void sollya_lib_set_diam_and_print(sollya_obj_t)`

```
void sollya_lib_set_diam(sollya_obj_t)
sollya_obj_t sollya_lib_get_diam()
```

Usage:

diam = *width* : constant → void
diam = *width* ! : constant → void
diam : constant

Parameters:

- *width* represents the maximal relative width of the intervals used

Description:

- **diam** is a global variable. Its value represents the maximal width allowed for intervals involved in safe algorithms of `Sollya` (namely **infnorm**, **checkinfnorm**, **accurateinfnorm**, **integral**, **findzeros**, **supnorm**).
- More precisely, **diam** is relative to the width of the input interval of the command. For instance, suppose that **diam**=1e-5: if **infnorm** is called on interval [0, 1], the maximal width of an interval will be 1e-5. But if it is called on interval [0, 1e−3], the maximal width will be 1e-8.

See also: **infnorm** (8.93), **checkinfnorm** (8.26), **accurateinfnorm** (8.3), **integral** (8.95), **findzeros** (8.69), **supnorm** (8.183)

8.41 dieonerrormode

Name: **dieonerrormode**
global variable controlling if `Sollya` is exited on an error or not.

Library names:

```
void sollya_lib_set_dieonerrormode_and_print(sollya_obj_t)
void sollya_lib_set_dieonerrormode(sollya_obj_t)
sollya_obj_t sollya_lib_get_dieonerrormode()
```

Usage:

dieonerrormode = *activation value* : on|off → void
dieonerrormode = *activation value* ! : on|off → void
dieonerrormode : on|off

Parameters:

- *activation value* controls if `Sollya` is exited on an error or not.

Description:

- **dieonerrormode** is a global variable. When its value is **off**, which is the default, `Sollya` will not exit on any syntax, typing, side-effect errors. These errors will be caught by the tool, even if a memory might be leaked at that point. On evaluation, the **error** special value will be produced.
- When the value of the **dieonerrormode** variable is **on**, `Sollya` will exit on any syntax, typing, side-effect errors. A warning message will be printed in these cases at appropriate **verbosity** levels.

Example 1:

```
> verbosity = 1!;
> dieonerrormode = off;
Die-on-error mode has been deactivated.
> for i from true to false do i + "Salut";
Warning: one of the arguments of the for loop does not evaluate to a constant.
The for loop will not be executed.
> exp(17);
Warning: rounding has happened. The value displayed is a faithful rounding to 16
5 bits of the true result.
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
2.41549527535752982147754351803858238798675673527224e7
```

Example 2:

```
> verbosity = 1!;
> dieonerrormode = off!;
> 5 */  4;
Warning: syntax error.
The last symbol read has been "/".
Will skip input until next semicolon after the unexpected token. May leak memory
.
  exp(17);
Warning: rounding has happened. The value displayed is a faithful rounding to 16
5 bits of the true result.
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
2.41549527535752982147754351803858238798675673527224e7
```

Example 3:

```
> verbosity = 1!;
> dieonerrormode;
off
> dieonerrormode = on!;
> dieonerrormode;
on
> for i from true to false do i + "Salut";
Warning: one of the arguments of the for loop does not evaluate to a constant.
The for loop will not be executed.
Warning: some syntax, typing or side-effect error has occurred.
As the die-on-error mode is activated, the tool will be exited.
```

Example 4:

```
> verbosity = 1!;
> dieonerrormode = on!;
> 5 */  4;
Warning: syntax error.
The last symbol read has been "/".
Will skip input until next semicolon after the unexpected token. May leak memory
.
Warning: some syntax, typing or side-effect error has occurred.
As the die-on-error mode is activated, the tool will be exited.
```

Example 5:

```
> verbosity = 0!;
> dieonerrormode = on!;
> 5 */  4;
```

See also: **on** (8.126), **off** (8.125), **verbosity** (8.198), **error** (8.57)

8.42 diff

Name: **diff**
differentiation operator

Library name:
`sollya_obj_t sollya_lib_diff(sollya_obj_t)`

Usage:

diff(*function*) : function → function

Parameters:

- *function* represents a function

Description:

- **diff**(*function*) returns the symbolic derivative of the function *function* by the global free variable.

  If *function* represents a function symbol that is externally bound to some code by **library**, the derivative is performed as a symbolic annotation to the returned expression tree.

Example 1:

```
> diff(sin(x));
cos(x)
```

Example 2:

```
> diff(x);
1
```

Example 3:

```
> diff(x^x);
x^x * (1 + log(x))
```

See also: **library** (8.101), **autodiff** (8.15), **taylor** (8.188), **taylorform** (8.189)

8.43 dirtyfindzeros

Name: **dirtyfindzeros**
gives a list of numerical values listing the zeros of a function on an interval.

Library name:
`sollya_obj_t sollya_lib_dirtyfindzeros(sollya_obj_t, sollya_obj_t)`

Usage:

dirtyfindzeros(*f*,*I*) : (function, range) → list

Parameters:

- f is a function.
- I is an interval.

Description:

- **dirtyfindzeros**(*f*,*I*) returns a list containing some zeros of f in the interval I. The values in the list are numerical approximation of the exact zeros. The precision of these approximations is approximately the precision stored in **prec**. If f does not have two zeros very close to each other, it can be expected that all zeros are listed. However, some zeros may be forgotten. This command should be considered as a numerical algorithm and should not be used if safety is critical.
- More precisely, the algorithm relies on global variables **prec** and **points** and it performs the following steps: let n be the value of variable **points** and t be the value of variable **prec**.
  - Evaluate $|f|$ at n evenly distributed points in the interval I. The working precision to be used is automatically chosen in order to ensure that the sign is correct.
  - Whenever f changes its sign for two consecutive points, find an approximation x of its zero with precision t using Newton's algorithm. The number of steps in Newton's iteration depends on t: the precision of the approximation is supposed to be doubled at each step.
  - Add this value to the list.
- The user should be aware that the list returned by **dirtyfindzeros** may contain a certain floating-point number twice. These repetitions reflect the existence of different zeros of the function the value of which rounds to the same floating-point number in the current working precision. In this case, increasing the working precision will end up making the two zeros distinguishable even in rounded floating-point arithmetic.

Example 1:

```
> dirtyfindzeros(sin(x),[-5;5]);
[|-3.14159265358979323846264338327950288419716939937751, 0, 3.1415926535897932384
626433832795028841971693993751|]
```

Example 2:

```
> L1=dirtyfindzeros(x^2*sin(1/x),[0;1]);
> points=1000!;
> L2=dirtyfindzeros(x^2*sin(1/x),[0;1]);
> length(L1); length(L2);
18
25
```

See also: **prec** (8.138), **points** (8.133), **findzeros** (8.69), **dirtyinfnorm** (8.44), **numberroots** (8.121)

8.44 dirtyinfnorm

Name: **dirtyinfnorm**
computes a numerical approximation of the infinity norm of a function on an interval.

Library name:
`sollya_obj_t sollya_lib_dirtyinfnorm(sollya_obj_t, sollya_obj_t)`

Usage:

dirtyinfnorm(*f*,*I*) : (function, range) → constant

Parameters:

- f is a function.
- I is an interval.

Description:

- **dirtyinfnorm**(f,I) computes an approximation of the infinity norm of the given function f on the interval I, e.g. $\max_{x \in I}\{|f(x)|\}$.
- The interval must be bound. If the interval contains one of -Inf or +Inf, the result of **dirtyinfnorm** is NaN.
- The result of this command depends on the global variables **prec** and **points**. Therefore, the returned result is generally a good approximation of the exact infinity norm, with precision **prec**. However, the result is generally underestimated and should not be used when safety is critical. Use **infnorm** instead.
- The following algorithm is used: let n be the value of variable **points** and t be the value of variable **prec**.
  - Evaluate $|f|$ at n evenly distributed points in the interval I. The evaluation are faithful roundings of the exact results at precision t.
  - Whenever the derivative of f changes its sign for two consecutive points, find an approximation x of its zero with precision t. Then compute a faithful rounding of $|f(x)|$ at precision t.
  - Return the maximum of all computed values.

Example 1:

```
> dirtyinfnorm(sin(x),[-10;10]);
1
```

Example 2:

```
> prec=15!;
> dirtyinfnorm(exp(cos(x))*sin(x),[0;5]);
1.45856
> prec=40!;
> dirtyinfnorm(exp(cos(x))*sin(x),[0;5]);
1.458528537136
> prec=100!;
> dirtyinfnorm(exp(cos(x))*sin(x),[0;5]);
1.458528537136237644438147455025
> prec=200!;
> dirtyinfnorm(exp(cos(x))*sin(x),[0;5]);
1.458528537136237644438147455023841718299214087993682374094153
```

Example 3:

```
> dirtyinfnorm(x^2, [log(0);log(1)]);
NaN
```

See also: **prec** (8.138), **points** (8.133), **infnorm** (8.93), **checkinfnorm** (8.26), **supnorm** (8.183)

8.45 dirtyintegral

Name: **dirtyintegral**
computes a numerical approximation of the integral of a function on an interval.

Library name:
`sollya_obj_t sollya_lib_dirtyintegral(sollya_obj_t, sollya_obj_t)`

Usage:

dirtyintegral(f,I) : (function, range) $\rightarrow$ constant

Parameters:

- f is a function.
- I is an interval.

Description:

- **dirtyintegral**(f,I) computes an approximation of the integral of f on I.
- The interval must be bound. If the interval contains one of -Inf or +Inf, the result of **dirtyintegral** is NaN, even if the integral has a meaning.
- The result of this command depends on the global variables **prec** and **points**. The method used is the trapezium rule applied at n evenly distributed points in the interval, where n is the value of global variable **points**.
- This command computes a numerical approximation of the exact value of the integral. It should not be used if safety is critical. In this case, use command **integral** instead.
- Warning: this command is currently known to be unsatisfactory. If you really need to compute integrals, think of using another tool or report a feature request to sylvain.chevillard@ens-lyon.org.

Example 1:

```
> sin(10);
-0.54402111088936981340474766185137728168364301291622
> dirtyintegral(cos(x),[0;10]);
-0.54400304905152629822448058882475382036536298356282
> points=2000!;
> dirtyintegral(cos(x),[0;10]);
-0.54401997751158321972222697312583199035995837926893
```

See also: **prec** (8.138), **points** (8.133), **integral** (8.95)

8.46 dirtysimplify

Name: **dirtysimplify**
simplifies an expression representing a function

Library name:
`sollya_obj_t sollya_lib_dirtysimplify(sollya_obj_t)`

Usage:

dirtysimplify(*function*) : function → function

Parameters:

- *function* represents the expression to be simplified

Description:

- The command **dirtysimplify** simplifies constant subexpressions of the expression given in argument representing the function *function*. Those constant subexpressions are evaluated using floating-point arithmetic with the global precision **prec**.

Example 1:

```
> print(dirtysimplify(sin(pi * x)));
sin(3.14159265358979323846264338327950288419716939937 51 * x)
> print(dirtysimplify(erf(exp(3) + x * log(4))));
erf(20.085536923187667740928529654581717896987907838554 4 + x * 1.386294361119890
6188344642429163531361510002687205)
```

Example 2:

```
> prec = 20!;
> t = erf(0.5);
> s = dirtysimplify(erf(0.5));
> prec = 200!;
> t;
0.52049987781304653768274665389196452873645157575796370005880 6
> s;
0.52050018310546875
```

See also: **simplify** (8.173), **autosimplify** (8.16), **prec** (8.138), **evaluate** (8.58), **horner** (8.87), **rationalmode** (8.153)

8.47 display

Name: **display**
sets or inspects the global variable specifying number notation

Library names:
`void sollya_lib_set_display_and_print(sollya_obj_t)`
`void sollya_lib_set_display(sollya_obj_t)`
`sollya_obj_t sollya_lib_get_display()`

Usage:

display = *notation value* : decimal|binary|dyadic|powers|hexadecimal → void
display = *notation value* ! : decimal|binary|dyadic|powers|hexadecimal → void
display : decimal|binary|dyadic|powers|hexadecimal

Parameters:

- *notation value* represents a variable of type decimal|binary|dyadic|powers|hexadecimal

Description:

- An assignment **display** = *notation value*, where *notation value* is one of **decimal**, **dyadic**, **powers**, **binary** or **hexadecimal**, activates the corresponding notation for output of values in **print**, **write** or at the `Sollya` prompt.

  If the global notation variable **display** is **decimal**, all numbers will be output in scientific decimal notation. If the global notation variable **display** is **dyadic**, all numbers will be output as dyadic numbers with Gappa notation. If the global notation variable **display** is **powers**, all numbers will be output as dyadic numbers with a notation compatible with Maple and PARI/GP. If the global notation variable **display** is **binary**, all numbers will be output in binary notation. If the global notation variable **display** is **hexadecimal**, all numbers will be output in C99/ IEEE754-2008 notation. All output notations can be parsed back by `Sollya`, inducing no error if the input and output precisions are the same (see **prec**).

  If the assignment **display** = *notation value* is followed by an exclamation mark, no message indicating the new state is displayed. Otherwise the user is informed of the new state of the global mode by an indication.

Example 1:

```
> display = decimal;
Display mode is decimal numbers.
> a = evaluate(sin(pi * x), 0.25);
> a;
0.707106781186547524400844362104849039284835937688 47
> display = binary;
Display mode is binary numbers.
> a;
1.011010100000100111100110011001111111001110111100110010010000100010110010111110
110001001101100110111010101001010101111101001111100011101011011110110000010111101
010001_2 * 2^(-1)
> display = hexadecimal;
Display mode is hexadecimal numbers.
> a;
0x1.6a09e667f3bcc908b2fb1366ea957d3e3adec1751p-1
> display = dyadic;
Display mode is dyadic numbers.
> a;
330700069911015586133239834882209443600671071332 65b-165
> display = powers;
Display mode is dyadic numbers in integer-power-of-2 notation.
> a;
330700069911015586133239834882209443600671071332 65 * 2^(-165)
```

See also: **print** (8.141), **write** (8.201), **decimal** (8.36), **dyadic** (8.53), **powers** (8.137), **binary** (8.20), **hexadecimal** (8.84), **prec** (8.138)

8.48 div

Name: **div**

Computes the euclidian division of polynomials or numbers and returns the quotient

Library name:

```
sollya_obj_t sollya_lib_euclidian_div(sollya_obj_t, sollya_obj_t)
```

Usage:

$$\textbf{div}(a,\ b)\ :\ (\mathsf{function},\ \mathsf{function}) \rightarrow \mathsf{function}$$

Parameters:

- a is a constant or a polynomial.
- b is a constant or a polynomial.

Description:

- When both a and b are constants, **div**(a,b) computes **floor**(a / b). In other words, it returns the quotient of the Euclidian division of a by b.
- When both a and b are polynomials with at least one being non-constant, **div**(a,b) computes a polynomial q such that the polynomial r equal to $a - qb$ is of degree strictly smaller than the degree of b (see exception below). In order to recover r, use the **mod** command.
- **div** works on polynomials whose coefficients are constant expressions that cannot be simplified (by the tool) to rational numbers. In most cases, the tool is able to perform the Euclidian polynomial division for such polynomials and stop the Euclidian division algorithm only when r is of degree strictly smaller than the degree of b. In certain cases, when the polynomials involve coefficients given as constant expressions that are mathematically zero but for which the tool is unable to detect this fact, the tool may be unable to correctly determine that r is actually of degree stricly

smaller than the degree of b. The issue arises in particular for polynomials whose leading coefficient is a constant expression which is zero without the tool being able to detect this. In these cases, **div**, together with **mod**, just guarantee that q and r, as returned by the two commands, satisfy that r added to the product of q and b yields a, and that r is of the smallest degree the tool can admit. However, there might exist another pair of a quotient and remainder polynomial for which the remainder polynomial is of a degree less than the one of r.

- When at least one of a or b is a function that is no polynomial, **div**(a,b) returns 0.

Example 1:

```
> div(1001, 231);
4
> div(13, 17);
0
> div(-14, 15);
-1
> div(-213, -5);
42
> div(23/13, 11/17);
2
> div(exp(13),-sin(17));
460177
```

Example 2:

```
> div(24 + 68 * x + 74 * x^2 + 39 * x^3 + 10 * x^4 + x^5, 4 + 4 * x + x^2);
6 + x * (11 + x * (6 + x))
> div(24 + 68 * x + 74 * x^2 + 39 * x^3 + 10 * x^4 + x^5, 2 * x^3);
19.5 + x * (5 + x * 0.5)
> div(x^2, x^3);
0
```

Example 3:

```
> div(exp(x), x^2);
0
```

See also: **gcd** (8.76), **mod** (8.115), **bezout** (8.19), **numberroots** (8.121)

8.49 /

Name: /
division function

Library names:

```
sollya_obj_t sollya_lib_div(sollya_obj_t, sollya_obj_t)
sollya_obj_t sollya_lib_build_function_div(sollya_obj_t, sollya_obj_t)
#define SOLLYA_DIV(x,y) sollya_lib_build_function_div((x), (y))
```

Usage:

function1 / *function2* : (function, function) → function
interval1 / *interval2* : (range, range) → range
interval1 / *constant* : (range, constant) → range
interval1 / *constant* : (constant, range) → range

Parameters:

- *function1* and *function2* represent functions

- *interval1* and *interval2* represent intervals (ranges)
- *constant* represents a constant or constant expression

Description:

- / represents the division (function) on reals. The expression *function1* / *function2* stands for the function composed of the division function and the two functions *function1* and *function2*, where *function1* is the numerator and *function2* the denominator.
- / can be used for interval arithmetic on intervals (ranges). / will evaluate to an interval that safely encompasses all images of the division function with arguments varying in the given intervals. If the intervals given contain points where the division function is not defined, infinities and NaNs will be produced in the output interval. Any combination of intervals with intervals or constants (resp. constant expressions) is supported. However, it is not possible to represent families of functions using an interval as one argument and a function (varying in the free variable) as the other one.

Example 1:

```
> 5 / 2;
2.5
```

Example 2:

```
> x / 2;
x * 0.5
```

Example 3:

```
> x / x;
1
```

Example 4:

```
> 3 / 0;
NaN
```

Example 5:

```
> diff(sin(x) / exp(x));
(exp(x) * cos(x) - sin(x) * exp(x)) / exp(x)^2
```

Example 6:

```
> [1;2] / [3;4];
[0.25;0.66666666666666666666666666666666666666666666666668]
> [1;2] / 17;
[5.88235294117647058823529411764705882352941176470588235294117647059e-2;0.11764705882352941176470
588235294117647058823529412]
> -13 / [4;17];
[-3.25;-0.76470588235294117647058823529411764705882352941175]
```

See also: + (8.132), − (8.114), ∗ (8.116), ^ (8.136)

8.50 double

Names: **double**, **D**
rounding to the nearest IEEE 754 double (binary64).

Library names:
```
sollya_obj_t sollya_lib_double(sollya_obj_t)
sollya_obj_t sollya_lib_double_obj()
int sollya_lib_is_double_obj(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_double(sollya_obj_t)
#define SOLLYA_D(x) sollya_lib_build_function_double(x)
```

Description:

- **double** is both a function and a constant.
- As a function, it rounds its argument to the nearest IEEE 754 double precision (i.e. IEEE754-2008 binary64) number. Subnormal numbers are supported as well as standard numbers: it is the real rounding described in the standard.
- As a constant, it symbolizes the double precision format. It is used in contexts when a precision format is necessary, e.g. in the commands **round**, **roundcoefficients** and **implementpoly**. See the corresponding help pages for examples.

Example 1:

```
> display=binary!;
> D(0.1);
1.10011001100110011001100110011001100110011001100110011001101_2 * 2^(-4)
> D(4.17);
1.00001010111000010100011110101110000101000111101011 1_2 * 2^(2)
> D(1.011_2 * 2^(-1073));
1.1_2 * 2^(-1073)
```

See also: **halfprecision** (8.82), **single** (8.175), **doubleextended** (8.52), **doubledouble** (8.51), **quad** (8.149), **tripledouble** (8.194), **roundcoefficients** (8.165), **implementpoly** (8.90), **round** (8.164), **printdouble** (8.142)

8.51 doubledouble

Names: **doubledouble**, **DD**
represents a number as the sum of two IEEE doubles.

Library names:
```
sollya_obj_t sollya_lib_double_double(sollya_obj_t)
sollya_obj_t sollya_lib_double_double_obj()
int sollya_lib_is_double_double_obj(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_double_double(sollya_obj_t)
#define SOLLYA_DD(x) sollya_lib_build_function_double_double(x)
```

Description:

- **doubledouble** is both a function and a constant.
- As a function, it rounds its argument to the nearest number that can be written as the sum of two double precision numbers.
- The algorithm used to compute **doubledouble**(x) is the following: let x_h = **double**(x) and let x_l = **double**($x - x_h$). Return the number $x_h + x_l$. Note that if the current precision is not sufficient to exactly represent $x_h + x_l$, a rounding will occur and the result of **doubledouble**(x) will be useless.

- As a constant, it symbolizes the double-double precision format. It is used in contexts when a precision format is necessary, e.g. in the commands **round**, **roundcoefficients** and **implementpoly**. See the corresponding help pages for examples.

Example 1:

```
> verbosity=1!;
> a = 1+ 2^(-100);
> DD(a);
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
1.00000000000000000000000000000078886090522101180541172856528278622967320643510
> prec=50!;
> DD(a);
1.00000000000000000000000000000078886090522101180541172856528278622967320643510
023004770278930664062 5
```

See also: **halfprecision** (8.82), **single** (8.175), **double** (8.50), **doubleextended** (8.52), **quad** (8.149), **tripledouble** (8.194), **roundcoefficients** (8.165), **implementpoly** (8.90), **round** (8.164)

8.52 doubleextended

Names: **doubleextended**, **DE**

computes the nearest number with 64 bits of mantissa.

Library names:

```
sollya_obj_t sollya_lib_doubleextended(sollya_obj_t)
sollya_obj_t sollya_lib_doubleextended_obj()
int sollya_lib_is_doubleextended_obj(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_doubleextended(sollya_obj_t)
#define SOLLYA_DE(x) sollya_lib_build_function_doubleextended(x)
```

Description:

- **doubleextended** is a function that computes the nearest floating-point number with 64 bits of mantissa to a given number. Since it is a function, it can be composed with other `Sollya` functions such as **exp**, **sin**, etc.
- **doubleextended** now does handle subnormal numbers for a presumed exponent width of the double-extended format of 15 bits. This means, with respect to rounding, **doubleextended** behaves as a IEEE 754-2008 binary79 with a 64 bit significand (with a hidden bit normal range), one sign bit and a 15 bit exponent field would behave. This behavior may be different from the one observed on Intel-based IA32/Intel64 processors (or compatible versions from other vendors). However it is the one seen on HP/Intel Itanium when the precision specifier is double-extended and pseudo-denormals are activated.
- Since it is a function and not a command, its behavior is a bit different from the behavior of **round**(x,64,RN) even if the result is exactly the same. **round**(x,64,RN) is immediately evaluated whereas **doubleextended**(x) can be composed with other functions (and thus be plotted and so on).

Example 1:

```
> display=binary!;
> DE(0.1);
1.10011001100110011001100110011001100110011001100110011001100110011001101_2 * 2^(-4)
> round(0.1,64,RN);
1.10011001100110011001100110011001100110011001100110011001100110011001101_2 * 2^(-4)
```

Example 2:

```
> D(2^(-2000));
0
> DE(2^(-20000));
0
```

Example 3:

```
> verbosity=1!;
> f = sin(DE(x));
> f(pi);
Warning: rounding has happened. The value displayed is a faithful rounding to 16
5 bits of the true result.
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
-5.0165576126683320235573270803307570138315616702 55e-20
> g = sin(round(x,64,RN));
Warning: at least one of the given expressions or a subexpression is not correct
ly typed
or its evaluation has failed because of some error on a side-effect.
```

See also: **roundcoefficients** (8.165), **halfprecision** (8.82), **single** (8.175), **double** (8.50), **doubledouble** (8.51), **quad** (8.149), **tripledouble** (8.194), **round** (8.164)

8.53 dyadic

Name: **dyadic**
special value for global state **display**

Library names:
`sollya_obj_t sollya_lib_dyadic()`
`int sollya_lib_is_dyadic(sollya_obj_t)`

Description:

- **dyadic** is a special value used for the global state **display**. If the global state **display** is equal to **dyadic**, all data will be output in dyadic notation with numbers displayed in Gappa format.

  As any value it can be affected to a variable and stored in lists.

See also: **decimal** (8.36), **powers** (8.137), **hexadecimal** (8.84), **binary** (8.20), **display** (8.47)

8.54 ==

Name: ==
equality test operator

Library name:
`sollya_obj_t sollya_lib_cmp_equal(sollya_obj_t, sollya_obj_t)`

Usage:

$$expr1 == expr2 : (\text{any type}, \text{any type}) \rightarrow \text{boolean}$$

Parameters:

- *expr1* and *expr2* represent expressions

Description:

- The test *expr1* == *expr2* returns **true** when *expr1* and *expr2* are syntactically equal and different from **error**, @NaN@ and [@NaN@, @NaN]. Conversely if *expr1* and *expr2* are objects that are mathematically different and `Sollya` manages to figure it out, the test returns **false**. In between

these two cases, there is the grey zone of expressions that are not syntactically equal but are mathematically equal. In such a case, `Sollya` normally tries to determine if the expressions are mathematically equal and if it manages to prove it, it returns **true**, without a warning. In the case when *expr1* and *expr2* are two constant expressions, `Sollya` will in particular try to evaluate their difference: in the case when the difference is 0 or is so small that `Sollya` does not manage to obtain a faithful rounding of the real value, it will return **true** (with a warning if it has not been possible to actually prove that the real value is 0). In any other case, when both expressions are not syntactically equal and `Sollya` has not been able to prove that they are mathematically equal, it returns **false**.

- The level of simplifications performed by `Sollya` to determine if expressions are mathematically equal depends on the value of **autosimplify**. If it is **off**, no formal simplification is performed, hence expression trees as simple as x+1 and 1+x will be considered not equal. Conversely, if **autosimplify** is set to **on**, polynomial subexpressions that are mathematically equal will in general be recognized as being equal.
- The user should always keep in mind that a litteral constant written in decimal arithmetic (such as 0.1 for instance) is not considered as an exact constant by `Sollya` (unless it is exactly representable in binary without requiring too much precision) and is first correctly rounded at precision **prec**, prior to any other operation. Of course, this leads to a rounding warning, but it is important to remember that this is done before the expression trees are compared, possibly leading to two expressions comparing equal, while they are obviously mathematically different, just because they contain different constants that have been rounded to the same value at precision **prec**. As a general rule, to avoid this behavior, the user should represent constants in an exact format such as hexadecimal or represent decimal constants as integer fractions (e.g., 0.1 represented by the constant expression 1/10).
- Notice that @NaN@, [@NaN, @NaN@] and **error** share the property that they compare not equal to anything, including themselves. This means if a variable *a* contains @NaN@, [@NaN, @NaN@] or **error** and whatever the content of variable *b* is, the test *a* == *b* returns **false**. The traditional way of testing if *a* contains @NaN@, [@NaN@, @NaN@] or **error** is indeed to check if *a* == *a* returns false. **error** can be distinguished from @NaN@ and [@NaN@, @NaN@] using the **!=** operator. In order to distinguish @NaN@ from [@NaN@, @NaN@], a match ... with ... construct must be used. In general, match ... with ... constructs work on all of @NaN@, [@NaN, @NaN@] and **error** and are able to distinguish between all of these.

Example 1:

```
> "Hello" == "Hello";
true
> "Hello" == "Salut";
false
> "Hello" == 5;
false
> 5 + x == 5 + x;
true
```

Example 2:

```
> verbosity = 1!;
> asin(1) * 2 == pi;
true
> cos(3)^2 == 1 - sin(3)^2;
Warning: the tool is unable to decide an equality test by evaluation even though
 faithful evaluation of the terms has been possible. The terms will be considere
d to be equal.
true
> exp(5) == log(4);
false
```

Example 3:

```
> autosimplify=off;
Automatic pure tree simplification has been deactivated.
> exp(1+x) == exp(x+1);
false
> autosimplify=on;
Automatic pure tree simplification has been activated.
> exp(1+x) == exp(x+1);
false
> (1/3+x)^2 == x^2 + 1/9 + (5-3)*x/3;
true
> log(x)/log(10) == log10(x);
false
```

Example 4:

```
> prec = 12;
The precision has been set to 12 bits.
> verbosity = 1!;
> 16384.1 == 16385.1;
Warning: Rounding occurred when converting the constant "16384.1" to floating-po
int with 12 bits.
If safe computation is needed, try to increase the precision.
Warning: Rounding occurred when converting the constant "16385.1" to floating-po
int with 12 bits.
If safe computation is needed, try to increase the precision.
true
> 16384 == 16384.25;
false
> 0.1 == 1/10;
Warning: Rounding occurred when converting the constant "0.1" to floating-point
with 12 bits.
If safe computation is needed, try to increase the precision.
false
> 0.1 == round(1/10, prec, RN);
Warning: Rounding occurred when converting the constant "0.1" to floating-point
with 12 bits.
If safe computation is needed, try to increase the precision.
true
```

Example 5:

```
> error == error;
false
> error != error;
false
> @NaN@ == @NaN@;
false
> @NaN@ != @NaN@;
true
> [@NaN@,@NaN@] == [@NaN@,@NaN@];
false
> [@NaN@,@NaN@] != [@NaN@,@NaN@];
true
> error == @NaN@;
false
> error != @NaN@;
false
> a = error;
> match a with
   @NaN@ : ("a contains @NaN@")
   [@NaN@, @NaN@] : ("a contains [@NaN@, @NaN@]")
   default:("a contains something else");
a contains something else
> a = @NaN@;
> match a with
   @NaN@ : ("a contains @NaN@")
   [@NaN@, @NaN@] : ("a contains [@NaN@, @NaN@]")
   default:("a contains something else");
a contains @NaN@
> a = [@NaN@, @NaN@];
> match a with
   @NaN@ : ("a contains @NaN@")
   [@NaN@, @NaN@] : ("a contains [@NaN@, @NaN@]")
   default:("a contains something else");
a contains [@NaN@, @NaN@]
```

See also: **!=** (8.118), > (8.80), >= (8.77), <= (8.99), < (8.108), **in** (8.91), **!** (8.120), && (8.6), || (8.127), **error** (8.57), **prec** (8.138), **autosimplify** (8.16)

8.55 erf

Name: **erf**
the error function.

Library names:
```
sollya_obj_t sollya_lib_erf(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_erf(sollya_obj_t)
#define SOLLYA_ERF(x) sollya_lib_build_function_erf(x)
```

Description:

- **erf** is the error function defined by:

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} \mathrm{d}t.$$

- It is defined for every real number x.

See also: **erfc** (8.56), **exp** (8.60)

8.56 erfc

Name: **erfc**
the complementary error function.

Library names:
```
sollya_obj_t sollya_lib_erfc(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_erfc(sollya_obj_t)
#define SOLLYA_ERFC(x) sollya_lib_build_function_erfc(x)
```

Description:

- **erfc** is the complementary error function defined by $\mathrm{erfc}(x) = 1 - \mathrm{erf}(x)$.
- It is defined for every real number x.

See also: **erf** (8.55)

8.57 error

Name: **error**
expression representing an input that is wrongly typed or that cannot be executed

Library names:
```
sollya_obj_t sollya_lib_error()
int sollya_lib_obj_is_error(sollya_obj_t)
```

Usage:

error : error

Description:

- The variable **error** represents an input during the evaluation of which a type or execution error has been detected or is to be detected. Inputs that are syntactically correct but wrongly typed evaluate to **error** at some stage. Inputs that are correctly typed but containing commands that depend on side-effects that cannot be performed or inputs that are wrongly typed at meta-level (cf. **parse**), evaluate to **error**.

  Remark that in contrast to all other elements of the `Sollya` language, **error** compares neither equal nor unequal to itself. This provides a means of detecting syntax errors inside the `Sollya` language itself without introducing issues of two different wrongly typed inputs being equal.

Example 1:

```
> print(5 + "foo");
error
```

Example 2:

```
> error;
error
```

Example 3:

```
> error == error;
false
> error != error;
false
```

Example 4:

```
> correct = 5 + 6;
> incorrect = 5 + "foo";
> correct == correct;
true
> incorrect == incorrect;
false
> errorhappened = !(incorrect == incorrect);
> errorhappened;
true
```

See also: **void** (8.199), **parse** (8.128), == (8.54), **!**= (8.118)

8.58 evaluate

Name: **evaluate**
evaluates a function at a constant point or in a range

Library name:
`sollya_obj_t sollya_lib_evaluate(sollya_obj_t, sollya_obj_t)`

Usage:

evaluate(*function*, *constant*) : (function, constant) → constant | range
evaluate(*function*, *range*) : (function, range) → range
evaluate(*function*, *function2*) : (function, function) → function

Parameters:

- *function* represents a function
- *constant* represents a constant point
- *range* represents a range
- *function2* represents a function that is not constant

Description:

- If its second argument is a constant *constant*, **evaluate** evaluates its first argument *function* at the point indicated by *constant*. This evaluation is performed in a way that the result is a faithful rounding of the real value of the *function* at *constant* to the current global precision. If such a faithful rounding is not possible, **evaluate** returns a range surely encompassing the real value of the function *function* at *constant*. If even interval evaluation is not possible because the expression is undefined or numerically unstable, NaN will be produced.
- If its second argument is a range *range*, **evaluate** evaluates its first argument *function* by interval evaluation on this range *range*. This ensures that the image domain of the function *function* on the preimage domain *range* is surely enclosed in the returned range.
- In the case when the second argument is a range that is reduced to a single point (such that [1; 1] for instance), the evaluation is performed in the same way as when the second argument is a constant but it produces a range as a result: **evaluate** automatically adjusts the precision of the intern computations and returns a range that contains at most three floating-point consecutive numbers in precision **prec**. This corresponds to the same accuracy as a faithful rounding of the actual result. If such a faithful rounding is not possible, **evaluate** has the same behavior as in the case when the second argument is a constant.
- If its second argument is a function *function2* that is not a constant, **evaluate** replaces all occurrences of the free variable in function *function* by function *function2*.

Example 1:

```
> midpointmode=on!;
> print(evaluate(sin(pi * x), 2.25));
0.70710678118654752440084436210484903928483593768847
> print(evaluate(sin(pi * x), [2.25; 2.25]));
0.70710678118654752440084436210484903928483593768847~4/5~
```

Example 2:

```
> print(evaluate(sin(pi * x), 2));
[-3.10036576513989761974912188739078952385417059655 8e-13490;5.300240158585712760
535084242602922324150077630252 8e-13489]
```

Example 3:

```
> print(evaluate(sin(pi * x), [2, 2.25]));
[-5.14339027267725463004699891996191240734922416542 1e-50;0.707106781186547524400
844362104849039284835937688 66]
```

Example 4:

```
> print(evaluate(sin(pi * x), 2 + 0.25 * x));
sin((pi) * 2 + x * (pi) * 0.25)
```

Example 5:

```
> print(evaluate(sin(pi * 1/x), 0));
[-1;1]
```

See also: **isevaluable** (8.98)

8.59 execute

Name: **execute**
executes the content of a file

Library name:
`void sollya_lib_execute(sollya_obj_t)`

Usage:

execute(*filename*) : string → void

Parameters:

- *filename* is a string representing a file name

Description:

- **execute** opens the file indicated by *filename*, and executes the sequence of commands it contains. This command is evaluated at execution time: this way you can modify the file *filename* (for instance using **bashexecute**) and execute it just after.
- If *filename* contains a command **execute**, it will be executed recursively.
- If *filename* contains a call to **restart**, it will be neglected.
- If *filename* contains a call to **quit**, the commands following **quit** in *filename* will be neglected.

Example 1:

```
> a=2;
> a;
2
> print("a=1;") > "example.sollya";
> execute("example.sollya");
> a;
1
```

Example 2:

```
> verbosity=1!;
> print("a=1; restart; a=2;") > "example.sollya";
> execute("example.sollya");
Warning: a restart command has been used in a file read into another.
This restart command will be neglected.
> a;
2
```

Example 3:

```
> verbosity=1!;
> print("a=1; quit; a=2;") > "example.sollya";
> execute("example.sollya");
Warning: the execution of a file read by execute demanded stopping the interpret
ation but it is not stopped.
> a;
1
```

See also: **parse** (8.128), **readfile** (8.155), **write** (8.201), **print** (8.141), **bashexecute** (8.18), **quit** (8.150), **restart** (8.160)

8.60 exp

Name: **exp**
the exponential function.

Library names:
`sollya_obj_t sollya_lib_exp(sollya_obj_t)`
`sollya_obj_t sollya_lib_build_function_exp(sollya_obj_t)`
`#define SOLLYA_EXP(x) sollya_lib_build_function_exp(x)`

Description:

- **exp** is the usual exponential function defined as the solution of the ordinary differential equation $y' = y$ with $y(0) = 1$.
- **exp**(x) is defined for every real number x.

See also: **exp** (8.60), **log** (8.104)

8.61 expand

Name: **expand**
expands polynomial subexpressions

Library name:
`sollya_obj_t sollya_lib_expand(sollya_obj_t)`

Usage:

expand(*function*) : function → function

Parameters:

- *function* represents a function

Description:

- **expand**(*function*) expands all polynomial subexpressions in function *function* as far as possible. Factors of sums are multiplied out, power operators with constant positive integer exponents are replaced by multiplications.

Example 1:

```
> print(expand(x^3));
x * x * x
```

Example 2:

```
> print(expand((x + 2)^3 + 2 * x));
8 + 12 * x + 6 * x * x + x * x * x + 2 * x
```

Example 3:

```
> print(expand(exp((x + (x + 3))^5)));
exp(243 + 405 * x + 270 * x * x + 90 * x * x * x + 15 * x * x * x * x + x * x *
x * x * x + x * 405 + 108 * x * 5 * x + 54 * x * x * 5 * x + 12 * x * x * x * 5
* x + x * x * x * x * 5 * x + x * x * 270 + 27 * x * x * x * 10 + 9 * x * x * x
* x * 10 + x * x * x * x * x * 10 + x * x * x * x * 90 + 6 * x * x * x * x * 10 + x
* x * x * x * x * 10 + x * x * x * x * 5 * x + 15 * x * x * x * x + x * x * x *
x * x)
```

See also: **dirtysimplify** (8.46), **simplify** (8.173), **horner** (8.87), **coeff** (8.27), **degree** (8.38)

8.62 expm1

Name: **expm1**
shifted exponential function.

Library names:
`sollya_obj_t sollya_lib_expm1(sollya_obj_t)`
`sollya_obj_t sollya_lib_build_function_expm1(sollya_obj_t)`
`#define SOLLYA_EXPM1(x) sollya_lib_build_function_expm1(x)`

Description:

- **expm1** is defined by $\mathrm{expm1}(x) = \exp(x) - 1$.
- It is defined for every real number x.

See also: **exp** (8.60)

8.63 exponent

Name: **exponent**
returns the scaled binary exponent of a number.

Library name:
`sollya_obj_t sollya_lib_exponent(sollya_obj_t)`

Usage:

exponent(x) : constant → integer

Parameters:

- x is a dyadic number.

Description:

- **exponent**(x) is by definition 0 if x is one of 0, NaN, or Inf.
- If x is not zero, it can be uniquely written as $x = m \cdot 2^e$ where m is an odd integer and e is an integer. **exponent**(x) returns e.

Example 1:

```
> a=round(Pi,20,RN);
> e=exponent(a);
> e;
-17
> m=mantissa(a);
> a-m*2^e;
0
```

See also: **mantissa** (8.109), **precision** (8.139)

8.64 externaldata

Name: **externaldata**
binds an external data symbol to a `Sollya` identifier

Library name:
`sollya_obj_t sollya_lib_externaldata(char *, void *, void (*)(void *));`

Usage:

externaldata(*identifier*, *filename*) : (identifier type, string) → void

Parameters:

- *identifier* represents the identifier the external symbol is to be bound to
- *filename* of type string represents the name of the object file where the external symbol can be found

Description:

- **externaldata** allows for binding the `Sollya` identifier *identifier* to an external symbol, looked up in a dynamically linked library. After this binding, `Sollya` can retrieve the bound object in its own symbol table, can copy it, can assign it to other variables, pass is as arguments to procedures and externalprocedures, compare it etc. However, `Sollya` stays agnostic of the symbol's meaning and type. Due to this reason, the use of **externaldata** makes no sense but in conjuction with the use of **externalproc**, where an external procedure is bound to `Sollya`. This external procedure may know the type and meaning of the object bound with **externaldata** and display it, handle it, execute it etc. The same functionality may also be used for tools that use `Sollya` thru its library interface to bind their own objects to `Sollya`, pass them thru `Sollya` code and retrieve them eventually.

Example 1:

```
> bashexecute("gcc -fPIC -Wall -I.. -I. -c externaldataexample.c");
> bashexecute("gcc -fPIC -shared -o externaldataexample externaldataexample.o");

> externaldata(magic, "./externaldataexample");
> magic;
magic
> print(magic);
magic
> magic == magic;
true
> magic == 17;
false
> zauberei = magic;
> zauberei;
magic
> zauberei == magic;
true
```

Example 2:

```
> bashexecute("gcc -fPIC -Wall -I.. -I. -c externaldataexample.c");
> bashexecute("gcc -fPIC -shared -o externaldataexample externaldataexample.o");

> externaldata(magic, "./externaldataexample");
> externalproc(printMagic, "./externaldataexample", object -> void);
> zauberei = magic;
> printMagic;
printMagic
> printMagic(magic);
The magic is 17
> printMagic(magic);
The magic is 18
> printMagic(zauberei);
The magic is 19
> procedure magicAsParameter(m) {
        m;
        printMagic(m);
        m;
  };
> magicAsParameter(magic);
magic
The magic is 20
magic
> magicAsParameter(zauberei);
magic
The magic is 21
magic
```

See also: **library** (8.101), **libraryconstant** (8.102), **externalproc** (8.66), **externalplot** (8.65), **bashexecute** (8.18), **void** (8.199), **constant** (8.30), **function** (8.75), **range** (8.151), **integer** (8.94), **string** (8.179), **boolean** (8.22), **list of** (8.103), **object** (8.123)

8.65 externalplot

Name: **externalplot**

plots the error of an external code with regard to a function

Library names:

```
void sollya_lib_externalplot(sollya_obj_t, sollya_obj_t, sollya_obj_t,
                             sollya_obj_t, sollya_obj_t, ...)
void sollya_lib_v_externalplot(sollya_obj_t, sollya_obj_t, sollya_obj_t,
                               sollya_obj_t, sollya_obj_t, va_list)
```

Usage:

externalplot(*filename*, *mode*, *function*, *range*, *precision*) : (string, absolute|relative, function, range, integer) → void

externalplot(*filename*, *mode*, *function*, *range*, *precision*, *perturb*) : (string, absolute|relative, function, range, integer, perturb) → void

externalplot(*filename*, *mode*, *function*, *range*, *precision*, *plot mode*, *result filename*) : (string, absolute|relative, function, range, integer, file|postscript|postscriptfile, string) → void

externalplot(*filename*, *mode*, *function*, *range*, *precision*, *perturb*, *plot mode*, *result filename*) : (string, absolute|relative, function, range, integer, perturb, file|postscript|postscriptfile, string) → void

Description:

- The command **externalplot** plots the error of an external function evaluation code sequence implemented in the object file named *filename* with regard to the function *function*. If *mode* evaluates to *absolute*, the difference of both functions is considered as an error function; if *mode* evaluates to *relative*, the difference is divided by the function *function*. The resulting error function is plotted on all floating-point numbers with *precision* significant mantissa bits in the range *range*.

  If the sixth argument of the command **externalplot** is given and evaluates to **perturb**, each of the floating-point numbers the function is evaluated at gets perturbed by a random value that is uniformly distributed in ±1 ulp around the original *precision* bit floating-point variable.

  If a sixth and seventh argument, respectively a seventh and eighth argument in the presence of **perturb** as a sixth argument, are given that evaluate to a variable of type file|postscript|postscriptfile respectively to a character sequence of type string, **externalplot** will plot (additionally) to a file in the same way as the command **plot** does. See **plot** for details.

  The external function evaluation code given in the object file name *filename* is supposed to define a function name `f` as follows (here in C syntax): `void f(mpfr_t rop, mpfr_t op)`. This function is supposed to evaluate `op` with an accuracy corresponding to the precision of `rop` and assign this value to `rop`.

Example 1:

```
> bashexecute("gcc -fPIC -c externalplotexample.c");
> bashexecute("gcc -shared -o externalplotexample externalplotexample.o -lgmp -l
mpfr");
> externalplot("./externalplotexample",relative,exp(x),[-1/2;1/2],12,perturb);
```

See also: **plot** (8.131), **asciiplot** (8.10), **perturb** (8.129), **absolute** (8.2), **relative** (8.157), **file** (8.68), **postscript** (8.134), **postscriptfile** (8.135), **bashexecute** (8.18), **externalproc** (8.66), **library** (8.101)

8.66 externalproc

Name: **externalproc**
binds an external code to a `Sollya` procedure

Library names:

```
sollya_obj_t sollya_lib_externalprocedure(sollya_externalprocedure_type_t,
                                          sollya_externalprocedure_type_t *,
                                          int, char *, void *);
sollya_obj_t sollya_lib_externalprocedure_with_data(
                                          sollya_externalprocedure_type_t,
                                          sollya_externalprocedure_type_t *,
```

```
int, char *, void *, void *,
void (*)(void *));
```

Usage:

externalproc(*identifier*, *filename*, *argumenttype* `->` *resulttype*) : (identifier type, string, type type, type type) → void

Parameters:

- *identifier* represents the identifier the code is to be bound to
- *filename* of type string represents the name of the object file where the code of procedure can be found
- *argumenttype* represents a definition of the types of the arguments of the `Sollya` procedure and the external code
- *resulttype* represents a definition of the result type of the external code

Description:

- **externalproc** allows for binding the `Sollya` identifier *identifier* to an external code. After this binding, when `Sollya` encounters *identifier* applied to a list of actual parameters, it will evaluate these parameters and call the external code with these parameters. If the external code indicated success, it will receive the result produced by the external code, transform it to `Sollya`'s internal representation and return it.

  In order to allow correct evaluation and typing of the data in parameter and in result to be passed to and received from the external code, **externalproc** has a third parameter *argumenttype* `->` *resulttype*. Both *argumenttype* and *resulttype* are one of **void**, **constant**, **function**, **object**, **range**, **integer**, **string**, **boolean**, **list of constant**, **list of function**, **list of object**, **list of range**, **list of integer**, **list of string**, **list of boolean**.

  It is worth mentioning that the difference between the data and result type **function** and the type **object** is minimal and due to support of legacy `Sollya` code. Both `Sollya` functions and `Sollya` objects are transferred from and to the external procedure thru the C type `sollya_obj_t`. The difference is that `Sollya` will check that a certain object is a mathematical function when **function** is used as a type, and will skip this test if the **object** type is used. Similarly, `Sollya` relies on an object produced by the external procedure to be a mathematical function when **function** is used and will not make this assumption for **object**.

  If upon a usage of a procedure bound to an external procedure the type of the actual parameters given or its number is not correct, `Sollya` produces a type error. An external function not applied to arguments represents itself and prints out with its argument and result types.

  The external function is supposed to return an integer indicating success. It returns its result depending on its `Sollya` result type as follows. Here, the external procedure is assumed to be implemented as a C function.

  - If the `Sollya` result type is void, the C function has no pointer argument for the result.
  - If the `Sollya` result type is **constant**, the first argument of the C function is of C type `mpfr_t *`, the result is returned by affecting the MPFR variable.
  - If the `Sollya` result type is **function**, the first argument of the C function is of C type `sollya_obj_t *`, the result is returned by affecting the `sollya_obj_t` variable.
  - If the `Sollya` result type is **object**, the first argument of the C function is of C type `sollya_obj_t *`, the result is returned by affecting the `sollya_obj_t` variable.
  - If the `Sollya` result type is **range**, the first argument of the C function is of C type `mpfi_t *`, the result is returned by affecting the MPFI variable.
  - If the `Sollya` result type is **integer**, the first argument of the C function is of C type `int *`, the result is returned by affecting the int variable.

- If the `Sollya` result type is **string**, the first argument of the C function is of C type `char **`, the result is returned by the `char *` pointed with a new `char *`.
- If the `Sollya` result type is **boolean**, the first argument of the C function is of C type `int *`, the result is returned by affecting the int variable with a boolean value.
- If the `Sollya` result type is **list of** type, the first argument of the C function is of a C type depending on the `Sollya` return type:
  * For a list of **constant**: `sollya_constant_list_t *`
  * For a list of **function**: `sollya_obj_list_t *`
  * For a list of **object**: `sollya_obj_list_t *`
  * For a list of **range**: `sollya_constant_list_t *`
  * For a list of **integer**: `sollya_int_list_t *`
  * For a list of **string**: `sollya_string_list_t *`
  * For a list of **boolean**: `sollya_boolean_list_t *`

The external procedure affects its possible pointer argument if and only if it succeeds. This means, if the function returns an integer indicating failure, it does not leak any memory to the encompassing environment.

The external procedure receives its arguments as follows: If the `Sollya` argument type is **void**, no argument array is given. Otherwise the C function receives a C `void **` argument representing an array of size equal to the arity of the function where each entry (of C type `void *`) represents a value with a C type depending on the corresponding `Sollya` type.

- If the `Sollya` type is **constant**, the `void *` is to be cast to `mpfr_t *`.
- If the `Sollya` type is **function**, the `void *` is to be cast to `sollya_obj_t`.
- If the `Sollya` type is **object**, the `void *` is to be cast to `sollya_obj_t`.
- If the `Sollya` type is **range**, the `void *` is to be cast to `mpfi_t *`.
- If the `Sollya` type is **integer**, the `void *` is to be cast to `int *`.
- If the `Sollya` type is **string**, the `void *` is to be cast to `char *`.
- If the `Sollya` type is **boolean**, the `void *` is to be cast to `int *`.
- If the `Sollya` type is **list of** type, the `void *` is to be cast to a list of a type depending on the type of the list argument:
  * For a list of **constant**: `sollya_constant_list_t`
  * For a list of **function**: `sollya_obj_list_t`
  * For a list of **object**: `sollya_obj_list_t`
  * For a list of **range**: `sollya_interval_list_t`
  * For a list of **integer**: `sollya_int_list_t`
  * For a list of **string**: `sollya_string_list_t`
  * For a list of **boolean**: `sollya_boolean_list_t`

The external procedure is not supposed to alter the memory pointed by its array argument `void **`.

In both directions (argument and result values), empty lists are represented by `NULL` pointers.

Similarly to internal procedures, externally bounded procedures can be considered to be objects inside `Sollya` that can be assigned to other variables, stored in list etc.

- The user should be aware that they may use the `Sollya` library in external codes to be dynamically bound to `Sollya` using **externalproc**. On most systems, it suffices to include the header of the `Sollya` library into the source code of the external procedure. Linking with the actual `Sollya` library is not necessary on most systems; as the interactive `Sollya` executable contains a superset of the `Sollya` library functions. On some systems, linking with the `Sollya` library or some of its dependencies may be necessary.

In particular, the `Sollya` library – and, of course, its header file – contain a certain set of functions to manipulate lists with elements of certain types, such as `sollya_constant_list_t`, `sollya_obj_list_t` and so on. As explained above, these types are passed in argument to (and received back thru a reference from) an external procedure. These list manipulation functions are not strictly necessary to the use of the `Sollya` library in free-standing applications that do not use the functionality provided with **externalproc**. They are therefore provided as-is without any further documentation, besides the comments given in the `Sollya` library header file.

- The dynamic object file whose name is given to **externalproc** for binding of an external procedure may also define a destructor function `int sollya_external_lib_close(void)`. If `Sollya` finds such a destructor function in the dynamic object file, it will call that function when closing the dynamic object file again. This happens when `Sollya` is terminated or when the current `Sollya` session is restarted using **restart**. The purpose of the destructor function is to allow the dynamically bound code to free any memory that it might have allocated before `Sollya` is terminated or restarted.

  The dynamic object file is not necessarily needed to define a destructor function. This ensure backward compatibility with older `Sollya` external library function object files.

  When defined, the destructor function is supposed to return an integer value indicating if an error has happened. Upon success, the destructor functions is to return a zero value, upon error a non-zero value.

Example 1:

```
> bashexecute("gcc -fPIC -Wall -c externalprocexample.c");
> bashexecute("gcc -fPIC -shared -o externalprocexample externalprocexample.o");

> externalproc(foo, "./externalprocexample", (integer, integer) -> integer);
> foo;
foo
> foo(5, 6);
11
> verbosity = 1!;
> foo();
Warning: at least one of the given expressions or a subexpression is not correct
ly typed
or its evaluation has failed because of some error on a side-effect.
error
> a = foo;
> a(5,6);
11
```

See also: **library** (8.101), **libraryconstant** (8.102), **externaldata** (8.64), **externalplot** (8.65), **bashexecute** (8.18), **void** (8.199), **constant** (8.30), **function** (8.75), **range** (8.151), **integer** (8.94), **string** (8.179), **boolean** (8.22), **list of** (8.103), **object** (8.123)

8.67 false

Name: **false**
the boolean value representing the false.

Library names:
`sollya_obj_t sollya_lib_false()`
`int sollya_lib_is_false(sollya_obj_t)`

Description:

- **false** is the usual boolean value.

Example 1:

```
> true && false;
false
> 2<1;
false
```

See also: **true** (8.195), && (8.6), || (8.127)

8.68 file

Name: **file**
special value for commands **plot** and **externalplot**

Library names:
`sollya_obj_t sollya_lib_file()`
`int sollya_lib_is_file(sollya_obj_t)`

Description:

- **file** is a special value used in commands **plot** and **externalplot** to save the result of the command in a data file.
- As any value it can be affected to a variable and stored in lists.

Example 1:

```
> savemode=file;
> name="plotSinCos";
> plot(sin(x),0,cos(x),[-Pi,Pi],savemode, name);
```

See also: **externalplot** (8.65), **plot** (8.131), **postscript** (8.134), **postscriptfile** (8.135)

8.69 findzeros

Name: **findzeros**
gives a list of intervals containing all zeros of a function on an interval.

Library name:
`sollya_obj_t sollya_lib_findzeros(sollya_obj_t, sollya_obj_t)`

Usage:

$$\textbf{findzeros}(f,I) : (\mathsf{function}, \mathsf{range}) \to \mathsf{list}$$

Parameters:

- f is a function.
- I is an interval.

Description:

- **findzeros**(f,I) returns a list of intervals $I_1, \ldots, I_n$ such that, for every zero z of f, there exists some k such that $z \in I_k$.
- The list may contain intervals I_k that do not contain any zero of f. An interval Ik may contain many zeros of f.
- This command is meant for cases when safety is critical. If you want to be sure not to forget any zero, use **findzeros**. However, if you just want to know numerical values for the zeros of f, **dirtyfindzeros** should be quite satisfactory and a lot faster.

- If δ denotes the value of global variable **diam**, the algorithm ensures that for each k, $|I_k| \leq \delta \cdot |I|$.
- The algorithm used is basically a bisection algorithm. It is the same algorithm that the one used for **infnorm**. See the help page of this command for more details. In short, the behavior of the algorithm depends on global variables **prec**, **diam**, **taylorrecursions** and **hopitalrecursions**.

Example 1:

```
> findzeros(sin(x),[-5;5]);
[|[-3.14208984375;-3.140869140625], [-1.220703125e-3;1.220703125e-3], [3.1408691
40625;3.14208984375]|]
> diam=1e-10!;
> findzeros(sin(x),[-5;5]);
[|[-3.14159265370108187198638916015625;-3.14159265253692865371704101562 5], [-1.1
64153218269348144531 25e-9;1.164153218269348144531 25e-9], [3.14159265253692865371
7041015625;3.14159265370108187198638916015625]|]
```

See also: **dirtyfindzeros** (8.43), **infnorm** (8.93), **prec** (8.138), **diam** (8.40), **taylorrecursions** (8.190), **hopitalrecursions** (8.86), **numberroots** (8.121)

8.70 fixed

Name: **fixed**
indicates that fixed-point formats should be used for **fpminimax**

Library names:
`sollya_obj_t sollya_lib_fixed()`
`int sollya_lib_is_fixed(sollya_obj_t)`

Usage:

fixed : fixed|floating

Description:

- The use of **fixed** in the command **fpminimax** indicates that the list of formats given as argument is to be considered to be a list of fixed-point formats. See **fpminimax** for details.

Example 1:

```
> fpminimax(cos(x),6,[|32,32,32,32,32,32,32|],[-1;1],fixed);
0.99999974807724356651306152343 75 + x^2 * (-0.49999286932870745658874511718 75 +
x^2 * (4.16335149202495813369750976562 5e-2 + x^2 * (-1.33822392672300338745117187
5e-3)))
```

See also: **fpminimax** (8.73), **floating** (8.71)

8.71 floating

Name: **floating**
indicates that floating-point formats should be used for **fpminimax**

Library names:
`sollya_obj_t sollya_lib_floating()`
`int sollya_lib_is_floating(sollya_obj_t)`

Usage:

floating : fixed|floating

Description:

- The use of **floating** in the command **fpminimax** indicates that the list of formats given as argument is to be considered to be a list of floating-point formats. See **fpminimax** for details.

Example 1:

```
> fpminimax(cos(x),6,[|D...|],[-1;1],floating);
0.99999974816012215939053930924274027347564697265625 + x * (-2.79593179695850233
44402306951076556592020898924 65e-15 + x * (-0.49999286980201401719980935922649223357439041137695 + x * (4.0484539189054105169841244454207387920433372507922e-14
+ x * (4.16335155289191682914662351322476752102375030517 58e-2 + x * (-4.01585881
87437337585789492184743637255073863551 18e-14 + x * (-1.33822408854837810246452001194938929984346032142 64e-3))))))
```

See also: **fpminimax** (8.73), **fixed** (8.70)

8.72 floor

Name: **floor**
the usual function floor.

Library names:
```
sollya_obj_t sollya_lib_floor(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_floor(sollya_obj_t)
#define SOLLYA_FLOOR(x) sollya_lib_build_function_floor(x)
```

Description:

- **floor** is defined as usual: **floor**(x) is the greatest integer y such that $y \leq x$.
- It is defined for every real number x.

See also: **ceil** (8.24), **nearestint** (8.117), **round** (8.164), **RD** (8.154)

8.73 fpminimax

Name: **fpminimax**
computes a good polynomial approximation with fixed-point or floating-point coefficients

Library names:
```
sollya_obj_t sollya_lib_fpminimax(sollya_obj_t, sollya_obj_t, sollya_obj_t,
                                  sollya_obj_t, ...)
sollya_obj_t sollya_lib_v_fpminimax(sollya_obj_t, sollya_obj_t,
                                    sollya_obj_t, sollya_obj_t, va_list)
```

Usage:

fpminimax(*f*, *n*, *formats*, *range*, *indic1*, *indic2*, *indic3*, *P*) : (function, integer, list, range, absolute|relative | fixed|floating | function, absolute|relative | fixed|floating | function, absolute|relative | fixed|floating | function, function) → function

fpminimax(*f*, *monomials*, *formats*, *range*, *indic1*, *indic2*, *indic3*, *P*) : (function, list, list, range, absolute|relative | fixed|floating | function, absolute|relative | fixed|floating | function, absolute|relative | fixed|floating | function, function) → function

fpminimax(*f*, *n*, *formats*, *L*, *indic1*, *indic2*, *indic3*, *P*) : (function, integer, list, list, absolute|relative | fixed|floating | function, absolute|relative | fixed|floating | function, absolute|relative | fixed|floating | function, function) → function

fpminimax(*f*, *monomials*, *formats*, *L*, *indic1*, *indic2*, *indic3*, *P*) : (function, list, list, list, absolute|relative | fixed|floating | function, absolute|relative | fixed|floating | function, absolute|relative | fixed|floating | function, function) → function

fpminimax(*f*, *n*, *formats*, *range*, *indic1*, *indic2*, *indic3*, *Lcoeffs*) : (function, integer, list, range, absolute|relative | fixed|floating | function, absolute|relative | fixed|floating | function, absolute|relative | fixed|floating | function, list) → function

fpminimax(*f*, *monomials*, *formats*, *range*, *indic1*, *indic2*, *indic3*, *Lcoeffs*) : (function, list, list, range, absolute|relative | fixed|floating | function, absolute|relative | fixed|floating | function, absolute|relative | fixed|floating | function, list) → function
fpminimax(*f*, *n*, *formats*, *L*, *indic1*, *indic2*, *indic3*, *Lcoeffs*) : (function, integer, list, list, absolute|relative | fixed|floating | function, absolute|relative | fixed|floating | function, absolute|relative | fixed|floating | function, list) → function
fpminimax(*f*, *monomials*, *formats*, *L*, *indic1*, *indic2*, *indic3*, *Lcoeffs*) : (function, list, list, list, absolute|relative | fixed|floating | function, absolute|relative | fixed|floating | function, absolute|relative | fixed|floating | function, list) → function

Parameters:

- f is the function to be approximated
- n is the degree of the polynomial that must approximate f
- *monomials* is a list of integers or a list of function. It indicates the basis for the approximation of f
- *formats* is a list indicating the formats that the coefficients of the polynomial must have
- *range* is the interval where the function must be approximated
- L is a list of interpolation points used by the method
- *indic1* (optional) is one of the optional indication parameters. See the detailed description below.
- *indic2* (optional) is one of the optional indication parameters. See the detailed description below.
- *indic3* (optional) is one of the optional indication parameters. See the detailed description below.
- P (optional) is the minimax polynomial to be considered for solving the problem.
- *Lcoeffs* (optional) is a list containing the coefficients of the minimax polynomial to be considered for solving the problem.

Description:

- **fpminimax** uses a heuristic (but practically efficient) method to find a good polynomial approximation of a function f on an interval *range*. It implements the method published in the article:
  Efficient polynomial L^∞-approximations
  Nicolas Brisebarre and Sylvain Chevillard
  Proceedings of the 18th IEEE Symposium on Computer Arithmetic (ARITH 18)
  pp. 169-176
- The basic usage of this command is **fpminimax**(*f*, *n*, *formats*, *range*). It computes a polynomial approximation of f with degree at most n on the interval *range*. *formats* is a list of integers or format types (such as **double**, **doubledouble**, etc.). The polynomial returned by the command has its coefficients that fit the formats indications. For instance, if formats[0] is 35, the coefficient of degree 0 of the polynomial will fit a floating-point format of 35 bits. If formats[1] is D, the coefficient of degree 1 will be representable by a floating-point number with a precision of 53 bits (which is not necessarily an IEEE 754 double precision number. See the remark below), etc.
- In its most general form, **fpminimax** takes a function f to be approximated, a list of basis functions $g_0, \ldots, g_n$, a list of points L, and a constrained part q. It returns an expression p of the form $p = q + \sum_{i=0}^{n} a_i g_i$ such that $(p - f)$ or $(p - f)/f$ (depending whether the error to be considered is the absolute of relative error) is as small as possible on the points of L and with coefficients $a_0, \ldots, a_n$ fitting on given formats. **fpminimax** admits several syntaxes listed above and described below to conveniently handle usual cases (such as when $g_0, \ldots, g_n$ is the standard monomial basis $X^0, \ldots, X^n$) and to automatically compute a suitable list of points L on a given range.

- The second argument may be either an integer, a list of integers or a list of functions. An integer indicates the degree of the desired polynomial approximation. A list of integers indicates the list of desired monomials. For instance, the list [|0, 2, 4, 6|] indicates that the polynomial must be even and of degree at most 6. Giving an integer n as second argument is equivalent as giving [|0, ..., n|]. Finally, a list of function g_k indicates that the desired approximation must be a linear combination of the g_k.
  The list of formats is interpreted with respect to the list of monomials. For instance, if the list of monomials is [|0, 2, 4, 6|] and the list of formats is [|161, 107, 53, 24|], the coefficients of degree 0 is searched as a floating-point number with precision 161, the coefficient of degree 2 is searched as a number of precision 107, and so on.

- The list of formats may contain either integers or format types (**halfprecision**, **single**, **double**, **doubledouble**, **tripledouble**, **doubleextended** and **quad**). The list may be too large or even infinite. Only the first indications will be considered. For instance, for a degree n polynomial, formats[$n+1$] and above will be discarded. This lets one use elliptical indications for the last coefficients. For historical reasons, and as opposed to the general rule within `Sollya`, the case of an end-elliptic list whose last defined element is numeric is interpreted as the infinite list where the last defined element is repeated: therefore, *e.g.*, [|24...|] is a perfect synonym for [|SG...|]. Because this behavior can be confusing, using for the formats an end-elliptic list whose last defined element is numeric is deprecated and will be forbidden in a future release.

- The floating-point coefficients considered by **fpminimax** do not have an exponent range. In particular, in the format list, **double** is an exact synonym for 53. Currently, **fpminimax** only ensures that the corresponding coefficient has at most 53 bits of mantissa. It does not imply that it is an IEEE-754 double.

- By default, the list of formats is interpreted as a list of floating-point formats. This may be changed by passing **fixed** as an optional argument (see below). Let us take an example: **fpminimax**(f,2, [|107, DD, 53|], [0;1]). Here the optional argument is missing (we could have set it to **floating**). Thus, **fpminimax** will search for a polynomial of degree 2 with a constant coefficient that is a 107 bits floating-point number, etc.
  Currently, **doubledouble** is just a synonym for 107 and **tripledouble** a synonym for 161. This behavior may change in the future (taking into account the fact that some double-doubles are not representable with 107 bits).
  Second example: **fpminimax**(f, 2, [|25, 18, 30|], [0;1], **fixed**). In this case, **fpminimax** will search for a polynomial of degree 2 with a constant coefficient of the form $m/2^{25}$ where m is an integer. In other words, it is a fixed-point number with 25 bits after the point. Note that even with argument **fixed**, the formats list is allowed to contain **halfprecision**, **single**, **double**, **doubleextended**, **doubledouble**, **quad** or **tripledouble**. In this this case, it is just a synonym for 11, 24, 53, 64, 107, 113 or 161. This is deprecated and may change in the future.

- The fourth argument may be a range or a list. Lists are for advanced users that know what they are doing. The core of the method is a kind of approximated interpolation. The list given here is a list of points that must be considered for the interpolation. It must contain at least as many points as unknown coefficients. If you give a list, it is also recommended that you provide the minimax polynomial as last argument. If you give a range, the list of points will be automatically computed.

- The fifth, sixth and seventh arguments are optional. By default, **fpminimax** will approximate f while optimizing the relative error, and interpreting the list of formats as a list of floating-point formats.
  This default behavior may be changed with these optional arguments. You may provide zero, one, two or three of the arguments in any order. This lets the user indicate only the non-default arguments.
  The three possible arguments are:
  - **relative** or **absolute**: the error to be optimized;
  - **floating** or **fixed**: formats of the coefficients;

– a constrained part q.

The constrained part lets the user assign in advance some of the coefficients. For instance, for approximating $\exp(x)$, it may be interesting to search for a polynomial p of the form

$$p = 1 + x + \frac{x^2}{2} + a_3x^3 + a_4x^4.$$

Thus, there is a constrained part $q = 1+x+x^2/2$ and the unknown polynomial should be considered in the monomial basis $[|3, 4|]$. Calling **fpminimax** with monomial basis [|3, 4|] and constrained part q, will return a polynomial with the right form.
Notice that it is perfectly allowed that the monomial basis contains monomials also present in the constrained part. This can be useful for instance to fix the upper part of a **doubledouble** and look only for the second **double** of the expansion. For instance, in the previous example, we could use the same constrained part $q = 1 + x + x^2/2$ but with the full monomial basis $[|0, ..., 4|]$. The resulting polynomial will be of the form $p = 1 + x + x^2/2 + a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4$, or equivalently $p = (1 + a_0) + (1 + a_1)x + (1/2 + a_2)x^2 + a_3x^3 + a_4x^4$. Therefore, if for instance the coefficients a_0 to a_4 are asked to be **double**, the resulting polynomial p has its three first coefficients that are **doubledouble** with fixed upper parts.

- The last argument is for advanced users. It is the minimax polynomial that approximates the function f in the given basis. If it is not given this polynomial will be automatically computed by **fpminimax**.
  This minimax polynomial is used to compute the list of interpolation points required by the method. It is also used, when floating-point coefficients are desired, to give an initial assumption for the exponents of the coeffcients. In general, you do not have to provide this argument. But if you want to obtain several polynomials of the same degree that approximate the same function on the same range, just changing the formats, you should probably consider computing only once the minimax polynomial and the list of points instead of letting **fpminimax** recompute them each time.
  Note that in the case when a constrained part is given, the minimax polynomial must take that into account. For instance, in the previous example, the minimax would be obtained by the following command:

```
P = remez(1-(1+x+x^2/2)/exp(x), [|3,4|], range, 1/exp(x));
```

  Note that the constrained part is not to be added to P.
  In the case when the second argument is an integer or a list of integers, there is no restriction for P, as long as it is a polynomial. However, when the second argument is a list of functions, and even if these functions are all polynomials, P must be expanded in the given basis. For instance, if the second argument is 2 or [|0, 1, 2|], P can be given in Horner form. However, if the second argument is $[|1, x, x^2|]$, P must be written as a linear combination of 1, x and x^2, otherwise, the algorithm will fail to recover the coefficients of P and will fail with an error message.
  Please also note that recovering the coefficients of P in an arbitrary basis is performed heuristically and no verification is performed to check that P does not contain other functions than the functions of the basis.
  For all these reasons, it is sometimes useful to provide a list of coefficients *Lcoeffs* instead of P. In this case, the convention is that the i-th element of *Lcoeffs* is the coefficient of the i-th basis function in the minimax polynomial. *Lcoeffs* might contain more elements than there are basis functions, in which case the extra elements will be ignored.

- For historical reasons, **fpminimax** and **remez** have different syntaxes. Indeed, the syntax of **fpminimax** has been designed to be higher level and closer to usual customer needs. However, the reader's attention is drawn on the fact that this could be confusing: the argument f in **fpminimax** is always the function to be approximated, regardless of the mode (**absolute** or **relative**) which is passed as a separate argument, while **remez** takes two arguments f and w and looks for the approximation p that minimizes $f - pw$ (with $w = 1$ by default). Therefore, in **remez** looking for the approximation p that minimizes the relative error with respect to a given function g is performed by taking $f = 1$ and $w = 1/g$.

It may seem at first sight that the syntax of **remez** is more powerful than **fpminimax** as one can imagine to simultaneously use a non trivial f and a non trivial w, hence performing a weighted approximation. The same result can actually be achieved with **fpminimax** (to the price of a slightly more involved syntax) by approximating f in the customed monomial basis formed of w, $w(x)x$, $w(x)x^2$, ..., $w(x)x^j$, etc.

- Note that **fpminimax** internally computes a minimax polynomial (using the same algorithm as **remez** command). Thus **fpminimax** may encounter the same problems as **remez**. In particular, it may be very slow when Haar condition is not fulfilled. Another consequence is that currently **fpminimax** has to be run with a sufficiently high working precision.

Example 1:

```
> P = fpminimax(cos(x),6,[|DD, DD, D...|],[-1b-5;1b-5]);
> printexpansion(P);
(0x3ff0000000000000 + 0xbc09fda15e029b00) + x * ((0x3af9eb57163024a8 + 0x37942c2
f3f3e3839) + x * (0xbfdfffffffffff98 + x * (0xbbd1693f9c028849 + x * (0x3fa55555
55145337 + x * (0x3c7a25f610ad9ebc + x * 0xbf56c138142da5b0)))))
```

Example 2:

```
> P = fpminimax(sin(x),6,[|32...|],[-1b-5;1b-5], fixed, absolute);
> display = powers!;
> P;
x * (1 + x^2 * (-357913941 * 2^(-31) + x^2 * (35789873 * 2^(-32))))
```

Example 3:

```
> P = fpminimax(exp(x), [|3,4|], [|D,24|], [-1/256; 1/246], 1+x+x^2/2);
> display = powers!;
> P;
1 + x * (1 + x * (1 * 2^(-1) + x * (375300225001191 * 2^(-51) + x * (5592621 * 2
^(-27)))))
```

Example 4:

```
> print("Weighted approximation: look for p such that (p-f)*w is small");
Weighted approximation: look for p such that (p-f)*w is small
> n = 3;
> f = exp(x);
> w = sqrt(1-x^2);
> monomials = [||];
> for i from 0 to n do { monomials = monomials :. x^i*sqrt(1-x^2); };
> q = fpminimax(f*w, monomials, [|D...|], [-99/100,99/100],absolute);
> display = dyadic!;
> print(q);
3194834406077049b-54 * x^3 * sqrt(1 - x^2) + 4792048811386295b-53 * x^2 * sqrt(1
 - x^2) + 8982938415377717b-53 * x * sqrt(1 - x^2) + 561436817847349b-49 * sqrt(
1 - x^2)
> leadingcoeff = proc(expr) {
    var res;
    match(expr) with
      a*b+c : {res = [|a,c|]; }
      a*b : {res = [|a,0|]; }
      default: {res = error; };
    return res;
  };
> p = 0;
> for i from n to 0 by -1 do {
    L = leadingcoeff(q);
    p = p + L[0]*x^i;
    q = L[1];
  };
> print(p);
3194834406077049b-54 * x^3 + 4792048811386295b-53 * x^2 + 8982938415377717b-53 *
 x + 561436817847349b-49 * x^0
> display=decimal!;
> dirtyinfnorm((p-f)*w, [-1,1]);
2.74160017175089881671602238121270074586325363075 17e-3
> r = remez(f*w,n,[-99/100,99/100],w);
> dirtyinfnorm((r-f)*w, [-1,1]);
2.74160017175087344135066807176752589743789653043 54e-3
```

Example 5:

```
> I = [-1b-20,  1b-22];
> P0 = fpminimax(exp(x), 4, [|DD,DD,DD,D,SG|], I);
> P1 = fpminimax(exp(x), 4, [|D,D,D,D,SG|], I, 1+x+x^2/2);
> for k from 0 to 2 do { DD(coeff(P0,k)) == coeff(P0,k); };
true
true
true
> for k from 0 to 2 do { DD(coeff(P1,k)) == coeff(P1,k); };
true
true
true
> dirtyinfnorm(P0/exp(x)-1, I);
6.27445652115921559286163876743451177021328445081 75e-35
> dirtyinfnorm(P1/exp(x)-1, I);
5.82847771248349593837942985271469650605248459529 98e-35
```

Example 6:

```
> f = cos(exp(x));
> pstar = remez(f, 5, [-1b-7;1b-7]);
> listpoints = dirtyfindzeros(f-pstar, [-1b-7; 1b-7]);
> P1 = fpminimax(f, 5, [|DD...|], listpoints, absolute, default, default, pstar)
;
> P2 = fpminimax(f, 5, [|D...|], listpoints, absolute, default, default, pstar);

> P3 = fpminimax(f, 5, [|D, D, D, 24...|], listpoints, absolute, default, defaul
t, pstar);
> print("Error of pstar: ", dirtyinfnorm(f-pstar, [-1b-7; 1b-7]));
Error of pstar:  7.90484412599030263325774360010600630998177261774 25e-16
> print("Error of P1:    ", dirtyinfnorm(f-P1, [-1b-7; 1b-7]));
Error of P1:     7.90484412599030265800812991234204639214796182020 64e-16
> print("Error of P2:    ", dirtyinfnorm(f-P2, [-1b-7; 1b-7]));
Error of P2:     8.24771445799508717371095735367913316863476209559 85e-16
> print("Error of P3:    ", dirtyinfnorm(f-P3, [-1b-7; 1b-7]));
Error of P3:     1.08454277156993282593701156841863009789063333951055e-15
```

Example 7:

```
> L = [|exp(x), sin(x), cos(x)-1, sin(x^3)|];
> g = (2^x-1)/x;
> p = fpminimax(g, L, [|D...|], [-1/16;1/16],absolute);
> display = powers!;
> p;
-3267884797436153 * 2^(-54) * sin(x^3) + 5247089102535885 * 2^(-53) * (cos(x) -
1) + -8159095033730771 * 2^(-54) * sin(x) + 6243315658446641 * 2^(-53) * exp(x)
```

Example 8:

```
> n = 9;
> T = [|1, x|];
> for i from 2 to n do T[i] = canonical(2*x*T[i-1]-T[i-2]);
> g = (2^x-1)/x;
> PCheb = fpminimax(g, T, [|DD,DE...|], [-1/16;1/16],absolute);
> display = dyadic!;
> print(PCheb);
11246956025386490542313786100826 1b-107 + x * (76130818934358736339243350051b-98
+ x * (34355379230514621195759363b-89 + x * (38101334801118260933451906 7b-95 + x
 * (82530727478018928634564 9b-89 + x * (30509835232506699542201 37b-94 + x * (118
012311663984525229 1b-86 + x * (65439920880394856570 53b-92 + x * (157504970467107
70365b-87 + x * 8733930098894247371b-90))))))))
```

Example 9:

```
> d = [467680523945888933825179146469210566289898 41b-165;1b-7];
> n = 4; f = 1; w = sqrt(x)/acos(1-x);
> p = remez(f, n, d, w);
> monomials = [||]; Lcoeffs = [||];
> for i from 0 to n do {
    monomials = monomials :. x^i*sqrt(x);
    Lcoeffs = Lcoeffs :. coeff(p, i);
  };
> pf = fpminimax(acos(1-x), monomials, [|D...|], d, relative, floating, 0, Lcoef
fs);
> display = dyadic!;
> print(pf);
6237591828287231b-61 * x^4 * sqrt(x) + 1137310727877715b-57 * x^3 * sqrt(x) + 76
42862123083659b-58 * x^2 * sqrt(x) + 1061508612083747b-53 * x * sqrt(x) + 636905
1672525773b-52 * sqrt(x)
```

See also: **remez** (8.158), **dirtyfindzeros** (8.43), **absolute** (8.2), **relative** (8.157), **fixed** (8.70), **floating** (8.71), **default** (8.37), **halfprecision** (8.82), **single** (8.175), **double** (8.50), **doubleextended** (8.52), **doubledouble** (8.51), **quad** (8.149), **tripledouble** (8.194), **implementpoly** (8.90), **coeff** (8.27), **degree** (8.38), **roundcoefficients** (8.165), **guessdegree** (8.81)

8.74 fullparentheses

Name: **fullparentheses**
activates, deactivates or inspects the state variable controlling output with full parenthesising

Library names:

```
void sollya_lib_set_fullparentheses_and_print(sollya_obj_t);
void sollya_lib_set_fullparentheses(sollya_obj_t);
sollya_obj_t sollya_lib_get_fullparentheses();
```

Usage:

fullparentheses = *activation value* : on|off → void
fullparentheses = *activation value* ! : on|off → void

Parameters:

- *activation value* represents **on** or **off**, i.e. activation or deactivation

Description:

- An assignment **fullparentheses** = *activation value*, where *activation value* is one of **on** or **off**, activates respectively deactivates the output of expressions with full parenthesising. In full parenthesising mode, `Sollya` commands like **print**, **write** and the implicit command when an expression is given at the prompt will output expressions with parenthesising at all places where it is necessary for expressions containing infix operators to be parsed back with the same result. Otherwise parentheses around associative operators are omitted.

  If the assignment **fullparentheses** = *activation value* is followed by an exclamation mark, no message indicating the new state is displayed. Otherwise the user is informed of the new state of the global mode by an indication.

Example 1:

```
> autosimplify = off!;
> fullparentheses = off;
Full parentheses mode has been deactivated.
> print(1 + 2 + 3);
1 + 2 + 3
> fullparentheses = on;
Full parentheses mode has been activated.
> print(1 + 2 + 3);
(1 + 2) + 3
```

See also: **print** (8.141), **write** (8.201), **autosimplify** (8.16)

8.75 function

Name: **function**
keyword for declaring a procedure-based function or a keyword representing a function type

Library names:
```
sollya_obj_t sollya_lib_procedurefunction(sollya_obj_t, sollya_obj_t)
sollya_obj_t sollya_lib_build_function_procedurefunction(sollya_obj_t,
                                                         sollya_obj_t)
SOLLYA_EXTERNALPROC_TYPE_FUNCTION
```

Usage:

function(*procedure*) : procedure → function
function : type type

Parameters:

- *procedure* is a procedure of type (range, integer, integer) → range

Description:

- For the sake of safety and mathematical consistency, `Sollya` distinguishes clearly between functions, seen in the mathematical sense of the term, i.e. mappings, and procedures, seen in the sense Computer Science gives to functions, i.e. pieces of code that compute results for arguments following an algorithm. In some cases however, it is interesting to use such Computer Science procedures as realisations of mathematical functions, e.g. in order to plot them or even to perform polynomial approximation on them. The **function** keyword allows for such a transformation of a `Sollya` procedure into a `Sollya` function.
- The procedure to be used as a function through **function**(*procedure*) must be of type (range, integer, integer) → range. This means it must take in argument an interval X, a degree of differentiation n and a working precision p. It must return in result an interval Y encompassing the image $f^{(n)}(X)$ of the n-th derivative of the implemented function f, i.e. $f^{(n)}(X) \subseteq Y$. In order to allow `Sollya`'s algorithms to work properly, the procedure must ensure that, whenever $(p, \mathrm{diam}(X))$ tends to $(+\infty, 0)$, the computed over-estimated bounding Y tends to the actual image $f^{(n)}(X)$.
- The user must be aware that they are responsible of the correctness of the procedure. If, for some n and X, *procedure* returns an interval Y such that $f^{(n)}(X) \not\subseteq Y$, **function** will successfully return a function without any complain, but this function might behave inconsistently in further computations.
- For cases when the procedure does not have the correct signature or does not return a finite interval as a result **function**(*procedure*) evaluates to Not-A-Number (resp. to an interval of Not-A-Numbers for interval evaluation).
- **function** also represents the function type for declarations of external procedures by means of **externalproc**.

  Remark that in contrast to other indicators, type indicators like **function** cannot be handled outside the **externalproc** context. In particular, they cannot be assigned to variables.

Example 1:

```
> procedure EXP(X,n,p) {
        var res, oldPrec;
        oldPrec = prec;
        prec = p!;

        res = exp(X);

        prec = oldPrec!;
        return res;
  };
> f = function(EXP);
> f(1);
2.71828182845904523536028747135266249775724709 37
> exp(1);
2.71828182845904523536028747135266249775724709 37
> f(x + 3);
(function(proc(X, n, p)
{
var res, oldPrec;
oldPrec = prec;
prec = p!;
res = exp(X);
prec = oldPrec!;
return res;
}))(3 + x)
> diff(f);
diff(function(proc(X, n, p)
{
var res, oldPrec;
oldPrec = prec;
prec = p!;
res = exp(X);
prec = oldPrec!;
return res;
}))
> (diff(f))(0);
1
> g = f(sin(x));
> g(17);
0.38235816999386683402690554641655641359573458342088
> diff(g);
(diff(function(proc(X, n, p)
{
var res, oldPrec;
oldPrec = prec;
prec = p!;
res = exp(X);
prec = oldPrec!;
return res;
}))(sin(x)) * cos(x)
> (diff(g))(1);
1.25338076749344683697237458088447611474812675164344
> p = remez(f,3,[-1/2;1/2]);
> p;
0.99967120901420646830315493949039176881764871951833 + x * (0.999737029835711401
34762682913614052309208076875596 + x * (0.51049729360282624921622721654643510358
3073053437 + x * 0.16981432460713328758889769474737038047910878586801 6))
```

See also: **proc** (8.146), **library** (8.101), **procedure** (8.147), **externalproc** (8.66), **boolean** (8.22), **constant** (8.30), **integer** (8.94), **list of** (8.103), **range** (8.151), **string** (8.179), **object** (8.123)

8.76 gcd

Name: **gcd**

Computes the greatest common divisor of polynomials or numbers.

Library name:

`sollya_obj_t sollya_lib_gcd(sollya_obj_t, sollya_obj_t)`

Usage:

$$\mathbf{gcd}(p,\ q) : (\mathsf{function}, \mathsf{function}) \to \mathsf{function}$$

Parameters:

- p is a constant or a polynomial.
- q is a constant or a polynomial.

Description:

- When both p and q are integers, **gcd**(p,q) computes the greatest common divisor of these two integers, i.e. the greatest non-negative integer dividing both p and q.
- When both p and q are rational numbers, say a/b and c/d, **gcd**(p,q) computes the greatest common divisor of $a \cdot d$ and $b \cdot c$, divided by the product of the denominators, $b \cdot d$.
- When both p and q are constants but at least one of them is no rational number, **gcd**(p,q) returns 1.
- When both p and q are polynomials with at least one being non-constant, **gcd**(p,q) returns the polynomial of greatest degree dividing both p and q, and whose leading coefficient is the greatest common divisor of the leading coefficients of p and q.
- Similarly to the cases documented for **div** and **mod**, **gcd** may fail to return the unique polynomial of largest degree dividing both p and q in cases when certain coefficients of either p or q are constant expressions for which the tool is unable to determine whether they are zero or not. These cases typically involve polynomials whose leading coefficient is zero but the tool is unable to detect this fact.
- When at least one of p or q is a function that is no polynomial, **gcd**(p,q) returns 1.

Example 1:

```
> gcd(1001, 231);
77
> gcd(13, 17);
1
> gcd(-210, 462);
42
```

Example 2:

```
> rationalmode = on!;
> gcd(6/7, 33/13);
3 / 91
```

Example 3:

```
> gcd(exp(13),sin(17));
1
```

Example 4:

```
> gcd(24 + 68 * x + 74 * x^2 + 39 * x^3 + 10 * x^4 + x^5, 480 + 776 * x + 476 *
x^2 + 138 * x^3 + 19 * x^4 + x^5);
4 + x * (4 + x)
> gcd(1001 * x^2, 231 * x);
x * 77
```

Example 5:

```
> gcd(exp(x), x^2);
1
```

See also: **bezout** (8.19), **div** (8.48), **mod** (8.115), **numberroots** (8.121)

8.77 >=

Name: >=
greater-than-or-equal-to operator

Library name:
`sollya_obj_t sollya_lib_cmp_greater_equal(sollya_obj_t, sollya_obj_t)`

Usage:

expr1 >= *expr2* : (constant, constant) → boolean

Parameters:

- *expr1* and *expr2* represent constant expressions

Description:

- The operator >= evaluates to true iff its operands *expr1* and *expr2* evaluate to two floating-point numbers a_1 respectively a_2 with the global precision **prec** and a_1 is greater than or equal to a_2. The user should be aware of the fact that because of floating-point evaluation, the operator >= is not exactly the same as the mathematical operation *greater-than-or-equal-to*.

Example 1:

```
> 5 >= 4;
true
> 5 >= 5;
true
> 5 >= 6;
false
> exp(2) >= exp(1);
true
> log(1) >= exp(2);
false
```

Example 2:

```
> prec = 12;
The precision has been set to 12 bits.
> 16384.1 >= 16385.1;
true
```

See also: == (8.54), **!**= (8.118), > (8.80), <= (8.99), < (8.108), **in** (8.91), **!** (8.120), && (8.6), || (8.127), **prec** (8.138), **max** (8.110), **min** (8.113)

8.78 getbacktrace

Name: **getbacktrace**
returns the list of `Sollya` procedures currently run

Library name:
`sollya_obj_t sollya_lib_getbacktrace();`

Usage:

getbacktrace() : void → list

Description:

- The **getbacktrace** command allows the stack of `Sollya` procedures that are currently run to be inspected. When called, **getbacktrace** returns an ordered list of structures, each of which contains an element `passed_args` and an element `called_proc`. The element `called_proc` contains the `Sollya` object representing the procedure being run. The element `passed_args` contains an ordered list of all effective arguments passed to the procedure when it was called. The procedure called last (*i.e.*, on top of the stack) comes first in the list returned by **getbacktrace**. When any of the procedure called takes no arguments, the `passed_args` element of the corresponding structure evaluates to an empty list.
- When called from outside any procedure (at toplevel), **getbacktrace** returns an empty list.
- When called for a stack containing a call to a variadic procedure that was called with an infinite number of effective arguments, the corresponding `passed_args` element evaluates to an end-elliptic list.

Example 1:

```
> procedure testA() {
        "Current backtrace:";
        getbacktrace();
  };
> procedure testB(X) {
        "X = ", X;
        testA();
  };
> procedure testC(X, Y) {
        "X = ", X, ", Y = ", Y;
        testB(Y);
  };
> testC(17, 42);
X = 17, Y = 42
X = 42
Current backtrace:
[|{ .passed_args = [| |], .called_proc = proc()
{
"Current backtrace:";
getbacktrace();
return void;
} }, { .passed_args = [|42|], .called_proc = proc(X)
{
"X = ", X;
testA();
return void;
} }, { .passed_args = [|17, 42|], .called_proc = proc(X, Y)
{
"X = ", X, ", Y = ", Y;
testB(Y);
return void;
} }|]
```

Example 2:

```
> getbacktrace();
[| |]
```

Example 3:

```
> procedure printnumargs(X) {
        var L, t;
        "number of arguments: ", X;
        L = getbacktrace();
        "Backtrace:";
        for t in L do {
          "  " @ objectname(t.called_proc) @ ", ", t.passed_args;
        };
  };
> procedure numargs(l = ...) {
        "l[17] = ", l[17];
        printnumargs(length(l));
  };
> procedure test() {
        numargs @ [|25, 26, 27 ...|];
  };
> test();
l[17] = 42
number of arguments: infty
Backtrace:
  printnumargs, [|infty|]
  numargs, [|25, 26, 27...|]
  test, [| |]
```

See also: **proc** (8.146), **procedure** (8.147), **objectname** (8.124), **bind** (8.21), **@** (8.29)

8.79 getsuppressedmessages

Name: **getsuppressedmessages**
returns a list of numbers of messages that have been suppressed from message output

Library name:
`sollya_obj_t sollya_lib_getsuppressedmessages();`

Usage:

getsuppressedmessages() : void → list

Description:

- The **getsuppressedmessages** command allows the user to inspect the state of warning and information message suppression. When called, **getsuppressedmessages** returns a list of integers numbers that stand for the warning and information messages that have been suppressed. If no message is suppressed, **getsuppressedmessages** returns an empty list.
- Every `Sollya` warning or information message (that is not fatal to the tool's execution) has a message number. By default, these numbers are not displayed when a message is output. When message number displaying is activated using **showmessagenumbers**, the message numbers are displayed together with the message. This allows the user to match the numbers returned in a list by **getsuppressedmessages** with the actual warning and information messages.
- The list of message numbers returned by **getsuppressedmessages** is suitable to be fed into the **unsuppressmessage** command. This way, the user may unsuppress all warning and information messages that have been suppressed.

Example 1:

```
> verbosity = 1;
The verbosity level has been set to 1.
> 0.1;
Warning: Rounding occurred when converting the constant "0.1" to floating-point
with 165 bits.
If safe computation is needed, try to increase the precision.
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
0.1
> suppressmessage(174);
> 0.1;
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
0.1
> suppressmessage(407);
> 0.1;
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
0.1
> getsuppressedmessages();
[|174, 407|]
> suppressmessage(207, 196);
> getsuppressedmessages();
[|174, 196, 207, 407|]
```

Example 2:

```
> suppressmessage(174, 209, 13, 24, 196);
> suppressmessage([| 16, 17 |]);
> suppressmessage(19);
> unsuppressmessage([| 13, 17 |]);
> getsuppressedmessages();
[|16, 19, 24, 174, 196, 209|]
> unsuppressmessage(getsuppressedmessages());
> getsuppressedmessages();
[| |]
```

Example 3:

```
> verbosity = 12;
The verbosity level has been set to 12.
> suppressmessage(174);
> exp(x * 0.1);
Information: no Horner simplification will be performed because the given tree i
s already in Horner form.
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
exp(x * 0.1)
> getsuppressedmessages();
[|174|]
> verbosity = 0;
The verbosity level has been set to 0.
> exp(x * 0.1);
exp(x * 0.1)
> getsuppressedmessages();
[|174|]
```

See also: **getsuppressedmessages** (8.79), **suppressmessage** (8.184), **unsuppressmessage** (8.196), **verbosity** (8.198), **roundingwarnings** (8.167)

8.80 >

Name: >
greater-than operator

Library name:
`sollya_obj_t sollya_lib_cmp_greater(sollya_obj_t, sollya_obj_t)`

Usage:

expr1 > *expr2* : (constant, constant) → boolean

Parameters:

- *expr1* and *expr2* represent constant expressions

Description:

- The operator > evaluates to true iff its operands *expr1* and *expr2* evaluate to two floating-point numbers a_1 respectively a_2 with the global precision **prec** and a_1 is greater than a_2. The user should be aware of the fact that because of floating-point evaluation, the operator > is not exactly the same as the mathematical operation *greater-than*.

Example 1:

```
> 5 > 4;
true
> 5 > 5;
false
> 5 > 6;
false
> exp(2) > exp(1);
true
> log(1) > exp(2);
false
```

Example 2:

```
> prec = 12;
The precision has been set to 12 bits.
> 16385.1 > 16384.1;
false
```

See also: == (8.54), != (8.118), >= (8.77), <= (8.99), < (8.108), **in** (8.91), **!** (8.120), && (8.6), || (8.127), **prec** (8.138), **max** (8.110), **min** (8.113)

8.81 guessdegree

Name: **guessdegree**
returns the minimal degree needed for a polynomial to approximate a function with a certain error on an interval.

Library names:
```
sollya_obj_t sollya_lib_guessdegree(sollya_obj_t, sollya_obj_t,
                                    sollya_obj_t, ...)
sollya_obj_t sollya_lib_v_guessdegree(sollya_obj_t, sollya_obj_t,
                                      sollya_obj_t, va_list)
```

Usage:

guessdegree(*f*,*I*,*eps*,*w*,*bound*) : (function, range, constant, function, constant) → range

Parameters:

- f is the function to be approximated.
- I is the interval where the function must be approximated.
- *eps* is the maximal acceptable error.
- w (optional) is a weight function. Default is 1.
- *bound* (optional) is a bound on the degree. Default is currently 128.

Description:

- **guessdegree** tries to find the minimal degree needed to approximate f on I by a polynomial with an error $\epsilon = pw - f$ whose infinity norm not greater than *eps*. More precisely, it finds n minimal such that there exists a polynomial p of degree n such that $\|pw - f\|_\infty <$ eps.
- **guessdegree** returns an interval: for common cases, this interval is reduced to a single number (i.e. the minimal degree). But in certain cases, **guessdegree** does not succeed in finding the minimal degree. In such cases the returned interval is of the form $[n, p]$ such that:
  - no polynomial of degree $n - 1$ gives an error less than *eps*.
  - there exists a polynomial of degree p giving an error less than *eps*.
- The fifth optional argument *bound* is used to prevent **guessdegree** from trying to find too large degrees. If **guessdegree** does not manage to find a degree n satisfying the error and such that $n \leq$ bound, an interval of the form $[\cdot, +\infty]$ is returned. Note that *bound* must be a positive integer.

Example 1:

```
> guessdegree(exp(x),[-1;1],1e-10);
[10;10]
```

Example 2:

```
> guessdegree(exp(x),[-1;1],1e-10,default, 9);
[10;infty]
```

Example 3:

```
> guessdegree(1, [-1;1], 1e-8, 1/exp(x));
[8;9]
```

See also: **dirtyinfnorm** (8.44), **remez** (8.158), **fpminimax** (8.73), **degree** (8.38)

8.82 halfprecision

Names: **halfprecision**, **HP**
rounding to the nearest IEEE 754 half-precision number (binary16).

Library names:

```
sollya_obj_t sollya_lib_halfprecision(sollya_obj_t)
sollya_obj_t sollya_lib_halfprecision_obj()
int sollya_lib_is_halfprecision_obj(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_halfprecision(sollya_obj_t)
#define SOLLYA_HP(x) sollya_lib_build_function_halfprecision(x)
```

Description:

- **halfprecision** is both a function and a constant.
- As a function, it rounds its argument to the nearest IEEE 754 half-precision (i.e. IEEE754-2008 binary16) number. Subnormal numbers are supported as well as standard numbers: it is the real rounding described in the standard.
- As a constant, it symbolizes the half-precision format. It is used in contexts when a precision format is necessary, e.g. in the commands **round** and **roundcoefficients**. It is not supported for **implementpoly**. See the corresponding help pages for examples.

Example 1:

```
> display=binary!;
> HP(0.1);
1.100110011_2 * 2^(-4)
> HP(4.17);
1.00001011_2 * 2^(2)
> HP(1.011_2 * 2^(-23));
1.1_2 * 2^(-23)
```

See also: **single** (8.175), **double** (8.50), **doubleextended** (8.52), **doubledouble** (8.51), **quad** (8.149), **tripledouble** (8.194), **roundcoefficients** (8.165), **fpminimax** (8.73), **implementpoly** (8.90), **round** (8.164), **printsingle** (8.144)

8.83 head

Name: **head**
gives the first element of a list.

Library name:
`sollya_obj_t sollya_lib_head(sollya_obj_t)`

Usage:

head(L) : list → any type

Parameters:

- L is a list.

Description:

- **head**(L) returns the first element of the list L. It is equivalent to L[0].
- If L is empty, the command will fail with an error.

Example 1:

```
> head([|1,2,3|]);
1
> head([|1,2...|]);
1
```

See also: **tail** (8.185), **revert** (8.162)

8.84 hexadecimal

Name: **hexadecimal**
special value for global state **display**

Library names:
`sollya_obj_t sollya_lib_hexadecimal()`
`int sollya_lib_is_hexadecimal(sollya_obj_t)`

Description:

- **hexadecimal** is a special value used for the global state **display**. If the global state **display** is equal to **hexadecimal**, all data will be output in hexadecimal C99/ IEEE 754-2008 notation.

  As any value it can be affected to a variable and stored in lists.

See also: **decimal** (8.36), **dyadic** (8.53), **powers** (8.137), **binary** (8.20), **display** (8.47)

8.85 honorcoeffprec

Name: **honorcoeffprec**
indicates the (forced) honoring the precision of the coefficients in **implementpoly**

Library names:
```
sollya_obj_t sollya_lib_honorcoeffprec()
int sollya_lib_is_honorcoeffprec(sollya_obj_t)
```

Usage:

honorcoeffprec : honorcoeffprec

Description:

- Used with command **implementpoly**, **honorcoeffprec** makes **implementpoly** honor the precision of the given polynomial. This means if a coefficient needs a double-double or a triple-double to be exactly stored, **implementpoly** will allocate appropriate space and use a double-double or triple-double operation even if the automatic (heuristic) determination implemented in command **implementpoly** indicates that the coefficient could be stored on less precision or, respectively, the operation could be performed with less precision. See **implementpoly** for details.

Example 1:

```
> verbosity = 1!;
> q = implementpoly(1 - dirtysimplify(TD(1/6)) * x^2,[-1b-10;1b-10],1b-60,DD,"p"
,"implementation.c");
Warning: at least one of the coefficients of the given polynomial has been round
ed in a way
that the target precision can be achieved at lower cost. Nevertheless, the imple
mented polynomial
is different from the given one.
> printexpansion(q);
0x3ff0000000000000 + x^2 * 0xbfc5555555555555
> r = implementpoly(1 - dirtysimplify(TD(1/6)) * x^2,[-1b-10;1b-10],1b-60,DD,"p"
,"implementation.c",honorcoeffprec);
Warning: the inferred precision of the 2th coefficient of the polynomial is grea
ter than
the necessary precision computed for this step. This may make the automatic dete
rmination
of precisions useless.
> printexpansion(r);
0x3ff0000000000000 + x^2 * (0xbfc5555555555555 + 0xbc65555555555555 + 0xb9055555
55555555)
```

See also: **implementpoly** (8.90), **printexpansion** (8.143), **fpminimax** (8.73)

8.86 hopitalrecursions

Name: **hopitalrecursions**
controls the number of recursion steps when applying L'Hopital's rule.

Library names:

```
void sollya_lib_set_hopitalrecursions_and_print(sollya_obj_t)
void sollya_lib_set_hopitalrecursions(sollya_obj_t)
sollya_obj_t sollya_lib_get_hopitalrecursions()
```

Usage:

hopitalrecursions = n : integer → void
hopitalrecursions = n ! : integer → void
hopitalrecursions : integer

Parameters:

- n represents the number of recursions

Description:

- **hopitalrecursions** is a global variable. Its value represents the number of steps of recursion that are tried when applying L'Hopital's rule. This rule is applied by the interval evaluator present in the core of `Sollya` (and particularly visible in commands like **infnorm**).
- If an expression of the form f/g has to be evaluated by interval arithmetic on an interval I and if f and g have a common zero in I, a direct evaluation leads to NaN. `Sollya` implements a safe heuristic to avoid this, based on L'Hopital's rule: in such a case, it can be shown that $(f/g)(I) \subseteq (f'/g')(I)$. Since the same problem may exist for f'/g', the rule is applied recursively. The number of step in this recursion process is controlled by **hopitalrecursions**.
- Setting **hopitalrecursions** to 0 makes `Sollya` use this rule only once; setting it to 1 makes `Sollya` use the rule twice, and so on. In particular: the rule is always applied at least once, if necessary.

Example 1:

```
> hopitalrecursions=0;
The number of recursions for Hopital's rule has been set to 0.
> evaluate(log(1+x)^2/x^2,[-1/2; 1]);
[NaN;NaN]
> hopitalrecursions=1;
The number of recursions for Hopital's rule has been set to 1.
> evaluate(log(1+x)^2/x^2,[-1/2; 1]);
[-2.52258872223978123766892848583270627230200053744 11;6.77258872223978123766892
8485832706272302000537441 2]
```

See also: **taylorrecursions** (8.190), **infnorm** (8.93), **findzeros** (8.69), **evaluate** (8.58)

8.87 horner

Name: **horner**
brings all polynomial subexpressions of an expression to Horner form

Library name:
```
sollya_obj_t sollya_lib_horner(sollya_obj_t)
```

Usage:

horner(*function*) : function → function

Parameters:

- *function* represents the expression to be rewritten in Horner form

Description:

- The command **horner** rewrites the expression representing the function *function* in a way such that all polynomial subexpressions (or the whole expression itself, if it is a polynomial) are written in Horner form. The command **horner** does not endanger the safety of computations even in `Sollya`'s floating-point environment: the function returned is mathematically equal to the function *function*.

Example 1:

```
> print(horner(1 + 2 * x + 3 * x^2));
1 + x * (2 + x * 3)
> print(horner((x + 1)^7));
1 + x * (7 + x * (21 + x * (35 + x * (35 + x * (21 + x * (7 + x))))))
```

Example 2:

```
> print(horner(exp((x + 1)^5) - log(asin(x + x^3) + x)));
exp(1 + x * (5 + x * (10 + x * (10 + x * (5 + x))))) - log(asin(x * (1 + x^2)) +
 x)
```

See also: **canonical** (8.23), **print** (8.141), **coeff** (8.27), **degree** (8.38), **autosimplify** (8.16), **simplify** (8.173)

8.88 HP

Name: **HP**
short form for **halfprecision**

See also: **halfprecision** (8.82)

8.89 implementconstant

Name: **implementconstant**
implements a constant in arbitrary precision

Library names:
`void sollya_lib_implementconstant(sollya_obj_t, ...);`
`void sollya_lib_v_implementconstant(sollya_obj_t, va_list);`

Usage:

implementconstant(*expr*) : constant → void
implementconstant(*expr*,*filename*) : (constant, string) → void
implementconstant(*expr*,*filename*,*functionname*) : (constant, string, string) → void

Description:

- The command **implementconstant** implements the constant expression *expr* in arbitrary precision. More precisely, it generates the source code (written in C, and using MPFR) of a C function `const_something` with the following signature:

  ```
  void const_something (mpfr_ptr y, mp_prec_t prec)
  ```

  Let us denote by c the exact mathematical value of the constant defined by the expression *expr*. When called with arguments y and prec (where the variable y is supposed to be already initialized), the function `mpfr_const_something` sets the precision of y to a suitable precision and stores in it an approximate value of c such that

  $$|y - c| \leq |c|\, 2^{1-\mathrm{prec}}.$$

- When no filename *filename* is given or if **default** is given as *filename*, the source code produced by **implementconstant** is printed on standard output. Otherwise, when *filename* is given as a string of characters, the source code is output to a file named *filename*. If that file cannot be opened and/or written to, **implementconstant** fails and has no other effect.

- When *functionname* is given as an argument to **implementconstant** and *functionname* evaluates to a string of characters, the default name for the C function `const_something` is replaced by *functionname*. When **default** is given as *functionname*, the default name is used nevertheless, as if no *functionname* argument were given. When choosing a character sequence for *functionname*, the user should keep attention to the fact that *functionname* must be a valid C identifier in order to enable error-free compilation of the produced code.

- If *expr* refers to a constant defined with **libraryconstant**, the produced code uses the external code implementing this constant. The user should keep in mind that it is up to them to make sure the symbol for that external code can get resolved when the newly generated code is to be loaded.

- If a subexpression of *expr* evaluates to 0, **implementconstant** will most likely fail with an error message.

- **implementconstant** is unable to implement constant expressions *expr* that contain procedure-based functions, i.e. functions created from `Sollya` procedures using the **function** construct. If *expr* contains such a procedure-based function, **implementconstant** prints a warning and fails silently. The reason for this lack of functionality is that the produced C source code, which is supposed to be compiled, would have to call back to the `Sollya` interpreter in order to evaluate the procedure-based function.

- Similarly, **implementconstant** is currently unable to implement constant expressions *expr* that contain library-based functions, i.e. functions dynamically bound to `Sollya` using the **library** construct. If *expr* contains such a library-based function, **implementconstant** prints a warning and fails silently. Support for this feature is in principle feasible from a technical standpoint and might be added in a future release of `Sollya`.

- Currently, non-differentiable functions such as **double**, **doubledouble**, **tripledouble**, **single**, **halfprecision**, **quad**, **doubleextended**, **floor**, **ceil**, **nearestint** are not supported by **implementconstant**. If **implementconstant** encounters one of them, a warning message is displayed and no code is produced. However, if **autosimplify** equals on, it is possible that `Sollya` silently simplifies subexpressions of *expr* containing such functions and that **implementconstant** successfully produces code for evaluating *expr*.

- While it produces an MPFR-based C source code for *expr*, **implementconstant** takes architectural and system-dependent parameters into account. For example, it checks whether literal constants figuring in *expr* can be represented on a C `long int` type or if they must be stored in a different manner not to affect their accuracy. These tests, performed by `Sollya` during execution of **implementconstant**, depend themselves on the architecture `Sollya` is running on. Users should keep this matter in mind, especially when trying to compile source code on one machine whilst it has been produced on another.

Example 1:

```
> implementconstant(exp(1)+log(2)/sqrt(1/10));
```

[The first 100 lines of the output have been removed]

```
    modify or redistribute this generated code itself, or its skeleton,
    you may (at your option) remove this special exception, which will
    cause this generated code and its skeleton and the resulting Sollya
    output files to be licensed under the CeCILL-C licence without this
    special exception.

    This special exception was added by the Sollya copyright holders in
    version 4.1 of Sollya.

*/


#include <mpfr.h>

void
const_something (mpfr_ptr y, mp_prec_t prec)
{
  /* Declarations */
  mpfr_t tmp1;
  mpfr_t tmp2;
  mpfr_t tmp3;
  mpfr_t tmp4;
  mpfr_t tmp5;
  mpfr_t tmp6;
  mpfr_t tmp7;

  /* Initializations */
  mpfr_init2 (tmp2, prec+5);
  mpfr_init2 (tmp1, prec+3);
  mpfr_init2 (tmp4, prec+8);
  mpfr_init2 (tmp3, prec+7);
  mpfr_init2 (tmp6, prec+11);
  mpfr_init2 (tmp7, prec+11);
  mpfr_init2 (tmp5, prec+11);

  /* Core */
  mpfr_set_prec (tmp2, prec+4);
  mpfr_set_ui (tmp2, 1, MPFR_RNDN);
  mpfr_set_prec (tmp1, prec+3);
  mpfr_exp (tmp1, tmp2, MPFR_RNDN);
  mpfr_set_prec (tmp4, prec+8);
  mpfr_set_ui (tmp4, 2, MPFR_RNDN);
  mpfr_set_prec (tmp3, prec+7);
  mpfr_log (tmp3, tmp4, MPFR_RNDN);
  mpfr_set_prec (tmp6, prec+11);
  mpfr_set_ui (tmp6, 1, MPFR_RNDN);
  mpfr_set_prec (tmp7, prec+11);
  mpfr_set_ui (tmp7, 10, MPFR_RNDN);
  mpfr_set_prec (tmp5, prec+11);
  mpfr_div (tmp5, tmp6, tmp7, MPFR_RNDN);
  mpfr_set_prec (tmp4, prec+7);
  mpfr_sqrt (tmp4, tmp5, MPFR_RNDN);
  mpfr_set_prec (tmp2, prec+5);
  mpfr_div (tmp2, tmp3, tmp4, MPFR_RNDN);
  mpfr_set_prec (y, prec+3);
  mpfr_add (y, tmp1, tmp2, MPFR_RNDN);

  /* Cleaning stuff */
  mpfr_clear(tmp1);
  mpfr_clear(tmp2);
  mpfr_clear(tmp3);
  mpfr_clear(tmp4);
```

Example 2:

```
> implementconstant(sin(13/17),"sine_of_thirteen_seventeenth.c");
> bashevaluate("tail -n 30 sine_of_thirteen_seventeenth.c");
#include <mpfr.h>

void
const_something (mpfr_ptr y, mp_prec_t prec)
{
  /* Declarations */
  mpfr_t tmp1;
  mpfr_t tmp2;
  mpfr_t tmp3;

  /* Initializations */
  mpfr_init2 (tmp2, prec+6);
  mpfr_init2 (tmp3, prec+6);
  mpfr_init2 (tmp1, prec+6);

  /* Core */
  mpfr_set_prec (tmp2, prec+6);
  mpfr_set_ui (tmp2, 13, MPFR_RNDN);
  mpfr_set_prec (tmp3, prec+6);
  mpfr_set_ui (tmp3, 17, MPFR_RNDN);
  mpfr_set_prec (tmp1, prec+6);
  mpfr_div (tmp1, tmp2, tmp3, MPFR_RNDN);
  mpfr_set_prec (y, prec+2);
  mpfr_sin (y, tmp1, MPFR_RNDN);

  /* Cleaning stuff */
  mpfr_clear(tmp1);
  mpfr_clear(tmp2);
  mpfr_clear(tmp3);
}
```

Example 3:

```
> implementconstant(asin(1/3 * pi),default,"arcsin_of_one_third_pi");
                    [ The first 100 lines of the output have been removed ]
    modify or redistribute this generated code itself, or its skeleton,
    you may (at your option) remove this special exception, which will
    cause this generated code and its skeleton and the resulting Sollya
    output files to be licensed under the CeCILL-C licence without this
    special exception.

    This special exception was added by the Sollya copyright holders in
    version 4.1 of Sollya.

*/


#include <mpfr.h>

void
arcsin_of_one_third_pi (mpfr_ptr y, mp_prec_t prec)
{
  /* Declarations */
  mpfr_t tmp1;
  mpfr_t tmp2;
  mpfr_t tmp3;

  /* Initializations */
  mpfr_init2 (tmp2, prec+8);
  mpfr_init2 (tmp3, prec+8);
  mpfr_init2 (tmp1, prec+8);

  /* Core */
  mpfr_set_prec (tmp2, prec+8);
  mpfr_const_pi (tmp2, MPFR_RNDN);
  mpfr_set_prec (tmp3, prec+8);
  mpfr_set_ui (tmp3, 3, MPFR_RNDN);
  mpfr_set_prec (tmp1, prec+8);
  mpfr_div (tmp1, tmp2, tmp3, MPFR_RNDN);
  mpfr_set_prec (y, prec+2);
  mpfr_asin (y, tmp1, MPFR_RNDN);

  /* Cleaning stuff */
  mpfr_clear(tmp1);
  mpfr_clear(tmp2);
  mpfr_clear(tmp3);
}
```

Example 4:

```
> implementconstant(ceil(log(19 + 1/3)),"constant_code.c","magic_constant");
> bashevaluate("tail -n -9 constant_code.c");
void
magic_constant (mpfr_ptr y, mp_prec_t prec)
{
  /* Initializations */

  /* Core */
  mpfr_set_prec (y, prec);
  mpfr_set_ui (y, 3, MPFR_RNDN);
}
```

Example 5:

```
> bashexecute("gcc -fPIC -Wall -c libraryconstantexample.c -I$HOME/.local/includ
e");
> bashexecute("gcc -shared -o libraryconstantexample libraryconstantexample.o -l
gmp -lmpfr");
> euler_gamma = libraryconstant("./libraryconstantexample");
> implementconstant(euler_gamma^(1/3), "euler.c");
> bashevaluate("tail -n -17 euler.c");
void
const_something (mpfr_ptr y, mp_prec_t prec)
{
  /* Declarations */
  mpfr_t tmp1;

  /* Initializations */
  mpfr_init2 (tmp1, prec+1);

  /* Core */
  euler_gamma (tmp1, prec+1);
  mpfr_set_prec (y, prec+2);
  mpfr_root (y, tmp1, 3, MPFR_RNDN);

  /* Cleaning stuff */
  mpfr_clear(tmp1);
}
```

See also: **implementpoly** (8.90), **libraryconstant** (8.102), **library** (8.101), **function** (8.75), **bashevaluate** (8.17)

8.90 implementpoly

Name: **implementpoly**

implements a polynomial using double, double-double and triple-double arithmetic and generates a Gappa proof

Library names:

```
sollya_obj_t sollya_lib_implementpoly(sollya_obj_t, sollya_obj_t,
                                      sollya_obj_t, sollya_obj_t,
                                      sollya_obj_t, sollya_obj_t, ...)
sollya_obj_t sollya_lib_v_implementpoly(sollya_obj_t, sollya_obj_t,
                                        sollya_obj_t, sollya_obj_t,
                                        sollya_obj_t, sollya_obj_t, va_list)
```

Usage:

implementpoly(*polynomial*, *range*, *error bound*, *format*, *functionname*, *filename*) : (function, range, constant, D|double|DD|doubledouble|TD|tripledouble, string, string) → function
implementpoly(*polynomial*, *range*, *error bound*, *format*, *functionname*, *filename*, *honor coefficient precisions*) : (function, range, constant, D|double|DD|doubledouble|TD|tripledouble, string, string, honorcoeffprec) → function
implementpoly(*polynomial*, *range*, *error bound*, *format*, *functionname*, *filename*, *proof filename*) : (function, range, constant, D|double|DD|doubledouble|TD|tripledouble, string, string, string) → function
implementpoly(*polynomial*, *range*, *error bound*, *format*, *functionname*, *filename*, *honor coefficient precisions*, *proof filename*) : (function, range, constant, D|double|DD|doubledouble|TD|tripledouble, string, string, honorcoeffprec, string) → function

Description:

- The command **implementpoly** implements the polynomial *polynomial* in range *range* as a function called *functionname* in `C` code using double, double-double and triple-double arithmetic in a way that the rounding error (estimated at its first order) is bounded by *error bound*. The produced code is output in a file named *filename*. The argument *format* indicates the double, double-double or triple-double format of the variable in which the polynomial varies, influencing also in the signature of the `C` function.

  If a seventh or eighth argument *proof filename* is given and if this argument evaluates to a variable of type string, the command **implementpoly** will produce a `Gappa` proof that the rounding error is less than the given bound. This proof will be output in `Gappa` syntax in a file name *proof filename*.

  The command **implementpoly** returns the polynomial that has been implemented. As the command **implementpoly** tries to adapt the precision needed in each evaluation step to its strict minimum and as it applies renormalization to double-double and triple-double precision coefficients to bring them to a round-to-nearest expansion form, the returned polynomial may differ from the polynomial *polynomial*. Nevertheless the difference will be small enough that the rounding error bound with regard to the polynomial *polynomial* (estimated at its first order) will be less than the given error bound.

  If a seventh argument *honor coefficient precisions* is given and evaluates to a variable **honorcoeffprec** of type honorcoeffprec, **implementpoly** will honor the precision of the given polynomial *polynomials*. This means if a coefficient needs a double-double or a triple-double to be exactly stored, **implementpoly** will allocate appropriate space and use a double-double or triple-double operation even if the automatic (heuristic) determination implemented in command **implementpoly** indicates that the coefficient could be stored on less precision or, respectively, the operation could be performed with less precision. The use of **honorcoeffprec** has advantages and disadvantages. If the polynomial *polynomial* given has not been determined by a process considering directly polynomials with floating-point coefficients, **honorcoeffprec** should not be indicated. The **implementpoly** command can then determine the needed precision using the same error estimation as used for the determination of the precisions of the operations. Generally, the coefficients will get rounded to double, double-double and triple-double precision in a way that minimizes their number and respects the rounding error bound *error bound*. Indicating **honorcoeffprec** may in this case short-circuit most precision estimations leading to sub-optimal code. On the other hand, if the polynomial *polynomial* has been determined with floating-point precisions in mind, **honorcoeffprec** should be indicated because such polynomials often are very sensitive in terms of error propagation with regard to their coefficients' values. Indicating **honorcoeffprec** prevents the **implementpoly** command from rounding the coefficients and altering by many orders of magnitude the approximation error of the polynomial with regard to the function it approximates.

  The implementer behind the **implementpoly** command makes some assumptions on its input and verifies them. If some assumption cannot be verified, the implementation will not succeed and **implementpoly** will evaluate to a variable **error** of type error. The same behaviour is observed if some file is not writable or some other side-effect fails, e.g. if the implementer runs out of memory.

  As error estimation is performed only on the first order, the code produced by the **implementpoly** command should be considered valid iff a `Gappa` proof has been produced and successfully run in `Gappa`.

Example 1:

```
> implementpoly(1 - 1/6 * x^2 + 1/120 * x^4, [-1b-10;1b-10], 1b-30, D, "p","impl
ementation.c");
1 + x^2 * (-0.16666666666666665741480812812369549646973609924316406250000
3333333333217685101601546193705871701240539550781250000
> bashevaluate("tail -n -29 implementation.c");
#define p_coeff_0h 1.0000000000000000000000000000000000000000000000000000000000
0000000000000000000e+00
#define p_coeff_2h -1.6666666666666665741480812812369549646973609924316406250000
0000000000000000000e-01
#define p_coeff_4h 8.3333333333333332176851016015461937058717012405395507812500
0000000000000000000e-03


void p(double *p_resh, double x) {
double p_x_0_pow2h;


p_x_0_pow2h = x * x;


double p_t_1_0h;
double p_t_2_0h;
double p_t_3_0h;
double p_t_4_0h;
double p_t_5_0h;



p_t_1_0h = p_coeff_4h;
p_t_2_0h = p_t_1_0h * p_x_0_pow2h;
p_t_3_0h = p_coeff_2h + p_t_2_0h;
p_t_4_0h = p_t_3_0h * p_x_0_pow2h;
p_t_5_0h = p_coeff_0h + p_t_4_0h;
*p_resh = p_t_5_0h;


}
```

Example 2:

```
> implementpoly(1 - 1/6 * x^2 + 1/120 * x^4, [-1b-10;1b-10], 1b-30, D, "p","impl
ementation.c","implementation.gappa");
1 + x^2 * (-0.16666666666666665741480812812369549646973609924316406250000
3333333333217685101601546193705871701240539550781250000)
```

Example 3:

```
> verbosity = 1!;
> q = implementpoly(1 - dirtysimplify(TD(1/6)) * x^2,[-1b-10;1b-10],1b-60,DD,"p"
,"implementation.c");
Warning: at least one of the coefficients of the given polynomial has been round
ed in a way
that the target precision can be achieved at lower cost. Nevertheless, the imple
mented polynomial
is different from the given one.
> printexpansion(q);
0x3ff0000000000000 + x^2 * 0xbfc5555555555555
> r = implementpoly(1 - dirtysimplify(TD(1/6)) * x^2,[-1b-10;1b-10],1b-60,DD,"p"
,"implementation.c",honorcoeffprec);
Warning: the inferred precision of the 2th coefficient of the polynomial is grea
ter than
the necessary precision computed for this step. This may make the automatic dete
rmination
of precisions useless.
> printexpansion(r);
0x3ff0000000000000 + x^2 * (0xbfc5555555555555 + 0xbc65555555555555 + 0xb9055555
55555555)
```

Example 4:

```
> p = 0x3ff0000000000000 + x * (0x3ff0000000000000 + x * (0x3fe0000000000000 + x
 * (0x3fc5555555555559 + x * (0x3fa55555555555bd + x * (0x3f811111111106e2 + x
 * (0x3f56c16c16bf5eb7 + x * (0x3f2a01a01a292dcd + x * (0x3efa01a0218a016a + x
 * (0x3ec71de360331aad + x * (0x3e927e42e3823bf3 + x * (0x3e5ae6b2710c2c9a + x
 * (0x3e2203730c0a7c1d + x * 0x3de5da557e0781df))))))))));
> q = implementpoly(p,[-1/2;1/2],1b-60,D,"p","implementation.c",honorcoeffprec,"
implementation.gappa");
> if (q != p) then print("During implementation, rounding has happened.") else p
rint("Polynomial implemented as given.");
Polynomial implemented as given.
```

See also: **honorcoeffprec** (8.85), **roundcoefficients** (8.165), **double** (8.50), **doubledouble** (8.51), **tripledouble** (8.194), **bashevaluate** (8.17), **printexpansion** (8.143), **error** (8.57), **remez** (8.158), **fpminimax** (8.73), **taylor** (8.188), **implementconstant** (8.89)

8.91 in

Name: **in**
containment test operator

Library name:
`sollya_obj_t sollya_lib_cmp_in(sollya_obj_t, sollya_obj_t)`

Usage:

expr **in** *range1* : (constant, range) → boolean
range1 **in** *range2* : (range, range) → boolean

Parameters:

- *expr* represents a constant expression
- *range1* and *range2* represent ranges (intervals)

Description:

- When its first operand is a constant expression *expr*, the operator **in** evaluates to true iff the constant value of the expression *expr* is contained in the interval *range1*.
- When both its operands are ranges (intervals), the operator **in** evaluates to true iff all values in *range1* are contained in the interval *range2*.
- **in** is also used as a keyword for loops over the different elements of a list.

Example 1:

```
> 5 in [-4;7];
true
> 4 in [-1;1];
false
> 0 in sin([-17;17]);
true
```

Example 2:

```
> [5;7] in [2;8];
true
> [2;3] in [4;5];
false
> [2;3] in [2.5;5];
false
```

Example 3:

```
> for i in [|1,...,5|] do print(i);
1
2
3
4
5
```

See also: == (8.54), **!**= (8.118), >= (8.77), > (8.80), <= (8.99), < (8.108), **!** (8.120), && (8.6), || (8.127), **prec** (8.138), **print** (8.141)

8.92 inf

Name: **inf**
gives the lower bound of an interval.

Library name:
`sollya_obj_t sollya_lib_inf(sollya_obj_t)`

Usage:

$$\mathbf{inf}(I) : \mathsf{range} \to \mathsf{constant}$$
$$\mathbf{inf}(x) : \mathsf{constant} \to \mathsf{constant}$$

Parameters:

- I is an interval.
- x is a real number.

Description:

- Returns the lower bound of the interval I. Each bound of an interval has its own precision, so this command is exact, even if the current precision is too small to represent the bound.

- When called on a real number x, **inf** behaves like the identity.

Example 1:

```
> inf([1;3]);
1
> inf(0);
0
```

Example 2:

```
> display=binary!;
> I=[0.111110000011111_2; 1];
> inf(I);
1.11110000011111_2 * 2^(-1)
> prec=12!;
> inf(I);
1.11110000011111_2 * 2^(-1)
```

See also: **mid** (8.111), **sup** (8.182), **max** (8.110), **min** (8.113)

8.93 infnorm

Name: **infnorm**
computes an interval bounding the infinity norm of a function on an interval.

Library names:
`sollya_obj_t sollya_lib_infnorm(sollya_obj_t, sollya_obj_t, ...)`
`sollya_obj_t sollya_lib_v_infnorm(sollya_obj_t, sollya_obj_t, va_list)`

Usage:

infnorm(*f*,*I*,*filename*,*Ilist*) : (function, range, string, list) → range

Parameters:

- f is a function.
- I is an interval.
- *filename* (optional) is the name of the file into a proof will be saved.
- *IList* (optional) is a list of intervals to be excluded.

Description:

- **infnorm**(*f*,*range*) computes an interval bounding the infinity norm of the given function f on the interval I, e.g. computes an interval J such that $\max_{x\in I}\{|f(x)|\} \subseteq J$.
- If *filename* is given, a proof in English will be produced (and stored in file called *filename*) proving that $\max_{x\in I}\{|f(x)|\} \subseteq J$. This feature of English proof generation is considered DEPRECATED in `Sollya`. The feature will be removed in future versions of `Sollya`.
- If a list *IList* of intervals $I_1, \ldots, I_n$ is given, the infinity norm will be computed on $I\backslash(I_1 \cup \ldots \cup I_n)$.
- The function f is assumed to be at least twice continuous on I. More generally, if f is $\mathcal{C}^k$, global variables **hopitalrecursions** and **taylorrecursions** must have values not greater than k.
- If the interval is reduced to a single point, the result of **infnorm** is an interval containing the exact absolute value of f at this point.
- If the interval is not bound, the result will be $[0, +\infty]$ which is correct but perfectly useless. **infnorm** is not meant to be used with infinite intervals.

- The result of this command depends on the global variables **prec**, **diam**, **taylorrecursions** and **hopitalrecursions**. The contribution of each variable is not easy even to analyse.
  - The algorithm uses interval arithmetic with precision **prec**. The precision should thus be set high enough to ensure that no critical cancellation will occur.
  - When an evaluation is performed on an interval $[a, b]$, if the result is considered being too large, the interval is split into $[a, \frac{a+b}{2}]$ and $[\frac{a+b}{2}, b]$ and so on recursively. This recursion step is not performed if the $(b-a) < \delta \cdot |I|$ where δ is the value of variable **diam**. In other words, **diam** controls the minimum length of an interval during the algorithm.
  - To perform the evaluation of a function on an interval, Taylor's rule is applied, e.g. $f([a,b]) \subseteq f(m) + [a-m, b-m] \cdot f'([a, b])$ where $m = \frac{a+b}{2}$. This rule is recursively applied n times where n is the value of variable **taylorrecursions**. Roughly speaking, the evaluations will avoid decorrelation up to order n.
  - When a function of the form $\frac{g}{h}$ has to be evaluated on an interval $[a, b]$ and when g and h vanish at a same point z of the interval, the ratio may be defined even if the expression $\frac{g(z)}{h(z)} = \frac{0}{0}$ does not make any sense. In this case, L'Hopital's rule may be used and $\left(\frac{g}{h}\right)([a, b]) \subseteq \left(\frac{g'}{h'}\right)([a, b])$. Since the same can occur with the ratio $\frac{g'}{h'}$, the rule is applied recursively. The variable **hopitalrecursions** controls the number of recursion steps.
- The algorithm used for this command is quite complex to be explained here. Please find a complete description in the following article:
  S. Chevillard and C. Lauter
  A certified infinite norm for the implementation of elementary functions
  QSIC 2007 - 7th International Conference on Quality Software, p.1-9
  https://hal.science/hal-02102507v2
- Users should be aware about the fact that the algorithm behind **infnorm** is inefficient in most cases and that other, better suited algorithms, such as **supnorm**, are available inside `Sollya`. As a matter of fact, while **infnorm** is maintained for compatibility reasons with legacy `Sollya` codes, users are advised to avoid using **infnorm** in new `Sollya` scripts and to replace it, where possible, by the **supnorm** command.

Example 1:

```
> infnorm(exp(x),[-2;3]);
[20.085536923187667740928529654581717896987907838554;20.085536923187667740928529
65458171789698790783855 44]
```

Example 2:

```
> infnorm(exp(x),[-2;3],[| [0;1], [2;2.5] |]);
[20.085536923187667740928529654581717896987907838554;20.085536923187667740928529
65458171789698790783855 44]
```

Example 3:

```
> infnorm(exp(x),[1;1]);
[2.71828182845904523536028747135266249775724709369 99;2.71828182845904523536028747
135266249775724709 37]
```

Example 4:

```
> infnorm(exp(x), [log(0);log(1)]);
[0;infty]
```

See also: **prec** (8.138), **diam** (8.40), **hopitalrecursions** (8.86), **taylorrecursions** (8.190), **dirtyinfnorm** (8.44), **checkinfnorm** (8.26), **supnorm** (8.183), **findzeros** (8.69), **diff** (8.42), **taylorrecursions** (8.190), **autodiff** (8.15), **numberroots** (8.121), **taylorform** (8.189)

8.94 integer

Name: **integer**
keyword representing a machine integer type

Library name:
`SOLLYA_EXTERNALPROC_TYPE_INTEGER`

Usage:

integer : type type

Description:

- **integer** represents the machine integer type for declarations of external procedures **externalproc**.

  Remark that in contrast to other indicators, type indicators like **integer** cannot be handled outside the **externalproc** context. In particular, they cannot be assigned to variables.

See also: **externalproc** (8.66), **boolean** (8.22), **constant** (8.30), **function** (8.75), **list of** (8.103), **range** (8.151), **string** (8.179), **object** (8.123)

8.95 integral

Name: **integral**
computes an interval bounding the integral of a function on an interval.

Library name:
`sollya_obj_t sollya_lib_integral(sollya_obj_t, sollya_obj_t)`

Usage:

integral(f,I) : (function, range) $\rightarrow$ range

Parameters:

- f is a function.
- I is an interval.

Description:

- **integral**(f,I) returns an interval J such that the exact value of the integral of f on I lies in J.
- This command is safe but very inefficient. Use **dirtyintegral** if you just want an approximate value.
- The result of this command depends on the global variable **diam**. The method used is the following: I is cut into intervals of length not greater then $\delta \cdot |I|$ where δ is the value of global variable **diam**. On each small interval J, an evaluation of f by interval is performed. The result is multiplied by the length of J. Finally all values are summed.

Example 1:

```
> sin(10);
-0.544021110889369813404747661851377281683643012916 22
> integral(cos(x),[0;10]);
[-0.547101979835796902240976371635259430756985992573 33;-0.540940151300131838481505408813733707440537411917 29]
> diam=1e-5!;
> integral(cos(x),[0;10]);
[-0.544329156859554271018577802959369567752938763827 77;-0.543713064012499695080396442219274890104258031735 55]
```

See also: **diam** (8.40), **dirtyintegral** (8.45), **prec** (8.138)

8.96 interpolate

Name: **interpolate**
computes an interpolation polynomial.

Library names:
```
sollya_obj_t sollya_lib_interpolate(sollya_obj_t, sollya_obj_t, ...)
sollya_obj_t sollya_lib_v_interpolate(sollya_obj_t, sollya_obj_t, va_list)
```

Usage:

interpolate(X, Y) : (list, list) → function
interpolate(X, Y, D) : (list, list, constant) → function
interpolate(X, Y, D, I) : (list, list, constant, range) → function
interpolate(X, f) : (list, function) → function
interpolate(X, f, D) : (list, function, constant) → function
interpolate(X, f, D, I) : (list, function, constant, range) → function

Parameters:

- X is a list of constants for the interpolation abscissas
- Y is a list of constants for the interpolation ordinates
- f is a function to be interpolated
- D is a constant defining the maximum permissible error
- I is an interval defining the domain to be considered for the error

Description:

- **interpolate** computes an interpolation polynomial p of minimal degree such that for all given abscissa points x_i in the list X the interpolation condition is fullfilled with $p(x_i) = y_i$, where y_i is the corresponding entry in the list of ordinates Y. All entries of the lists X and Y need to be constant expressions; the entries in the list of abscissas X need to be pairwise distinct. If no parameter D is given, the interpolation is exact, *i.e.*, an interpolation polynomial (expression) is formed such that the interpolation condition is satisfied without error.
- **interpolate** accepts a function f instead of a list of ordinates. When called in this manner, **interpolate** computes a polynomial p such that for all given abscissa points x_i in the list X the interpolation condition is fullfilled with $p(x_i) = f(x_i)$. All entries in the list X still need to be constant expressions and they must be pairwise distinct. If no parameter D is given, the interpolation stays exact, even if this means forming constant expressions for the ordinates $f(x_i)$.
- As exact interpolation is often not needed and in order to speed up computations, **interpolate** accepts an optional parameter D, a positive constant, that defines the maximum permissible error. In this case, instead of computing the exact interpolation polynomial p as it is defined above, **interpolate** computes and returns an approximate interpolation polynomial $\tilde{p}$ that does not differ from p more than the amount D with supremum norm over the interval I that is given by the interval of all abscissa points in X.
  Formally, let I be the convex hull of all abscissa points in X; typically, the lower bound of I is the minimum value amongst all X, the upper bound is the maximum value amongst all X. Let p be the unique polynomial such that $p(x_i) = y_i$ for all x_i in the list X and y_i either the corresponding entry in the list Y or $y_i = f(x_i)$. Then **interpolate** returns $\tilde{p}$ such that

  $$\|\tilde{p} - p\|_{\infty,I} \leq D.$$

  The support for D works for any list of distinct abscissa points X and for any list of ordinate points Y or function f. The algorithm is however optimized for the case when X is a list of floating-point (or rational) constants and a function f. When f is a polynomial of degree n and X contains at most $n + 1$ abscissa points, **interpolate** may return f as-is, even though D has been provided; typically without performing any rounding on the coefficients of the polynomial f.

- When a permissible-error argument D is given to **interpolate**, the command additionnally supports an optional interval parameter I. If this parameter is specified, the interval I to consider the error between $\tilde{p}$ and p on is not taken as the convex hull of the abscissa points X but assumed to be the interval I. This parameter is supported to both allow for interpolation over intervals where the first (or last) interpolation point does not lie on the lower (resp. upper) bound of the interval and to speed up computation: determining the minimum and maximum values in X may be expensive.
- **interpolate** is completely insensible to the tool's working precision **prec**. It adapts its own working precision entirely automatically. However the user should be aware that precision adaption comes at a price: if the interpolation problem requires high precision to be solved, the execution time of **interpolate** will be high. This can typically be observed when the list of abscissas X contains two values that are mathematically different but for which high precision is needed to distinguish them.
- In case of error, *e.g.*, when the abscissa points in X are not all distinct, when the function does not evaluate at one of the abscissa points or when the permissible error D or domain interval I are not acceptable, **interpolate** evaluates to **error**.

Example 1:

```
> p = interpolate([| 1, 2, 3 |], [| 3, 5, 9 |]);
> write("p = ", p, "\n");
p = 3 + _x_ * (-1 + _x_)
```

Example 2:

```
> p = interpolate([| 1, 2, 3, 4 |], [| 1, -1, 2, -3 |]);
> write("p = ", p, "\n");
p = 21 + _x_ * (-100 / 3 + _x_ * (15.5 + _x_ * -13 / 6))
> p = interpolate([| 1, 2, 3, 4 |], [| 1, -1, 2, -3 |], 2^-17);
> write("p = ", p, "\n");
p = 21 + _x_ * (-33.333333492279052734375 + _x_ * (15.5 + _x_ * (-2.166666656732
5592041015625)))
> p = interpolate([| 1, 2, 3, 4 |], [| 1, -1, 2, -3 |], 2^-17, [0; 2^42]);
> write("p = ", p, "\n");
p = 21 + _x_ * (-33.333333333333333333333333333333333333333333363228 + _x_ * (15
.5 + _x_ * (-2.166666666666666666666666666666666666666666647983)))
```

Example 3:

```
> p = interpolate([| 1, 2, 3, 4 |], [| 1, -1, 2, -3 |]);
> q = interpolate([| 1, 2, 3, 4 |], [| 1, -1, 2, -3 |], 2^-17);
> delta = horner(p - q);
> write("delta = ", delta, "\n");
delta = _x_ * (1 / 6291456 + _x_^2 * -1 / 100663296)
> Delta = dirtyinfnorm(delta,[1;4]);
> "Delta = ", Delta, " = 2^(", log2(Delta), ")";
Delta = 2.44712943687280343165566692530133801132209571287 9e-7 = 2^(-21.96240625
18028904536343473598695412718995360192 31)
> p = interpolate([| 1, 2, 3, 4 |], [| 1, -1, 2, -3 |]);
> q = interpolate([| 1, 2, 3, 4 |], [| 1, -1, 2, -3 |], 2^-2000);
> delta = horner(p - q);
> write("delta = ", delta, "\n");
delta = _x_ * (1 / 4.40882186985313737305407949252229881864569997602 06e604 + _x_
^2 * -1 / 7.05411499176501979688652718803567810983311996163 3e605)
> Delta = dirtyinfnorm(delta,[1;4]);
> "Delta = ", Delta, " = 2^(", log2(Delta), ")";
Delta = 3.49209100137739927444769482451237519884019335776 84e-605 = 2^(-2007.9624
06251802890453634347359869541271899536019231 2)
```

Example 4:

```
> p = interpolate([| 1, 2, 3, 4 |], [| 1, -1, 2, -3 |]);
> q = interpolate([| 1, 2, 3, 4 |], [| 1, -1, 2, -3 |], 2^-17, [0;2^10]);
> delta = horner(p - q);
> write("delta = ", delta, "\n");
delta = _x_ * (1 / 422212465065984 + _x_^2 * -1 / 6755399441055744)
> Delta = dirtyinfnorm(delta,[0;2^10]);
> "Delta = ", Delta, " = 2^(", log2(Delta), ")";
Delta = 1.589432940818369388580322265625e-7 = 2^(-22.584984514668420136955755457
026138868402791643092)
> p = interpolate([| 1, 2, 3, 4 |], [| 1, -1, 2, -3 |]);
> q = interpolate([| 1, 2, 3, 4 |], [| 1, -1, 2, -3 |], 2^-2000, [0;2^10]);
> delta = horner(p - q);
> write("delta = ", delta, "\n");
delta = _x_ * (-1 / 3.6983878408024986992700435663608496048481867984446e611 + _x
_^2 * 1 / 5.9174205452839979188320697061773593677570988775114e612)
> Delta = dirtyinfnorm(delta,[0;2^10]);
> "Delta = ", Delta, " = 2^(", log2(Delta), ")";
Delta = 1.8145160239721784427448462541908690004638874174061e-604 = 2^(-2005.5849
845146684201369557554570261388684027916430 7)
```

Example 5:

```
> p = interpolate([| 1, 2, 3, 4 |], 17 + _x_ * (42 + _x_ * (23 + _x_ * 1664)));
> write("p = ", p, "\n");
p = 17 + _x_ * (42 + _x_ * (23 + _x_ * 1664))
```

Example 6:

```
> p = interpolate([| 1, 2, 3, 4 |], exp(_x_));
> write("p = ", p, "\n");
p = -1 * (-2 * (-3 * ((exp(4) - exp(3) - (exp(3) - exp(2))) / 2 - (exp(3) - exp(
2) - (exp(2) - exp(1))) / 2) / 3 + (exp(3) - exp(2) - (exp(2) - exp(1))) / 2) +
exp(2) - exp(1)) + exp(1) + _x_ * (-1 * (-2 * ((exp(4) - exp(3) - (exp(3) - exp(
2))) / 2 - (exp(3) - exp(2) - (exp(2) - exp(1))) / 2) / 3 + -3 * ((exp(4) - exp(
3) - (exp(3) - exp(2))) / 2 - (exp(3) - exp(2) - (exp(2) - exp(1))) / 2) / 3 + (
exp(3) - exp(2) - (exp(2) - exp(1))) / 2) + -2 * (-3 * ((exp(4) - exp(3) - (exp(
3) - exp(2))) / 2 - (exp(3) - exp(2) - (exp(2) - exp(1))) / 2) / 3 + (exp(3) - e
xp(2) - (exp(2) - exp(1))) / 2) + exp(2) - exp(1) + _x_ * (-1 * ((exp(4) - exp(3
) - (exp(3) - exp(2))) / 2 - (exp(3) - exp(2) - (exp(2) - exp(1))) / 2) / 3 + -2
 * ((exp(4) - exp(3) - (exp(3) - exp(2))) / 2 - (exp(3) - exp(2) - (exp(2) - exp
(1))) / 2) / 3 + -3 * ((exp(4) - exp(3) - (exp(3) - exp(2))) / 2 - (exp(3) - exp
(2) - (exp(2) - exp(1))) / 2) / 3 + (exp(3) - exp(2) - (exp(2) - exp(1))) / 2 +
_x_ * ((exp(4) - exp(3) - (exp(3) - exp(2))) / 2 - (exp(3) - exp(2) - (exp(2) -
exp(1))) / 2) / 3))
> p = interpolate([| 1, 2, 3, 4 |], exp(_x_), 2^-17);
> write("p = ", p, "\n");
p = -7.71721162274479866027832031 25 + _x_ * (17.914661619812250137329101562 5 + _
x_ * (-9.777572456747293472290039062 5 + _x_ * 2.298404288943856954574584960937 5)
)
> p = interpolate([| 1, 2, 3, 4 |], exp(_x_), 2^-17, [0; 2^42]);
> write("p = ", p, "\n");
p = -7.717211620141288536337557462573403702892010733189 + _x_ * (17.914661614969
411928830253122773955194465923548481 + _x_ * (-9.777572455021435040741686047095 9
032694184512464826 + _x_ * 2.298404288652356883609277858248014275601785523829 3))
```

Example 7:

```
> p = interpolate([| 1, 2, 3, 4 |], exp(_x_));
> q = interpolate([| 1, 2, 3, 4 |], exp(_x_), 2^-17);
> delta = horner(p - q);
> Delta = dirtyinfnorm(delta,[1;4]);
> "Delta = ", Delta, " = 2^(", log2(Delta), ")";
Delta = 7.810112347060637234639121597209262961385931274 1733e-9 = 2^(-26.93200955
256119984275780277821143295432153816 1928)
> p = interpolate([| 1, 2, 3, 4 |], exp(_x_));
> q = interpolate([| 1, 2, 3, 4 |], exp(_x_), 2^-2000);
> delta = horner(p - q);
> Delta = dirtyinfnorm(delta,[1;4]);
> "Delta = ", Delta, " = 2^(", log2(Delta), ")";
Delta = 3.57473788609725532267698406859678634948023806 0537e-606 = 2^(-2011.25058
804007182581755580347361219488621898097 02)
```

Example 8:

```
> p = interpolate([| 1, 2, 3, 4 |], exp(_x_));
> q = interpolate([| 1, 2, 3, 4 |], exp(_x_), 2^-17, [0;2^10]);
> delta = horner(p - q);
> Delta = dirtyinfnorm(delta,[0;2^10]);
> "Delta = ", Delta, " = 2^(", log2(Delta), ")";
Delta = 3.31749920226547169261202017379302575142433563899 91e-9 = 2^(-28.16725673
530497071077326569601846519127335178 6528)
> p = interpolate([| 1, 2, 3, 4 |], exp(_x_));
> q = interpolate([| 1, 2, 3, 4 |], exp(_x_), 2^-2000, [0;2^10]);
> delta = horner(p - q);
> Delta = dirtyinfnorm(delta,[0;2^10]);
> "Delta = ", Delta, " = 2^(", log2(Delta), ")";
Delta = 1.67480290063655159679957323422134460429582811153817e-605 = 2^(-2009.022
506085018547722587665527832736251964064 2038)
```

Example 9:

```
> f = exp(_x_);
> I = [-1/2;1/2];
> n = 17;
> X = [||]; for i from 0 to n do X = (~(1/2 * (inf(I) + sup(I)) + 1/2 * (sup(I)
- inf(I)) * cos(pi * ((2 * i + 1)/(2 * (n + 1)))))) .: X; X = revert(X);
> p = interpolate(X, f, 2^-110);
> q = remez(f, n, I);
> Delta = dirtyinfnorm(p-f,I);
> "||p-f|| = ", Delta, " = 2^(", log2(Delta), ")";
||p-f|| = 4.68228236487407954157902095723249905405327155207 87e-27 = 2^(-87.46484
662321150600413054025010034112695842423389 6)
> Delta = dirtyinfnorm(q-f,I);
> "||q-f|| = ", Delta, " = 2^(", log2(Delta), ")";
||q-f|| = 4.56155601044629644311741814836035675196724569110 95e-27 = 2^(-87.50253
253019238300469442897864638537892102159012)
> Delta = dirtyinfnorm(horner(p-q),I);
> "||p-q|| = ", Delta, " = 2^(", log2(Delta), ")";
||p-q|| = 1.20729201910945452306513389470528343007256343110 758e-28 = 2^(-92.7422
119802845233892604657112185260321526210955 53)
```

Example 10:

```
> interpolate([| 1, 1 + 2^-1000 |], 17 + 42 * x);
17 + x * 42
> interpolate([| 1, 1 + 2^-10000 |], 17 + 42 * x);
17 + x * 42
> interpolate([| 1, 1 |], 17 + 42 * x);
error
```

See also: **remez** (8.158), **dirtyinfnorm** (8.44), **supnorm** (8.183), **prec** (8.138), **error** (8.57)

8.97 isbound

Name: **isbound**
indicates whether a variable is bound or not.

Usage:

isbound(*ident*) : boolean

Parameters:

- *ident* is a name.

Description:

- **isbound**(*ident*) returns a boolean value indicating whether the name *ident* is used or not to represent a variable. It returns true when *ident* is the name used to represent the global variable or if the name is currently used to refer to a (possibly local) variable.
- When a variable is defined in a block and has not been defined outside, **isbound** returns true when called inside the block, and false outside. Note that **isbound** returns true as soon as a variable has been declared with **var**, even if no value is actually stored in it.
- If *ident1* is bound to a variable and if *ident2* refers to the global variable, the command **rename**(*ident2*, *ident1*) hides the value of *ident1* which becomes the global variable. However, if the global variable is again renamed, *ident1* gets its value back. In this case, **isbound**(*ident1*) returns true. If *ident1* was not bound before, **isbound**(*ident1*) returns false after that *ident1* has been renamed.

Example 1:

```
> isbound(x);
false
> isbound(f);
false
> isbound(g);
false
> f=sin(x);
> isbound(x);
true
> isbound(f);
true
> isbound(g);
false
```

Example 2:

```
> isbound(a);
false
> { var a; isbound(a); };
true
> isbound(a);
false
```

Example 3:

```
> f=sin(x);
> isbound(x);
true
> rename(x,y);
> isbound(x);
false
```

Example 4:

```
> x=1;
> f=sin(y);
> rename(y,x);
> f;
sin(x)
> x;
x
> isbound(x);
true
> rename(x,y);
> isbound(x);
true
> x;
1
```

See also: **rename** (8.159)

8.98 isevaluable

Name: **isevaluable**
tests whether a function can be evaluated at a point

Usage:

isevaluable(*function*, *constant*) : (function, constant) → boolean

Parameters:

- *function* represents a function
- *constant* represents a constant point

Description:

- **isevaluable** applied to function *function* and a constant *constant* returns a boolean indicating whether or not a subsequent call to **evaluate** on the same function *function* and constant *constant* will produce a numerical result or NaN. This means **isevaluable** returns false iff **evaluate** will return NaN.
- The command **isevaluable** is now considered DEPRECATED in `Sollya`. As checks for NaNs are now possible in Sollya, the command **isevaluable** can be fully emulated with a call to evaluate and a couple of tests, as shown below in the last example.

Example 1:

```
> isevaluable(sin(pi * log(x)), 0.5);
true
> print(evaluate(sin(pi * log(x)), 0.5));
-0.82148283122563882875872566228649962370813607461095
```

Example 2:

```
> isevaluable(sin(pi * log(x)), 0);
true
> print(evaluate(sin(pi * log(x)), 0));
[-1;1]
```

Example 3:

```
> isevaluable(sin(pi * 1/x), 0.5);
true
> print(evaluate(sin(pi * 1/x), 0.5));
[-3.100365765139897619749121887390789523854170596558e-13490;5.300240158585712760
535084242602922324150077630252 8e-13489]
```

Example 4:

```
> procedure isEvaluableEmulation(f, c) {
        return match evaluate(f, c) with
                    NaN : (false)
                 [NaN;NaN] : (false)
                 default : (true);
  };
> isEvaluableEmulation(sin(pi * log(x)), 0.5);
true
> isEvaluableEmulation(sin(pi * log(x)), 0);
true
> isEvaluableEmulation(sin(pi * log(x)), -1);
false
```

See also: **evaluate** (8.58)

8.99 <=

Name: <=
less-than-or-equal-to operator

Library name:
`sollya_obj_t sollya_lib_cmp_less_equal(sollya_obj_t, sollya_obj_t)`

Usage:

expr1 <= *expr2* : (constant, constant) → boolean

Parameters:

- *expr1* and *expr2* represent constant expressions

Description:

- The operator <= evaluates to true iff its operands *expr1* and *expr2* evaluate to two floating-point numbers a_1 respectively a_2 with the global precision **prec** and a_1 is less than or equal to a_2. The user should be aware of the fact that because of floating-point evaluation, the operator <= is not exactly the same as the mathematical operation *less-than-or-equal-to*.

Example 1:

```
> 5 <= 4;
false
> 5 <= 5;
true
> 5 <= 6;
true
> exp(2) <= exp(1);
false
> log(1) <= exp(2);
true
```

Example 2:

```
> prec = 12;
The precision has been set to 12 bits.
> 16385.1 <= 16384.1;
true
```

See also: == (8.54), **!**= (8.118), >= (8.77), > (8.80), < (8.108), **in** (8.91), **!** (8.120), && (8.6), || (8.127), **prec** (8.138), **max** (8.110), **min** (8.113)

8.100 length

Name: **length**
computes the length of a list or string.

Library name:
`sollya_obj_t sollya_lib_length(sollya_obj_t)`

Usage:

$$\textbf{length}(L) : \mathsf{list} \to \mathsf{integer}$$
$$\textbf{length}(s) : \mathsf{string} \to \mathsf{integer}$$

Parameters:

- L is a list.
- s is a string.

Description:

- **length** returns the length of a list or a string, e.g. the number of elements or letters.
- The empty list or string have length 0. If L is an end-elliptic list, **length** returns +Inf.

Example 1:

```
> length("Hello World!");
12
```

Example 2:

```
> length([|1,...,5|]);
5
```

Example 3:

```
> length([| |]);
0
```

Example 4:

```
> length([|1,2...|]);
infty
```

8.101 library

Name: **library**
binds an external mathematical function to a variable in `Sollya`

Library names:

```
sollya_obj_t sollya_lib_libraryfunction(sollya_obj_t, char *,
                                        int (*)(mpfi_t, mpfi_t, int))
sollya_obj_t sollya_lib_build_function_libraryfunction(sollya_obj_t, char *,
                                                       int (*)(mpfi_t,
                                                               mpfi_t, int))
sollya_obj_t sollya_lib_libraryfunction_with_data(
                                        sollya_obj_t, char *,
                                        int (*)(mpfi_t, mpfi_t, int, void *),
                                        void *, void (*)(void *))
sollya_obj_t sollya_lib_build_function_libraryfunction_with_data(
                                        sollya_obj_t, char *,
                                        int (*)(mpfi_t,
                                                mpfi_t, int, void *),
                                        void *, void (*)(void *))
```

Usage:

library(*path*) : string → function

Description:

- The command **library** lets you extend the set of mathematical functions known to `Sollya`. By default, `Sollya` knows the most common mathematical functions such as **exp**, **sin**, **erf**, etc. Within `Sollya`, these functions may be composed. This way, `Sollya` should satisfy the needs of a lot of users. However, for particular applications, one may want to manipulate other functions such as Bessel functions, or functions defined by an integral or even a particular solution of an ODE.
- **library** makes it possible to let `Sollya` know about new functions. In order to let it know, you have to provide an implementation of the function you are interested in. This implementation is a C file containing a function of the form:

  ```
  int my_ident(sollya_mpfi_t result, sollya_mpfi_t op, int n)
  ```

  The semantic of this function is the following: it is an implementation of the function and its derivatives in interval arithmetic. `my_ident(result, I, n)` shall store in `result` an enclosure of the image set of the n-th derivative of the function f over `I`: $f^{(n)}(I) \subseteq$ result.
- The integer value returned by the function implementation currently has no meaning.
- You do not need to provide a working implementation for any `n`. Most functions of `Sollya` requires a relevant implementation only for f, f' and f''. For higher derivatives, its is not so critical and the implementation may just store $[-\infty, +\infty]$ in result whenever $n > 2$.
- Note that you should respect somehow interval-arithmetic standards in your implementation: `result` has its own precision and you should perform the intermediate computations so that `result` is as tight as possible.
- You can include sollya.h in your implementation and use library functionnalities of `Sollya` for your implementation. However, this requires to have compiled `Sollya` with `-fPIC` in order to make the `Sollya` executable code position independent and to use a system on with programs, using `dlopen` to open dynamic routines can dynamically open themselves. **Important notice:** as the code will be run in a context where a sollya session is already opened, the library functions must be used directly, without calling `sollya_lib_init` and `sollya_lib_close` (calling these functions would conflict with the current session, leading to weird and hard to debug behaviors).

- To bind your function into `Sollya`, you must use the same identifier as the function name used in your implementation file (`my_ident` in the previous example). Once the function code has been bound to an identifier, you can use a simple assignment to assign the bound identifier to yet another identifier. This way, you may use convenient names inside `Sollya` even if your implementation environment requires you to use a less convenient name.
- The dynamic object file whose name is given to **library** for binding of an external library function may also define a destructor function `int sollya_external_lib_close(void)`. If `Sollya` finds such a destructor function in the dynamic object file, it will call that function when closing the dynamic object file again. This happens when `Sollya` is terminated or when the current `Sollya` session is restarted using **restart**. The purpose of the destructor function is to allow the dynamically bound code to free any memory that it might have allocated before `Sollya` is terminated or restarted. The dynamic object file is not necessarily needed to define a destructor function. This ensure backward compatibility with older `Sollya` external library function object files. When defined, the destructor function is supposed to return an integer value indicating if an error has happened. Upon success, the destructor functions is to return a zero value, upon error a non-zero value.

Example 1:

```
> bashexecute("gcc -fPIC -Wall -c libraryexample.c -I$HOME/.local/include");
> bashexecute("gcc -shared -o libraryexample libraryexample.o -lgmp -lmpfr");
> myownlog = library("./libraryexample");
> evaluate(log(x), 2);
0.693147180559945309417232121458176568075500134360 25
> evaluate(myownlog(x), 2);
0.693147180559945309417232121458176568075500134360 25
```

See also: **function** (8.75), **bashexecute** (8.18), **externalproc** (8.66), **externaldata** (8.64), **externalplot** (8.65), **diff** (8.42), **evaluate** (8.58), **libraryconstant** (8.102)

8.102 libraryconstant

Name: **libraryconstant**
binds an external mathematical constant to a variable in `Sollya`

Library names:

```
sollya_obj_t sollya_lib_libraryconstant(char *, void (*)(mpfr_t, mp_prec_t))
sollya_obj_t sollya_lib_build_function_libraryconstant(char *,
                                                       void (*)(mpfr_t,
                                                                mp_prec_t))
sollya_obj_t sollya_lib_libraryconstant_with_data(char *,
                                                  void (*)(mpfr_t,
                                                           mp_prec_t,
                                                           void *),
                                                  void *,
                                                  void (*)(void *))
sollya_obj_t sollya_lib_build_function_libraryconstant_with_data(
                                                  char *,
                                                  void (*)(mpfr_t,
                                                           mp_prec_t,
                                                           void *),
                                                  void *,
                                                  void (*)(void *))
```

Usage:

libraryconstant(*path*) : string → function

Description:

- The command **libraryconstant** lets you extend the set of mathematical constants known to `Sollya`. By default, the only mathematical constant known by `Sollya` is **pi**. For particular applications, one may want to manipulate other constants, such as Euler's gamma constant, for instance.
- **libraryconstant** makes it possible to let `Sollya` know about new constants. In order to let it know, you have to provide an implementation of the constant you are interested in. This implementation is a C file containing a function of the form:

  ```
  void my_ident(mpfr_t result, mp_prec_t prec)
  ```

  The semantic of this function is the following: it is an implementation of the constant in arbitrary precision. `my_ident(result, prec)` shall set the precision of the variable result to a suitable precision (the variable is assumed to be already initialized) and store in result an approximate value of the constant with a relative error not greater than $2^{1-\mathrm{prec}}$. More precisely, if c is the exact value of the constant, the value stored in result should satisfy

  $$|\mathrm{result} - c| \leq |c|\, 2^{1-\mathrm{prec}}.$$

- You can include sollya.h in your implementation and use library functionnalities of `Sollya` for your implementation. However, this requires to have compiled `Sollya` with `-fPIC` in order to make the `Sollya` executable code position independent and to use a system on with programs, using `dlopen` to open dynamic routines can dynamically open themselves.
- To bind your constant into `Sollya`, you must use the same identifier as the function name used in your implementation file (`my_ident` in the previous example). Once the function code has been bound to an identifier, you can use a simple assignment to assign the bound identifier to yet another identifier. This way, you may use convenient names inside `Sollya` even if your implementation environment requires you to use a less convenient name.
- Once your constant is bound, it is considered by `Sollya` as an infinitely accurate constant (i.e. a 0-ary function, exactly like **pi**).
- The dynamic object file whose name is given to **libraryconstant** for binding of an external library constant may also define a destructor function `int sollya_external_lib_close(void)`. If `Sollya` finds such a destructor function in the dynamic object file, it will call that function when closing the dynamic object file again. This happens when `Sollya` is terminated or when the current `Sollya` session is restarted using **restart**. The purpose of the destructor function is to allow the dynamically bound code to free any memory that it might have allocated before `Sollya` is terminated or restarted. The dynamic object file is not necessarily needed to define a destructor function. This ensure backward compatibility with older `Sollya` external library function object files. When defined, the destructor function is supposed to return an integer value indicating if an error has happened. Upon success, the destructor functions is to return a zero value, upon error a non-zero value.

Example 1:

```
> bashexecute("gcc -fPIC -Wall -c libraryconstantexample.c -I$HOME/.local/includ
e");
> bashexecute("gcc -shared -o libraryconstantexample libraryconstantexample.o -l
gmp -lmpfr");
> euler_gamma = libraryconstant("./libraryconstantexample");
> prec = 20!;
> euler_gamma;
0.577215
> prec = 100!;
> euler_gamma;
0.57721566490153286060651209082
> midpointmode = on;
Midpoint mode has been activated.
> [euler_gamma];
0.57721566490153286060651209008~2/4~
```

See also: **bashexecute** (8.18), **externalproc** (8.66), **externaldata** (8.64), **externalplot** (8.65), **pi** (8.130), **library** (8.101), **evaluate** (8.58), **implementconstant** (8.89)

8.103 list of

Name: **list of**
keyword used in combination with a type keyword

Library names:
```
SOLLYA_EXTERNALPROC_TYPE_CONSTANT_LIST
SOLLYA_EXTERNALPROC_TYPE_FUNCTION_LIST
SOLLYA_EXTERNALPROC_TYPE_RANGE_LIST
SOLLYA_EXTERNALPROC_TYPE_INTEGER_LIST
SOLLYA_EXTERNALPROC_TYPE_STRING_LIST
SOLLYA_EXTERNALPROC_TYPE_BOOLEAN_LIST
SOLLYA_EXTERNALPROC_TYPE_OBJECT_LIST
```

Description:

- **list of** is used in combination with one of the following keywords for indicating lists of the respective type in declarations of external procedures using **externalproc**: **boolean**, **constant**, **function**, **integer**, **range**, **object** and **string**.

See also: **externalproc** (8.66), **boolean** (8.22), **constant** (8.30), **function** (8.75), **integer** (8.94), **range** (8.151), **string** (8.179), **object** (8.123)

8.104 log

Name: **log**
natural logarithm.

Library names:
```
sollya_obj_t sollya_lib_log(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_log(sollya_obj_t)
#define SOLLYA_LOG(x) sollya_lib_build_function_log(x)
```

Description:

- **log** is the natural logarithm defined as the inverse of the exponential function: $\log(y)$ is the unique real number x such that $\exp(x) = y$.
- It is defined only for $y \in [0; +\infty]$.

See also: **exp** (8.60), **log2** (8.107), **log10** (8.105)

8.105 log10

Name: **log10**
decimal logarithm.

Library names:
```
sollya_obj_t sollya_lib_log10(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_log10(sollya_obj_t)
#define SOLLYA_LOG10(x) sollya_lib_build_function_log10(x)
```

Description:

- **log10** is the decimal logarithm defined by: $\log10(x) = \log(x)/\log(10)$.
- It is defined only for $x \in [0; +\infty]$.

See also: **log** (8.104), **log2** (8.107)

8.106 log1p

Name: **log1p**
translated logarithm.

Library names:
```
sollya_obj_t sollya_lib_log1p(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_log1p(sollya_obj_t)
#define SOLLYA_LOG1P(x) sollya_lib_build_function_log1p(x)
```

Description:

- **log1p** is the function defined by $\log1p(x) = \log(1 + x)$.
- It is defined only for $x \in [-1; +\infty]$.

See also: **log** (8.104)

8.107 log2

Name: **log2**
binary logarithm.

Library names:
```
sollya_obj_t sollya_lib_log2(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_log2(sollya_obj_t)
#define SOLLYA_LOG2(x) sollya_lib_build_function_log2(x)
```

Description:

- **log2** is the binary logarithm defined by: $\log2(x) = \log(x)/\log(2)$.
- It is defined only for $x \in [0; +\infty]$.

See also: **log** (8.104), **log10** (8.105)

8.108 <

Name: <
less-than operator

Library name:
```
sollya_obj_t sollya_lib_cmp_less(sollya_obj_t, sollya_obj_t)
```

Usage:

expr1 < *expr2* : (constant, constant) → boolean

Parameters:

- *expr1* and *expr2* represent constant expressions

Description:

- The operator < evaluates to true iff its operands *expr1* and *expr2* evaluate to two floating-point numbers a_1 respectively a_2 with the global precision **prec** and a_1 is less than a_2. The user should be aware of the fact that because of floating-point evaluation, the operator < is not exactly the same as the mathematical operation *less-than*.

Example 1:

```
> 5 < 4;
false
> 5 < 5;
false
> 5 < 6;
true
> exp(2) < exp(1);
false
> log(1) < exp(2);
true
```

Example 2:

```
> prec = 12;
The precision has been set to 12 bits.
> 16384.1 < 16385.1;
false
```

See also: == (8.54), **!**= (8.118), >= (8.77), > (8.80), <= (8.99), **in** (8.91), **!** (8.120), && (8.6), || (8.127), **prec** (8.138), **max** (8.110), **min** (8.113)

8.109 mantissa

Name: **mantissa**
returns the integer mantissa of a number.

Library name:
`sollya_obj_t sollya_lib_mantissa(sollya_obj_t)`

Usage:

mantissa(x) : constant → integer

Parameters:

- x is a dyadic number.

Description:

- **mantissa**(x) is by definition x if x equals 0, NaN, or Inf.
- If x is not zero, it can be uniquely written as $x = m \cdot 2^e$ where m is an odd integer and e is an integer. **mantissa**(x) returns m.

Example 1:

```
> a=round(Pi,20,RN);
> e=exponent(a);
> m=mantissa(a);
> m;
411775
> a-m*2^e;
0
```

See also: **exponent** (8.63), **precision** (8.139)

8.110 max

Name: **max**
determines which of given constant expressions has maximum value

Library names:
`sollya_obj_t sollya_lib_max(sollya_obj_t, ...)`
`sollya_obj_t sollya_lib_v_max(sollya_obj_t, va_list)`

Usage:

$$\textbf{max}(\textit{expr1},\textit{expr2},...,\textit{exprn}) : (\mathsf{constant}, \mathsf{constant}, ..., \mathsf{constant}) \to \mathsf{constant}$$
$$\textbf{max}(l) : \mathsf{list} \to \mathsf{constant}$$

Parameters:

- *expr* are constant expressions.
- l is a list of constant expressions.

Description:

- **max** determines which of a given set of constant expressions *expr* has maximum value. To do so, **max** tries to increase the precision used for evaluation until it can decide the ordering or some maximum precision is reached. In the latter case, a warning is printed indicating that there might actually be another expression that has a greater value.
- Even though **max** determines the maximum expression by evaluation, it returns the expression that is maximum as is, i.e. as an expression tree that might be evaluated to any accuracy afterwards.
- **max** can be given either an arbitrary number of constant expressions in argument or a list of constant expressions. The list however must not be end-elliptic.
- Users should be aware that the behavior of **max** follows the IEEE 754-2008 standard with respect to NaNs. In particular, **max** evaluates to NaN if and only if all arguments of **max** are NaNs. This means that NaNs may disappear during computations.

Example 1:

```
> max(1,2,3,exp(5),log(0.25));
148.413159102576603421115580040552279623487667593 88
> max(17);
17
```

Example 2:

```
> l = [|1,2,3,exp(5),log(0.25)|];
> max(l);
148.413159102576603421115580040552279623487667593 88
```

Example 3:

```
> print(max(exp(17),sin(62)));
exp(17)
```

Example 4:

```
> verbosity = 1!;
> print(max(17 + log2(13)/log2(9),17 + log(13)/log(9)));
Warning: the tool is unable to decide a maximum computation by evaluation even t
hough faithful evaluation of the terms has been possible. The terms will be cons
idered to be equal.
17 + log2(13) / log2(9)
```

See also: **min** (8.113), == (8.54), **!**= (8.118), >= (8.77), > (8.80), < (8.108), <= (8.99), **in** (8.91), **inf** (8.92), **sup** (8.182)

8.111 mid

Name: **mid**
gives the middle of an interval.

Library name:
`sollya_obj_t sollya_lib_mid(sollya_obj_t)`

Usage:

$$\textbf{mid}(I) : \mathsf{range} \to \mathsf{constant}$$
$$\textbf{mid}(x) : \mathsf{constant} \to \mathsf{constant}$$

Parameters:

- I is an interval.
- x is a real number.

Description:

- Returns the middle of the interval I. If the middle is not exactly representable at the current precision, the value is returned as an unevaluated expression.
- When called on a real number x, **mid** behaves like the identity.

Example 1:

```
> mid([1;3]);
2
> mid(17);
17
```

See also: **inf** (8.92), **sup** (8.182)

8.112 midpointmode

Name: **midpointmode**
global variable controlling the way intervals are displayed.

Library names:
`void sollya_lib_set_midpointmode_and_print(sollya_obj_t)`
`void sollya_lib_set_midpointmode(sollya_obj_t)`
`sollya_obj_t sollya_lib_get_midpointmode()`

Usage:

midpointmode = *activation value* : on|off → void
midpointmode = *activation value* ! : on|off → void
midpointmode : on|off

Parameters:

- *activation value* enables or disables the mode.

Description:

- **midpointmode** is a global variable. When its value is **off**, intervals are displayed as usual (in the form $[a;b]$). When its value is **on**, and if a and b have the same first significant digits, the interval in displayed in a way that lets one immediately see the common digits of the two bounds.
- This mode is supported only with **display** set to **decimal**. In other modes of display, **midpointmode** value is simply ignored.

Example 1:

```
> a = round(Pi,30,RD);
> b = round(Pi,30,RU);
> d = [a,b];
> d;
[3.14159265160560607910156250;3.14159265533089637756347656250]
> midpointmode=on!;
> d;
0.314159265~1/6~e1
```

See also: **on** (8.126), **off** (8.125), **roundingwarnings** (8.167), **display** (8.47), **decimal** (8.36)

8.113 min

Name: **min**
determines which of given constant expressions has minimum value

Library names:
`sollya_obj_t sollya_lib_min(sollya_obj_t, ...)`
`sollya_obj_t sollya_lib_v_min(sollya_obj_t, va_list)`

Usage:

min(*expr1*,*expr2*,...,*exprn*) : (constant, constant, ..., constant) → constant
min(l) : list → constant

Parameters:

- *expr* are constant expressions.
- l is a list of constant expressions.

Description:

- **min** determines which of a given set of constant expressions *expr* has minimum value. To do so, **min** tries to increase the precision used for evaluation until it can decide the ordering or some maximum precision is reached. In the latter case, a warning is printed indicating that there might actually be another expression that has a lesser value.
- Even though **min** determines the minimum expression by evaluation, it returns the expression that is minimum as is, i.e. as an expression tree that might be evaluated to any accuracy afterwards.
- **min** can be given either an arbitrary number of constant expressions in argument or a list of constant expressions. The list however must not be end-elliptic.

- Users should be aware that the behavior of **min** follows the IEEE 754-2008 standard with respect to NaNs. In particular, **min** evaluates to NaN if and only if all arguments of **min** are NaNs. This means that NaNs may disappear during computations.

Example 1:

```
> min(1,2,3,exp(5),log(0.25));
-1.3862943611198906188344642429163531361510002687205
> min(17);
17
```

Example 2:

```
> l = [|1,2,3,exp(5),log(0.25)|];
> min(l);
-1.3862943611198906188344642429163531361510002687205
```

Example 3:

```
> print(min(exp(17),sin(62)));
sin(62)
```

Example 4:

```
> verbosity = 1!;
> print(min(17 + log2(13)/log2(9),17 + log(13)/log(9)));
Warning: the tool is unable to decide a minimum computation by evaluation even t
hough faithful evaluation of the terms has been possible. The terms will be cons
idered to be equal.
17 + log(13) / log(9)
```

See also: **max** (8.110), == (8.54), **!**= (8.118), >= (8.77), > (8.80), < (8.108), <= (8.99), **in** (8.91), **inf** (8.92), **sup** (8.182)

8.114 –

Name: –
subtraction function

Library names:

```
sollya_obj_t sollya_lib_sub(sollya_obj_t, sollya_obj_t)
sollya_obj_t sollya_lib_build_function_sub(sollya_obj_t, sollya_obj_t)
#define SOLLYA_SUB(x,y) sollya_lib_build_function_sub((x), (y))
sollya_obj_t sollya_lib_neg(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_neg(sollya_obj_t)
#define SOLLYA_NEG(x) sollya_lib_build_function_neg(x)
```

Usage:

function1 – *function2* : (function, function) → function
interval1 – *interval2* : (range, range) → range
interval1 – *constant* : (range, constant) → range
interval1 – *constant* : (constant, range) → range
– *function1* : function → function
– *interval1* : range → range

Parameters:

- *function1* and *function2* represent functions

- *interval1* and *interval2* represent intervals (ranges)
- *constant* represents a constant or constant expression

Description:

- − represents the subtraction (function) on reals. The expression *function1* − *function2* stands for the function composed of the subtraction function and the two functions *function1* and *function2*, where *function1* is the subtrahend and *function2* the subtractor.
- − can be used for interval arithmetic on intervals (ranges). − will evaluate to an interval that safely encompasses all images of the subtraction function with arguments varying in the given intervals. Any combination of intervals with intervals or constants (resp. constant expressions) is supported. However, it is not possible to represent families of functions using an interval as one argument and a function (varying in the free variable) as the other one.
- − stands also for the negation function.

Example 1:

```
> 5 - 2;
3
```

Example 2:

```
> x - 2;
-2 + x
```

Example 3:

```
> x - x;
0
```

Example 4:

```
> diff(sin(x) - exp(x));
cos(x) - exp(x)
```

Example 5:

```
> [1;2] - [3;4];
[-3;-1]
> [1;2] - 17;
[-16;-15]
> 13 - [-4;17];
[-4;17]
```

Example 6:

```
> -exp(x);
-exp(x)
> -13;
-13
> -[13;17];
[-17;-13]
```

See also: + (8.132), ∗ (8.116), / (8.49), ^ (8.136)

8.115 mod

Name: **mod**

Computes the euclidian division of polynomials or numbers and returns the rest

Library name:

```
sollya_obj_t sollya_lib_euclidian_mod(sollya_obj_t, sollya_obj_t)
```

Usage:

$$\textbf{mod}(a,\ b) : (\text{function}, \text{function}) \to \text{function}$$

Parameters:

- a is a constant or a polynomial.
- b is a constant or a polynomial.

Description:

- **mod**(a,b) computes $a - (b * \textbf{div}(a,b))$. In other words, it returns the remainder of the Euclidian division of a by b.
- See **div** for subtle cases involving polynomials whose degree can not easily be computed by the tool as their leading coefficient is given as a constant expression that is mathematically zero but for which the tool is unable to detect this fact.

Example 1:

```
> mod(1001, 231);
77
> mod(13, 17);
13
> mod(-14, 15);
1
> mod(-213, -5);
-3
> print(mod(23/13, 11/17));
105 / 221
> print(mod(exp(13),-sin(17)));
exp(13) + 460177 * sin(17)
```

Example 2:

```
> mod(24 + 68 * x + 74 * x^2 + 39 * x^3 + 10 * x^4 + x^5, 4 + 4 * x + x^2);
0
> mod(24 + 68 * x + 74 * x^2 + 39 * x^3 + 10 * x^4 + x^5, 2 * x^3);
24 + x * (68 + x * 74)
> mod(x^2, x^3);
x^2
```

Example 3:

```
> mod(exp(x), x^2);
exp(x)
> mod(x^3, sin(x));
x^3
```

See also: **gcd** (8.76), **div** (8.48), **bezout** (8.19), **numberroots** (8.121)

8.116 ∗

Name: ∗
multiplication function

Library names:
```
sollya_obj_t sollya_lib_mul(sollya_obj_t, sollya_obj_t)
sollya_obj_t sollya_lib_build_function_mul(sollya_obj_t, sollya_obj_t)
#define SOLLYA_MUL(x,y) sollya_lib_build_function_mul((x), (y))
```

Usage:

function1 ∗ *function2* : (function, function) → function
interval1 ∗ *interval2* : (range, range) → range
interval1 ∗ *constant* : (range, constant) → range
interval1 ∗ *constant* : (constant, range) → range

Parameters:

- *function1* and *function2* represent functions
- *interval1* and *interval2* represent intervals (ranges)
- *constant* represents a constant or constant expression

Description:

- ∗ represents the multiplication (function) on reals. The expression *function1* ∗ *function2* stands for the function composed of the multiplication function and the two functions *function1* and *function2*.
- ∗ can be used for interval arithmetic on intervals (ranges). ∗ will evaluate to an interval that safely encompasses all images of the multiplication function with arguments varying in the given intervals. Any combination of intervals with intervals or constants (resp. constant expressions) is supported. However, it is not possible to represent families of functions using an interval as one argument and a function (varying in the free variable) as the other one.

Example 1:
```
> 5 * 2;
10
```

Example 2:
```
> x * 2;
x * 2
```

Example 3:
```
> x * x;
x^2
```

Example 4:
```
> diff(sin(x) * exp(x));
sin(x) * exp(x) + exp(x) * cos(x)
```

Example 5:
```
> [1;2] * [3;4];
[3;8]
> [1;2] * 17;
[17;34]
> 13 * [-4;17];
[-52;221]
```

See also: + (8.132), − (8.114), / (8.49), ^ (8.136)

8.117 nearestint

Name: **nearestint**
the function mapping the reals to the integers nearest to them.

Library names:
```
sollya_obj_t sollya_lib_nearestint(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_nearestint(sollya_obj_t)
#define SOLLYA_NEARESTINT(x) sollya_lib_build_function_nearestint(x)
```

Description:

- **nearestint** is defined as usual: **nearestint**(x) is the integer nearest to x, with the special rule that the even integer is chosen if there exist two integers equally near to x.
- It is defined for every real number x.

See also: **ceil** (8.24), **floor** (8.72), **round** (8.164), **RN** (8.163)

8.118 !=

Name: **!=**
negated equality test operator

Library name:
```
sollya_obj_t sollya_lib_cmp_not_equal(sollya_obj_t, sollya_obj_t)
```

Usage:

expr1 **!=** *expr2* : (any type, any type) → boolean

Parameters:

- *expr1* and *expr2* represent expressions

Description:

- The operator **!=** evaluates to true iff its operands *expr1* and *expr2* are syntactically unequal and both different from **error**, constant expressions that are not constants and that evaluate to two different floating-point number with the global precision **prec** or polynomials that are unequal while automatic expression simplification is activated. The user should be aware of the fact that because of floating-point evaluation, the operator **!=** is not exactly the same as the negation of the mathematical equality. Further, expressions that are polynomials may not be structurally equal when **!=** evaluates to **false**; in order to obtain purely structural tests, the user should deactivate automatic simplification using **autosimplify**.

  Following the IEEE 754 standard, NaN compares unequal to itself, even though this corresponds to a case when *expr1* and *expr2* are syntactically equal and different from error. Accordingly, the interval [NaN, NaN] compares unequal to itself.

  Note that the expressions **!**(*expr1* **!=** *expr2*) and *expr1* == *expr2* do not always evaluate to the same boolean value. See **error** for details.
- Notice that @NaN@, [@NaN, @NaN@] and **error** have special properties with respect to comparison for equality with == and **!=**. See documentation for == for a detailed description.

Example 1:

```
> "Hello" != "Hello";
false
> "Hello" != "Salut";
true
> "Hello" != 5;
true
> 5 + x != 5 + x;
false
```

Example 2:

```
> 1 != exp(0);
false
> asin(1) * 2 != pi;
false
> exp(5) != log(4);
true
```

Example 3:

```
> sin(pi/6) != 1/2 * sqrt(3);
true
```

Example 4:

```
> prec = 12;
The precision has been set to 12 bits.
> 16384.1 != 16385.1;
false
```

Example 5:

```
> NaN != NaN;
true
> [NaN,NaN] != [NaN,NaN];
true
> error != error;
false
```

Example 6:

```
> p = x + x^2;
> q = x * (1 + x);
> autosimplify = on;
Automatic pure tree simplification has been activated.
> p != q;
false
> autosimplify = off;
Automatic pure tree simplification has been deactivated.
> p != q;
true
```

See also: == (8.54), > (8.80), >= (8.77), <= (8.99), < (8.108), **in** (8.91), **!** (8.120), && (8.6), || (8.127), **error** (8.57), **prec** (8.138), **autosimplify** (8.16)

8.119 nop

Name: **nop**
no operation

Usage:

$$\mathbf{nop} : \mathsf{void} \to \mathsf{void}$$
$$\mathbf{nop}() : \mathsf{void} \to \mathsf{void}$$
$$\mathbf{nop}(n) : \mathsf{integer} \to \mathsf{void}$$

Description:

- The command **nop** does nothing. This means it is an explicit parse element in the `Sollya` language that finally does not produce any result or side-effect.
- The command **nop** may take an optional positive integer argument *n*. The argument controls how much (useless) multiprecision floating-point multiplications `Sollya` performs while doing nothing. With this behaviour, **nop** can be used for calibration of timing tests.
- The keyword **nop** is implicit in some procedure definitions. Procedures without imperative body get parsed as if they had an imperative body containing one **nop** statement.

Example 1:

```
> nop;
```

Example 2:

```
> nop(100);
```

Example 3:

```
> succ = proc(n) { return n + 1; };
> succ;
proc(n)
{
nop;
return (n) + (1);
}
> succ(5);
6
```

See also: **proc** (8.146), **time** (8.192)

8.120 !

Name: **!**
boolean NOT operator

Library name:
`sollya_obj_t sollya_lib_negate(sollya_obj_t)`

Usage:

! *expr* : boolean → boolean

Parameters:

- *expr* represents a boolean expression

Description:

- **!** evaluates to the boolean NOT of the boolean expression *expr*. **!** *expr* evaluates to true iff *expr* does not evaluate to true.

Example 1:

```
> ! false;
true
```

Example 2:

```
> ! (1 == exp(0));
false
```

See also: && (8.6), || (8.127)

8.121 numberroots

Name: **numberroots**
Computes the number of roots of a polynomial in a given range.

Library name:
`sollya_obj_t sollya_lib_numberroots(sollya_obj_t, sollya_obj_t)`

Usage:

numberroots(p, I) : (function, range) → integer

Parameters:

- p is a polynomial.
- I is an interval.

Description:

- **numberroots** rigorously computes the number of roots of polynomial the p in the interval I. The technique used is Sturm's algorithm. The value returned is not just a numerical estimation of the number of roots of p in I: it is the exact number of roots.
- The command **findzeros** computes safe enclosures of all the zeros of a function, without forgetting any, but it is not guaranteed to separate them all in distinct intervals. **numberroots** is more accurate since it guarantees the exact number of roots. However, it does not compute them. It may be used, for instance, to certify that **findzeros** did not put two distinct roots in the same interval.
- Multiple roots are counted only once.
- The interval I must be bounded. The algorithm cannot handle unbounded intervals. Moreover, the interval is considered as a closed interval: if one (or both) of the endpoints of I are roots of p, they are counted.
- The argument p can be any expression, but if `Sollya` fails to prove that it is a polynomial an error is produced. Also, please note that if the coefficients of p or the endpoints of I are not exactly representable, they are first numerically evaluated, before the algorithm is used. In that case, the counted number of roots corresponds to the rounded polynomial on the rounded interval **and not** to the exact parameters given by the user. A warning is displayed to inform the user.

Example 1:

```
> numberroots(1+x-x^2, [1,2]);
1
> findzeros(1+x-x^2, [1,2]);
[|[1.617919921875;1.6180419921875]|]
```

Example 2:

```
> numberroots((1+x)*(1-x), [-1,1]);
2
> numberroots(x^2, [-1,1]);
1
```

Example 3:

```
> verbosity = 1!;
> numberroots(x-pi, [0,4]);
Warning: the 0th coefficient of the polynomial is neither a floating point
constant nor can be evaluated without rounding to a floating point constant.
Will faithfully evaluate it with the current precision (165 bits)
1
```

Example 4:

```
> verbosity = 1!;
> numberroots(1+x-x^2, [0, @Inf@]);
Warning: the given interval must have finite bounds.
Warning: at least one of the given expressions or a subexpression is not correct
ly typed
or its evaluation has failed because of some error on a side-effect.
error
> numberroots(exp(x), [0, 1]);
Warning: the given function must be a polynomial in this context.
Warning: at least one of the given expressions or a subexpression is not correct
ly typed
or its evaluation has failed because of some error on a side-effect.
error
```

See also: **dirtyfindzeros** (8.43), **findzeros** (8.69), **gcd** (8.76)

8.122 numerator

Name: **numerator**
gives the numerator of an expression

Library name:
`sollya_obj_t sollya_lib_numerator(sollya_obj_t)`

Usage:

numerator(*expr*) : function → function

Parameters:

- *expr* represents an expression

Description:

- If *expr* represents a fraction *expr1*/*expr2*, **numerator**(*expr*) returns the numerator of this fraction, i.e. *expr1*.

  If *expr* represents something else, **numerator**(*expr*) returns the expression itself, i.e. *expr*.

  Note that for all expressions *expr*, **numerator**(*expr*) **/ denominator**(*expr*) is equal to *expr*.

Example 1:

```
> numerator(5/3);
5
```

Example 2:

```
> numerator(exp(x));
exp(x)
```

Example 3:

```
> a = 5/3;
> b = numerator(a)/denominator(a);
> print(a);
5 / 3
> print(b);
5 / 3
```

Example 4:

```
> a = exp(x/3);
> b = numerator(a)/denominator(a);
> print(a);
exp(x / 3)
> print(b);
exp(x / 3)
```

See also: **denominator** (8.39), **rationalmode** (8.153)

8.123 object

Name: **object**
keyword representing a `Sollya` object type

Library name:
`SOLLYA_EXTERNALPROC_TYPE_OBJECT`

Usage:

object : type type

Description:

- **object** represents the `Sollya` object type for declarations of external procedures **externalproc**.

  Remark that in contrast to other indicators, type indicators like **object** cannot be handled outside the **externalproc** context. In particular, they cannot be assigned to variables.

See also: **externalproc** (8.66), **boolean** (8.22), **constant** (8.30), **function** (8.75), **list of** (8.103), **range** (8.151), **string** (8.179), **integer** (8.94)

8.124 objectname

Name: **objectname**
returns, given a `Sollya` object, a string that can be reparsed to the object

Library name:
`sollya_obj_t sollya_lib_objectname(sollya_obj_t);`

Usage:

objectname(*obj*) : any type → string

Description:

- **objectname**(*obj*) queries the `Sollya` symbol table in order to recover the name of an identifier the object *obj* is assigned to. If it succeeds, it returns a string containing the recovered identifier. In contrast, if it does not succeed, it returns a string simply containing a textual representation of *obj*.
- The only condition for an identifier to be eligible to be returned by **objectname**(*obj*) is to be accessible in the scope **objectname** is executed in, *i.e.*, not to be shadowed by an identifier of the same name which does not hold the object *obj*.
- In any case, if the string returned by **objectname** is given to the **parse** command in the same scope, the original object *obj* is recovered.
- **objectname** is particularly useful in combination with **getbacktrace**, when the `Sollya` procedure stack is to be displayed in a fashion, where procedures are identified by their name and not their procedural content.

- **objectname** may also be used to get a string representation of the free mathematical variable.
- If an object is simply to be cast into a string, without trying to retrieve an identifier for it, **objectname** is not appropriate. In this case, it suffices to concatenate it to an empty string with the **@** operator.

Example 1:

```
> s = "Hello";
> objectname("Hello");
s
```

Example 2:

```
> f = exp(x);
> g = sin(x);
> [| objectname(exp(x)), objectname(sin(x)), objectname(cos(x)) |];
[|"f", "g", "cos(x)"|]
```

Example 3:

```
> o = { .f = exp(x), .I = [-1;1] };
> s1 = o@""; s1;
{ .f = exp(x), .I = [-1;1] }
> s2 = objectname({ .I = [-1;1], .f = exp(x)}); s2;
o
> parse(s1) == parse(s2);
true
> write("s2 = \"", s2, "\" parses to ", parse(s2), "\n");
s2 = "o" parses to { .f = exp(x), .I = [-1;1] }
```

Example 4:

```
> n = 1664;
> objectname(n);
n
```

Example 5:

```
> f = exp(x);
> g = sin(x);
> procedure test() {
      var f;
      var h;
      f = tan(x);
      h = cos(x);
      [| objectname(exp(x)), objectname(sin(x)), objectname(cos(x)), objectname(
tan(x)) |];
  };
> test();
[|"exp(x)", "g", "h", "f"|]
```

Example 6:

```
> procedure apply_proc(p, a, b) {
      return p(a, b);
  };
> procedure show_trace_and_add(n, m) {
      var i, bt;
      bt = getbacktrace();
      write("Procedure stack:\n");
      for i from 0 to length(bt) - 1 do {
          write("   Procedure ", objectname((bt[i]).called_proc), " called with
", length((bt[i]).passed_args), " arguments\n");
      };
      write("\n");
      return n + m;
  };
> procedure show_and_succ(u) {
        return apply_proc(show_trace_and_add, u, 1);
  };
> show_and_succ(16);
Procedure stack:
   Procedure show_trace_and_add called with 2 arguments
   Procedure apply_proc called with 3 arguments
   Procedure show_and_succ called with 1 arguments

17
```

Example 7:

```
> f = exp(three_decker_sauerkraut_and_toadstool_sandwich_with_arsenic_sauce);
> g = sin(_x_);
> h = f(g);
> h;
exp(sin(three_decker_sauerkraut_and_toadstool_sandwich_with_arsenic_sauce))
> objectname(_x_);
three_decker_sauerkraut_and_toadstool_sandwich_with_arsenic_sauce
```

See also: **parse** (8.128), **var** (8.197), **getbacktrace** (8.78), **proc** (8.146), **procedure** (8.147), **@** (8.29)

8.125 off

Name: **off**
special value for certain global variables.

Library names:
`sollya_obj_t sollya_lib_off()`
`int sollya_lib_is_off(sollya_obj_t)`

Description:

- **off** is a special value used to deactivate certain functionnalities of `Sollya`.
- As any value it can be affected to a variable and stored in lists.

Example 1:

```
> canonical=on;
Canonical automatic printing output has been activated.
> p=1+x+x^2;
> mode=off;
> p;
1 + x + x^2
> canonical=mode;
Canonical automatic printing output has been deactivated.
> p;
1 + x * (1 + x)
```

See also: **on** (8.126), **autosimplify** (8.16), **canonical** (8.23), **timing** (8.193), **fullparentheses** (8.74), **midpointmode** (8.112), **rationalmode** (8.153), **roundingwarnings** (8.167), **timing** (8.193), **dieonerrormode** (8.41)

8.126 on

Name: **on**
special value for certain global variables.

Library names:
`sollya_obj_t sollya_lib_on()`
`int sollya_lib_is_on(sollya_obj_t)`

Description:

- **on** is a special value used to activate certain functionnalities of `Sollya`.
- As any value it can be affected to a variable and stored in lists.

Example 1:

```
> p=1+x+x^2;
> mode=on;
> p;
1 + x * (1 + x)
> canonical=mode;
Canonical automatic printing output has been activated.
> p;
1 + x + x^2
```

See also: **off** (8.125), **autosimplify** (8.16), **canonical** (8.23), **timing** (8.193), **fullparentheses** (8.74), **midpointmode** (8.112), **rationalmode** (8.153), **roundingwarnings** (8.167), **timing** (8.193), **dieonerrormode** (8.41)

8.127 ||

Name: ||
boolean OR operator

Library name:
`sollya_obj_t sollya_lib_or(sollya_obj_t, sollya_obj_t)`

Usage:

$$expr1 \;||\; expr2 : (\mathsf{boolean}, \mathsf{boolean}) \to \mathsf{boolean}$$

Parameters:

- *expr1* and *expr2* represent boolean expressions

Description:

- || evaluates to the boolean OR of the two boolean expressions *expr1* and *expr2*. || evaluates to true iff at least one of *expr1* or *expr2* evaluates to true.

Example 1:

```
> false || false;
false
```

Example 2:

```
> (1 == exp(0)) || (0 == log(1));
true
```

See also: && (8.6), **!** (8.120)

8.128 parse

Name: **parse**
parses an expression contained in a string

Library name:
`sollya_obj_t sollya_lib_parse(sollya_obj_t)`

Usage:

parse(*string*) : string → function | error

Parameters:

- *string* represents a character sequence

Description:

- **parse**(*string*) parses the character sequence *string* containing an expression built on constants and base functions.

  If the character sequence does not contain a well-defined expression, a warning is displayed indicating a syntax error and **parse** returns a **error** of type error.

- The character sequence to be parsed by **parse** may contain commands that return expressions, including **parse** itself. Those commands get executed after the string has been parsed. **parse**(*string*) will return the expression computed by the commands contained in the character sequence *string*.

- **parse** expects an expression (or a command returning an expression) but cannot be used to evaluate a command producing a side effect without returning a result, nor to evaluate a sequence of instructions. However, such a behavior may be obtained by encapsulating the side effects into an anonymous procedure that parse can handle and by immediately applying it (see examples below).

Example 1:

```
> parse("exp(x)");
exp(x)
```

Example 2:

```
> text = "remez(exp(x),5,[-1;1])";
> print("The string", text, "gives", parse(text));
The string remez(exp(x),5,[-1;1]) gives 8.738190988275620367686831573168760490396
64388498642e-3 * x^5 + 4.379369637959601547823317126536527289379500558838 1e-2 *
x^4 + 0.166424656149527681851294338440121939256540657559 05 * x^3 + 0.49919698262
963614991826575452094101562044819693772 * x^2 + 1.000038346505998154663400680582
31011540878088492516 * x + 1.000044750295595026062037128165582433840775229322 13
```

Example 3:

```
> verbosity = 1!;
> parse("5 + * 3");
Warning: syntax error. Will try to continue parsing (expecting ";"). May leak me
mory.
Warning: the string "5 + * 3" could not be parsed by the miniparser.
Warning: at least one of the given expressions or a subexpression is not correct
ly typed
or its evaluation has failed because of some error on a side-effect.
error
```

Example 4:

```
> i = 17;
> parse("i = 42");
error
> i;
17
> (parse("proc () { i = 42; }"))();
> i;
42
```

See also: **execute** (8.59), **readfile** (8.155), **print** (8.141), **error** (8.57), **dieonerrormode** (8.41), **proc** (8.146)

8.129 perturb

Name: **perturb**
indicates random perturbation of sampling points for **externalplot**

Library names:
`sollya_obj_t sollya_lib_perturb()`
`int sollya_lib_is_perturb(sollya_obj_t)`

Usage:

perturb : perturb

Description:

- The use of **perturb** in the command **externalplot** enables the addition of some random noise around each sampling point in **externalplot**.

  See **externalplot** for details.

Example 1:

```
> bashexecute("gcc -fPIC -c externalplotexample.c");
> bashexecute("gcc -shared -o externalplotexample externalplotexample.o -lgmp -l
mpfr");
> externalplot("./externalplotexample",relative,exp(x),[-1/2;1/2],12,perturb);
```

See also: **externalplot** (8.65), **absolute** (8.2), **relative** (8.157), **bashexecute** (8.18)

8.130 pi

Name: **pi**
the constant π.

Library names:

```
sollya_obj_t sollya_lib_pi()
int sollya_lib_is_pi(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_pi()
#define SOLLYA_PI (sollya_lib_build_function_pi())
```

Description:

- **pi** is the constant π, defined as half the period of sine and cosine.
- In `Sollya`, **pi** is considered a 0-ary function. This way, the constant is not evaluated at the time of its definition but at the time of its use. For instance, when you define a constant or a function relating to π, the current precision at the time of the definition does not matter. What is important is the current precision when you evaluate the function or the constant value.
- Remark that when you define an interval, the bounds are first evaluated and then the interval is defined. In this case, **pi** will be evaluated as any other constant value at the definition time of the interval, thus using the current precision at this time.

Example 1:

```
> verbosity=1!; prec=12!;
> a = 2*pi;
> a;
Warning: rounding has happened. The value displayed is a faithful rounding to 12
 bits of the true result.
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
6.283
> prec=20!;
> a;
Warning: rounding has happened. The value displayed is a faithful rounding to 20
 bits of the true result.
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
6.28319
```

Example 2:

```
> display=binary;
Display mode is binary numbers.
> prec=12!;
> d = [pi; 5];
> d;
[1.1001001_2 * 2^(1);1.01_2 * 2^(2)]
> prec=20!;
> d;
[1.1001001_2 * 2^(1);1.01_2 * 2^(2)]
```

See also: **cos** (8.31), **sin** (8.174), **tan** (8.186), **asin** (8.11), **acos** (8.4), **atan** (8.13), **evaluate** (8.58), **prec** (8.138), **libraryconstant** (8.102)

8.131 plot

Name: **plot**
plots one or several functions

Library names:

```
void sollya_lib_plot(sollya_obj_t, sollya_obj_t, ...)
void sollya_lib_v_plot(sollya_obj_t, sollya_obj_t, va_list)
```

Usage:

plot(*f1*, ... ,*fn*, *I*) : (function, ... ,function, range) → void
plot(*f1*, ... ,*fn*, *I*, **file**, *name*) : (function, ... ,function, range, **file**, string) → void
plot(*f1*, ... ,*fn*, *I*, **postscript**, *name*) : (function, ... ,function, range, **postscript**, string) → void
plot(*f1*, ... ,*fn*, *I*, **postscriptfile**, *name*) : (function, ... ,function, range, **postscriptfile**, string) → void
plot(*L*, *I*) : (list, range) → void
plot(*L*, *I*, **file**, *name*) : (list, range, **file**, string) → void
plot(*L*, *I*, **postscript**, *name*) : (list, range, **postscript**, string) → void
plot(*L*, *I*, **postscriptfile**, *name*) : (list, range, **postscriptfile**, string) → void

Parameters:

- *f1*, ..., *fn* are functions to be plotted.
- *L* is a list of functions to be plotted.
- *I* is the interval where the functions have to be plotted.
- *name* is a string representing the name of a file.

Description:

- This command plots one or several functions *f1*, ... ,*fn* on an interval *I*. Functions can be either given as parameters of **plot** or as a list *L* which elements are functions. The functions are drawn on the same plot with different colors.
- If *L* contains an element that is not a function (or a constant), an error occurs.
- **plot** relies on the value of global variable **points**. Let *n* be the value of this variable. The algorithm is the following: each function is evaluated at *n* evenly distributed points in *I*. At each point, the computed value is a faithful rounding of the exact value with a sufficiently high precision. Each point is finally plotted. This should avoid numerical artefacts such as critical cancellations.
- The plot can be saved either as a data file or as a postscript file.
- If you use argument **file** with a string *name*, `Sollya` will save a data file called name.dat and a gnuplot directives file called name.p. Invoking gnuplot on name.p will plot the data stored in name.dat.
- If you use argument **postscript** with a string *name*, `Sollya` will save a postscript file called name.eps representing your plot.
- If you use argument **postscriptfile** with a string *name*, `Sollya` will produce the corresponding name.dat, name.p and name.eps.
- By default, this command uses gnuplot to produce the final plot. If `Sollya` is run while the environment variable `SOLLYA_GNUPLOT` is set, the content of that variable is used as the gnuplot binary. If your terminal is not graphic (typically if you use `Sollya` through ssh without -X) gnuplot should be able to detect that and produce an ASCII-art version on the standard output. If it is not the case, you can either store the plot in a postscript file to view it locally, or use **asciiplot** command.
- If every function is constant, **plot** will not plot them but just display their value.
- If the interval is reduced to a single point, **plot** will just display the value of the functions at this point.

Example 1:

```
> plot(sin(x),0,cos(x),[-Pi,Pi]);
```

Example 2:

```
> plot(sin(x),0,cos(x),[-Pi,Pi],postscriptfile,"plotSinCos");
```

Example 3:

```
> plot(exp(0), sin(1), [0;1]);
1
0.84147098480789650665250232163029899962256306079837
```

Example 4:

```
> plot(sin(x), cos(x), [1;1]);
0.84147098480789650665250232163029899962256306079837
0.54030230586813971740093660744297660373231042061792
```

See also: **externalplot** (8.65), **asciiplot** (8.10), **file** (8.68), **postscript** (8.134), **postscriptfile** (8.135), **points** (8.133)

8.132 +

Name: +
addition function

Library names:

```
sollya_obj_t sollya_lib_add(sollya_obj_t, sollya_obj_t)
sollya_obj_t sollya_lib_build_function_add(sollya_obj_t, sollya_obj_t)
#define SOLLYA_ADD(x,y) sollya_lib_build_function_add((x), (y))
```

Usage:

$$\begin{aligned} \textit{function1} + \textit{function2} &: (\mathsf{function}, \mathsf{function}) \to \mathsf{function} \\ \textit{interval1} + \textit{interval2} &: (\mathsf{range}, \mathsf{range}) \to \mathsf{range} \\ \textit{interval1} + \textit{constant} &: (\mathsf{range}, \mathsf{constant}) \to \mathsf{range} \\ \textit{interval1} + \textit{constant} &: (\mathsf{constant}, \mathsf{range}) \to \mathsf{range} \end{aligned}$$

Parameters:

- *function1* and *function2* represent functions
- *interval1* and *interval2* represent intervals (ranges)
- *constant* represents a constant or constant expression

Description:

- + represents the addition (function) on reals. The expression *function1* + *function2* stands for the function composed of the addition function and the two functions *function1* and *function2*.
- + can be used for interval arithmetic on intervals (ranges). + will evaluate to an interval that safely encompasses all images of the addition function with arguments varying in the given intervals. Any combination of intervals with intervals or constants (resp. constant expressions) is supported. However, it is not possible to represent families of functions using an interval as one argument and a function (varying in the free variable) as the other one.

Example 1:

```
> 1 + 2;
3
```

Example 2:

```
> x + 2;
2 + x
```

Example 3:

```
> x + x;
x * 2
```

Example 4:

```
> diff(sin(x) + exp(x));
cos(x) + exp(x)
```

Example 5:

```
> [1;2] + [3;4];
[4;6]
> [1;2] + 17;
[18;19]
> 13 + [-4;17];
[9;30]
```

See also: − (8.114), ∗ (8.116), / (8.49), ^ (8.136)

8.133 points

Name: **points**
controls the number of points chosen by `Sollya` in certain commands.

Library names:
`void sollya_lib_set_points_and_print(sollya_obj_t)`
`void sollya_lib_set_points(sollya_obj_t)`
`sollya_obj_t sollya_lib_get_points()`

Usage:

$$\textbf{points} = n : \text{integer} \rightarrow \text{void}$$
$$\textbf{points} = n\ ! : \text{integer} \rightarrow \text{void}$$
$$\textbf{points} : \text{constant}$$

Parameters:

- n represents the number of points

Description:

- **points** is a global variable. Its value represents the number of points used in numerical algorithms of `Sollya` (namely **dirtyinfnorm**, **dirtyintegral**, **dirtyfindzeros**, **plot**).

Example 1:

```
> f=x^2*sin(1/x);
> points=10;
The number of points has been set to 10.
> dirtyfindzeros(f, [0;1]);
[|0, 0.31830988618379067153776752674502872406891929148092|]
> points=100;
The number of points has been set to 100.
> dirtyfindzeros(f, [0;1]);
[|0, 2.448537586029159011828980974961759415914763780622 4e-2, 3.97887357729738339
42220940843128590508614911435115e-2, 4.54728408833986673625382181064326748669884
70211559e-2, 5.30516476972984452562945877908381206781532152468 19e-2, 6.366197723
67581343075535053490057448137838582961 84e-2, 7.95774715459476678844418816862571 8
101722982287023e-2, 0.10610329539459689051258917558167624135630643049363 8, 0.159
15494309189533576888376337251436203445964574046, 0.31830988618379067153776752674
502872406891929148092|]
```

See also: **dirtyinfnorm** (8.44), **dirtyintegral** (8.45), **dirtyfindzeros** (8.43), **plot** (8.131), **diam** (8.40), **prec** (8.138)

8.134 postscript

Name: **postscript**
special value for commands **plot** and **externalplot**

Library names:
```
sollya_obj_t sollya_lib_postscript()
int sollya_lib_is_postscript(sollya_obj_t)
```

Description:

- **postscript** is a special value used in commands **plot** and **externalplot** to save the result of the command in a postscript file.
- As any value it can be affected to a variable and stored in lists.

Example 1:

```
> savemode=postscript;
> name="plotSinCos";
> plot(sin(x),0,cos(x),[-Pi,Pi],savemode, name);
```

See also: **externalplot** (8.65), **plot** (8.131), **file** (8.68), **postscriptfile** (8.135)

8.135 postscriptfile

Name: **postscriptfile**
special value for commands **plot** and **externalplot**

Library names:
```
sollya_obj_t sollya_lib_postscriptfile()
int sollya_lib_is_postscriptfile(sollya_obj_t)
```

Description:

- **postscriptfile** is a special value used in commands **plot** and **externalplot** to save the result of the command in a data file and a postscript file.
- As any value it can be affected to a variable and stored in lists.

Example 1:

```
> savemode=postscriptfile;
> name="plotSinCos";
> plot(sin(x),0,cos(x),[-Pi,Pi],savemode, name);
```

See also: **externalplot** (8.65), **plot** (8.131), **file** (8.68), **postscript** (8.134)

8.136 ^

Name: ^
power function

Library names:
```
sollya_obj_t sollya_lib_pow(sollya_obj_t, sollya_obj_t)
sollya_obj_t sollya_lib_build_function_pow(sollya_obj_t, sollya_obj_t)
#define SOLLYA_POW(x,y) sollya_lib_build_function_pow((x), (y))
```

Usage:

function1 ^ *function2* : (function, function) → function
interval1 ^ *interval2* : (range, range) → range
interval1 ^ *constant* : (range, constant) → range
interval1 ^ *constant* : (constant, range) → range

Parameters:

- *function1* and *function2* represent functions
- *interval1* and *interval2* represent intervals (ranges)
- *constant* represents a constant or constant expression

Description:

- ^ represents the power (function) on reals. The expression *function1* ^ *function2* stands for the function composed of the power function and the two functions *function1* and *function2*, where *function1* is the base and *function2* the exponent. If *function2* is a constant integer, ^ is defined on negative values of *function1*. Otherwise ^ is defined as $e^{y \cdot \ln x}$.
- Note that whenever several ^ are composed, the priority goes to the last ^. This corresponds to the natural way of thinking when a tower of powers is written on a paper. Thus, `2^3^5` is read as 2^{3^5} and is interpreted as $2^{(3^5)}$.
- ^ can be used for interval arithmetic on intervals (ranges). ^ will evaluate to an interval that safely encompasses all images of the power function with arguments varying in the given intervals. If the intervals given contain points where the power function is not defined, infinities and NaNs will be produced in the output interval. Any combination of intervals with intervals or constants (resp. constant expressions) is supported. However, it is not possible to represent families of functions using an interval as one argument and a function (varying in the free variable) as the other one.

Example 1:

```
> 5 ^ 2;
25
```

Example 2:

```
> x ^ 2;
x^2
```

Example 3:

```
> 3 ^ (-5);
4.11522633744855967078189300411522633744855967078l9e-3
```

Example 4:

```
> (-3) ^ (-2.5);
NaN
```

Example 5:

```
> diff(sin(x) ^ exp(x));
sin(x)^exp(x) * ((cos(x) * exp(x)) / sin(x) + exp(x) * log(sin(x)))
```

Example 6:

```
> 2^3^5;
14134776518227074636666380005943348126619871175004951664972849610340958208
> (2^3)^5;
32768
> 2^(3^5);
14134776518227074636666380005943348126619871175004951664972849610340958208
```

Example 7:

```
> [1;2] ^ [3;4];
[1;16.0000000000000000000000000000000000000000000000000001]
> [1;2] ^ 17;
[1;131072]
> 13 ^ [-4;17];
[3.501277966457757081334687160813696999404782745702e-5;8650415919381337933]
```

See also: + (8.132), − (8.114), ∗ (8.116), / (8.49)

8.137 powers

Name: **powers**
special value for global state **display**

Library names:
`sollya_obj_t sollya_lib_powers()`
`int sollya_lib_is_powers(sollya_obj_t)`

Description:

- **powers** is a special value used for the global state **display**. If the global state **display** is equal to **powers**, all data will be output in dyadic notation with numbers displayed in a Maple and PARI/GP compatible format.

  As any value it can be affected to a variable and stored in lists.

See also: **decimal** (8.36), **dyadic** (8.53), **hexadecimal** (8.84), **binary** (8.20), **display** (8.47)

8.138 prec

Name: **prec**
controls the precision used in numerical computations.

Library names:
`void sollya_lib_set_prec_and_print(sollya_obj_t)`
`void sollya_lib_set_prec(sollya_obj_t)`
`sollya_obj_t sollya_lib_get_prec()`

Description:

- **prec** is a global variable. Its value represents the precision of the floating-point format used in numerical computations.
- Many commands try to adapt their working precision in order to have approximately n correct bits in output, where n is the value of **prec**.

Example 1:

```
> display=binary!;
> prec=50;
The precision has been set to 50 bits.
> dirtyinfnorm(exp(x),[1;2]);
1.11011000111001100100101110001101010011011101 1_2 * 2^(2)
> prec=100;
The precision has been set to 100 bits.
> dirtyinfnorm(exp(x),[1;2]);
1.11011000111001100100101110001101010011011101101011011100110000110011101000111
11101000100000011011_2 * 2^(2)
```

See also: **evaluate** (8.58), **diam** (8.40)

8.139 precision

Name: **precision**
returns the precision necessary to represent a number.

Library name:
`sollya_obj_t sollya_lib_precision(sollya_obj_t)`

Usage:

$$\textbf{precision}(x) : \text{constant} \rightarrow \text{integer}$$

Parameters:

- x is a dyadic number.

Description:

- **precision**(x) is by definition $|x|$ if x equals 0, NaN, or Inf.
- If x is not zero, it can be uniquely written as $x = m \cdot 2^e$ where m is an odd integer and e is an integer. **precision**(x) returns the number of bits necessary to write m in binary (i.e. $1 + \lfloor \log_2(m) \rfloor$).

Example 1:

```
> a=round(Pi,20,RN);
> precision(a);
19
> m=mantissa(a);
> 1+floor(log2(m));
19
```

Example 2:

```
> a=255;
> precision(a);
8
> m=mantissa(a);
> 1+floor(log2(m));
8
```

Example 3:

```
> a=256;
> precision(a);
1
> m=mantissa(a);
> 1+floor(log2(m));
1
```

See also: **mantissa** (8.109), **exponent** (8.63), **round** (8.164)

8.140 .:

Name: **.:**
add an element at the beginning of a list.

Library name:
`sollya_obj_t sollya_lib_prepend(sollya_obj_t, sollya_obj_t)`

Usage:

$$x\textbf{.:}L : (\text{any type, list}) \to \text{list}$$

Parameters:

- x is an object of any type.
- L is a list (possibly empty).

Description:

- **.:** adds the element x at the beginning of the list L.
- Note that since x may be of any type, it can be in particular a list.

Example 1:

```
> 1.:[|2,3,4|];
[|1, 2, 3, 4|]
```

Example 2:

```
> [|1,2,3|].:[|4,5,6|];
[|[|1, 2, 3|], 4, 5, 6|]
```

Example 3:

```
> 1.:[||];
[|1|]
```

See also: **:.** (8.8), **@** (8.29)

8.141 print

Name: **print**
prints an expression

Usage:

$$\textbf{print}(\textit{expr1},...,\textit{exprn}) : (\text{any type},..., \text{any type}) \to \text{void}$$
$$\textbf{print}(\textit{expr1},...,\textit{exprn}) > \textit{filename} : (\text{any type},..., \text{any type}, \text{string}) \to \text{void}$$
$$\textbf{print}(\textit{expr1},...,\textit{exprn}) >> \textit{filename} : (\text{any type},...,\text{any type}, \text{string}) \to \text{void}$$

Parameters:

- *expr* represents an expression
- *filename* represents a character sequence indicating a file name

Description:

- **print**(*expr1*,...,*exprn*) prints the expressions *expr1* through *exprn* separated by spaces and followed by a newline.

  If a second argument *filename* is given after a single ">", the displaying is not output on the standard output of `Sollya` but if in the file *filename* that get newly created or overwritten. If a double ">>" is given, the output will be appended to the file *filename*.

  The global variables **display**, **midpointmode** and **fullparentheses** have some influence on the formatting of the output (see **display**, **midpointmode** and **fullparentheses**).

  Remark that if one of the expressions *expri* given in argument is of type string, the character sequence *expri* evaluates to is displayed. However, if *expri* is of type list and this list contains a variable of type string, the expression for the list is displayed, i.e. all character sequences get displayed surrounded by double quotes ("). Nevertheless, escape sequences used upon defining character sequences are interpreted immediately.

Example 1:

```
> print(x + 2 + exp(sin(x)));
x + 2 + exp(sin(x))
> print("Hello","world");
Hello world
> print("Hello","you", 4 + 3, "other persons.");
Hello you 7 other persons.
```

Example 2:

```
> print("Hello");
Hello
> print([|"Hello"|]);
[|"Hello"|]
> s = "Hello";
> print(s,[|s|]);
Hello [|"Hello"|]
> t = "Hello\tyou";
> print(t,[|t|]);
Hello    you [|"Hello\tyou"|]
```

Example 3:

```
> print(x + 2 + exp(sin(x))) > "foo.sol";
> readfile("foo.sol");
x + 2 + exp(sin(x))
```

Example 4:

```
> print(x + 2 + exp(sin(x))) >> "foo.sol";
```

Example 5:

```
> display = decimal;
Display mode is decimal numbers.
> a = evaluate(sin(pi * x), 0.25);
> b = evaluate(sin(pi * x), [0.25; 0.25 + 1b-50]);
> print(a);
0.707106781186547524400844362104849039284835937688 47
> display = binary;
Display mode is binary numbers.
> print(a);
1.0110101000001001111001100110011111111001110111100110010010000100010110010111110
1100010011011001101110101010010101011111010011111000111010110111101100000101110101
010001_2 * 2^(-1)
> display = hexadecimal;
Display mode is hexadecimal numbers.
> print(a);
0x1.6a09e667f3bcc908b2fb1366ea957d3e3adec1751p-1
> display = dyadic;
Display mode is dyadic numbers.
> print(a);
330700069911015586133239834882209443600671071332 65b-165
> display = powers;
Display mode is dyadic numbers in integer-power-of-2 notation.
> print(a);
330700069911015586133239834882209443600671071332 65 * 2^(-165)
> display = decimal;
Display mode is decimal numbers.
> midpointmode = off;
Midpoint mode has been deactivated.
> print(b);
[0.707106781186547524400844362104849039284835937688 45;0.707106781186549497437217
825175573477826462744170 49]
> midpointmode = on;
Midpoint mode has been activated.
> print(b);
0.7071067811865~4/5~
> display = dyadic;
Display mode is dyadic numbers.
> print(b);
[206687543694384741333274896801380902250419419582 9b-161;165350034955508254441962
370193859364144326751565 71b-164]
> display = decimal;
Display mode is decimal numbers.
> autosimplify = off;
Automatic pure tree simplification has been deactivated.
> fullparentheses = off;
Full parentheses mode has been deactivated.
> print(x + x * ((x + 1) + 1));
x + x * (x + 1 + 1)
> fullparentheses = on;
Full parentheses mode has been activated.
> print(x + x * ((x + 1) + 1));
x + (x * ((x + 1) + 1))
```

See also: **write** (8.201), **printexpansion** (8.143), **printdouble** (8.142), **printsingle** (8.144), **printxml** (8.145), **readfile** (8.155), **autosimplify** (8.16), **display** (8.47), **midpointmode** (8.112), **fullparentheses** (8.74), **evaluate** (8.58), **rationalmode** (8.153)

8.142 printdouble

Name: **printdouble**
prints a constant value as a hexadecimal double precision number

Library name:
`void sollya_lib_printdouble(sollya_obj_t)`

Usage:

printdouble(*constant*) : constant → void

Parameters:

- *constant* represents a constant

Description:

- Prints a constant value as a hexadecimal number on 16 hexadecimal digits. The hexadecimal number represents the integer equivalent to the 64 bit memory representation of the constant considered as a double precision number.

  If the constant value does not hold on a double precision number, it is first rounded to the nearest double precision number before displayed. A warning is displayed in this case.

Example 1:

```
> printdouble(3);
0x4008000000000000
```

Example 2:

```
> prec=100!;
> verbosity = 1!;
> printdouble(exp(5));
Warning: the given expression is not a constant but an expression to evaluate. A
 faithful evaluation to 100 bits will be used.
Warning: rounding down occurred before printing a value as a double.
0x40628d389970338f
```

See also: **printsingle** (8.144), **printexpansion** (8.143), **double** (8.50)

8.143 printexpansion

Name: **printexpansion**
prints a polynomial in Horner form with its coefficients written as a expansions of double precision numbers

Library name:
`void sollya_lib_printexpansion(sollya_obj_t)`

Usage:

printexpansion(*polynomial*) : function → void

Parameters:

- *polynomial* represents the polynomial to be printed

Description:

- The command **printexpansion** prints the polynomial *polynomial* in Horner form writing its coefficients as expansions of double precision numbers. The double precision numbers themselves are displayed in hexadecimal memory notation (see **printdouble**).

  If some of the coefficients of the polynomial *polynomial* are not floating-point constants but constant expressions, they are evaluated to floating-point constants using the global precision **prec**. If a rounding occurs in this evaluation, a warning is displayed.

  If the exponent range of double precision is not sufficient to display all the mantissa bits of a coefficient, the coefficient is displayed rounded and a warning is displayed.

  If the argument *polynomial* does not a polynomial, nothing but a warning or a newline is displayed. Constants can be displayed using **printexpansion** since they are polynomials of degree 0.

Example 1:

```
> printexpansion(roundcoefficients(taylor(exp(x),5,0),[|DD...|]));
0x3ff0000000000000 + x * (0x3ff0000000000000 + x * (0x3fe0000000000000 + x * ((0
x3fc5555555555555 + 0x3c65555555555555) + x * ((0x3fa5555555555555 + 0x3c4555555
5555555) + x * (0x3f81111111111111 + 0x3c01111111111111)))))
```

Example 2:

```
> printexpansion(remez(exp(x),5,[-1;1]));
(0x3ff0002eec90e5a6 + 0x3c9ea6a6a0087757 + 0xb8eb3e644ef44998) + x * ((0x3ff0002
8358fd3ac + 0x3c8ffa7d96c95f7a + 0xb91da9809b13dd54 + 0x35c0000000000000) + x *
((0x3fdff2d7e6a9fea5 + 0x3c74460e4c0e4fe2 + 0x38fcd1b6b4e85bb0 + 0x3590000000000
000) + x * ((0x3fc54d6733b4839e + 0x3c6654e4d8614a44 + 0xb905c7a26b66ea92 + 0xb5
98000000000000) + x * ((0x3fa66c209b7150a8 + 0x3c34b1bba8f78092 + 0xb8c75f6eb90d
ae02 + 0x3560000000000000) + x * (0x3f81e554242ab128 + 0xbc23e920a76e760c + 0x38
c0589c2cae6caf + 0x3564000000000000)))))
```

Example 3:

```
> verbosity = 1!;
> prec = 3500!;
> printexpansion(pi);
(0x400921fb54442d18 + 0x3ca1a62633145c07 + 0xb92f1976b7ed8fbc + 0x35c4cf98e80417
7d + 0x32631d89cd9128a5 + 0x2ec0f31c6809bbdf + 0x2b5519b3cd3a431b + 0x27e8158536
f92f8a + 0x246ba7f09ab6b6a9 + 0xa0eedd0dbd2544cf + 0x1d779fb1bd1310ba + 0x1a1a63
7ed6b0bff6 + 0x96aa485fca40908e + 0x933e501295d98169 + 0x8fd160dbee83b4e0 + 0x8c
59b6d799ae131c + 0x08f6cf70801f2e28 + 0x05963bf0598da483 + 0x023871574e69a459 +
0x8000000005702db3 + 0x8000000000000000)
Warning: the expansion is not complete because of the limited exponent range of
double precision.
Warning: rounding occurred while printing.
```

See also: **printdouble** (8.142), **horner** (8.87), **print** (8.141), **prec** (8.138), **remez** (8.158), **taylor** (8.188), **roundcoefficients** (8.165), **fpminimax** (8.73), **implementpoly** (8.90)

8.144 printsingle

Name: **printsingle**
prints a constant value as a hexadecimal single precision number

Library name:
`void sollya_lib_printsingle(sollya_obj_t)`

Usage:

printsingle(*constant*) : constant → void

Parameters:

- *constant* represents a constant

Description:

- Prints a constant value as a hexadecimal number on 8 hexadecimal digits. The hexadecimal number represents the integer equivalent to the 32 bit memory representation of the constant considered as a single precision number.

  If the constant value does not hold on a single precision number, it is first rounded to the nearest single precision number before it is displayed. A warning is displayed in this case.

Example 1:

```
> printsingle(3);
0x40400000
```

Example 2:

```
> prec=100!;
> verbosity = 1!;
> printsingle(exp(5));
Warning: the given expression is not a constant but an expression to evaluate. A
 faithful evaluation to 100 bits will be used.
Warning: rounding up occurred before printing a value as a single.
0x431469c5
```

See also: **printdouble** (8.142), **single** (8.175)

8.145 printxml

Name: **printxml**
prints an expression as an MathML-Content-Tree

Library names:
`void sollya_lib_printxml(sollya_obj_t)`
`void sollya_lib_printxml_newfile(sollya_obj_t, sollya_obj_t)`
`void sollya_lib_printxml_appendfile(sollya_obj_t, sollya_obj_t)`

Usage:

printxml(*expr*) : function → void
printxml(*expr*) > *filename* : (function, string) → void
printxml(*expr*) > > *filename* : (function, string) → void

Parameters:

- *expr* represents a functional expression
- *filename* represents a character sequence indicating a file name

Description:

- The command **printxml** is now considered DEPRECATED in `Sollya`.
- **printxml**(*expr*) prints the functional expression *expr* as a tree of MathML Content Definition Markups. This XML tree can be re-read in external tools or by usage of the **readxml** command.

  If a second argument *filename* is given after a single >, the MathML tree is not output on the standard output of `Sollya` but if in the file *filename* that get newly created or overwritten. If a double > > is given, the output will be appended to the file *filename*.

Example 1:

```
> printxml(x + 2 + exp(sin(x)));

<?xml version="1.0" encoding="UTF-8"?>
<!-- generated by sollya: http://sollya.org/ -->
<!-- syntax: printxml(...);   example: printxml(x^2-2*x+5); -->
<?xml-stylesheet type="text/xsl" href="http://sollya.org/mathmlc2p-web.xsl"?>
<?xml-stylesheet type="text/xsl" href="mathmlc2p-web.xsl"?>
<!-- This stylesheet allows direct web browsing of MathML-c XML files (http:// o
r file://) -->

<math xmlns="http://www.w3.org/1998/Math/MathML">
<semantics>
<annotation-xml encoding="MathML-Content">
<lambda>
<bvar><ci> x </ci></bvar>
<apply>
<apply>
<plus/>
<apply>
<plus/>
<ci> x </ci>
<cn type="integer" base="10"> 2 </cn>
</apply>
<apply>
<exp/>
<apply>
<sin/>
<ci> x </ci>
</apply>
</apply>
</apply>
</apply>
</lambda>
</annotation-xml>
<annotation encoding="sollya/text">(x + 1b1) + exp(sin(x))</annotation>
</semantics>
</math>
```

Example 2:

```
> printxml(x + 2 + exp(sin(x))) > "foo.xml";
```

Example 3:

```
> printxml(x + 2 + exp(sin(x))) >> "foo.xml";
```

See also: **readxml** (8.156), **print** (8.141), **write** (8.201)

8.146 proc

Name: **proc**
defines a `Sollya` procedure

Usage:

proc(*formal parameter1*, *formal parameter2*,..., *formal parameter n*) { *procedure body* } : void → procedure

proc(*formal parameter1*, *formal parameter2*,..., *formal parameter n*) { *procedure body* **return** *expression*; } : void → procedure

proc(*formal list parameter* = ...) { *procedure body* } : void → procedure

proc(*formal list parameter* = ...) { *procedure body* **return** *expression*; } : void → procedure

Parameters:

- *formal parameter1*, *formal parameter2* through *formal parameter n* represent identifiers used as formal parameters
- *formal list parameter* represents an identifier used as a formal parameter for the list of an arbitrary number of parameters
- *procedure body* represents the imperative statements in the body of the procedure
- *expression* represents the expression **proc** shall evaluate to

Description:

- The **proc** keyword allows for defining procedures in the `Sollya` language. These procedures are common `Sollya` objects that can be applied to actual parameters after definition. Upon such an application, the `Sollya` interpreter applies the actual parameters to the formal parameters *formal parameter1* through *formal parameter n* (resp. builds up the list of arguments and applies it to the list *formal list parameter*) and executes the *procedure body*. The procedure applied to actual parameters evaluates then to the expression *expression* in the **return** statement after the *procedure body* or to **void**, if no return statement is given (i.e. a **return void** statement is implicitly given).
- `Sollya` procedures defined by **proc** have no name. They can be bound to an identifier by assigning the procedure object a **proc** expression produces to an identifier. However, it is possible to use procedures without giving them any name. For instance, `Sollya` procedures, i.e. procedure objects, can be elements of lists. They can even be given as an argument to other internal `Sollya` procedures. See also **procedure** on this subject.
- Upon definition of a `Sollya` procedure using **proc**, no type check is performed. More precisely, the statements in *procedure body* are merely parsed but not interpreted upon procedure definition with **proc**. Type checks are performed once the procedure is applied to actual parameters or to **void**. At this time, if the procedure was defined using several different formal parameters *formal parameter 1* through *formal parameter n*, it is checked whether the number of actual parameters corresponds to the number of formal parameters. If the procedure was defined using the syntax for a procedure with an arbitrary number of parameters by giving a *formal list parameter*, the number of actual arguments is not checked but only a list *formal list parameter* of appropriate length is built up. Type checks are further performed upon execution of each statement in *procedure body* and upon evaluation of the expression *expression* to be returned.

  Procedures defined by **proc** containing a **quit** or **restart** command cannot be executed (i.e. applied). Upon application of a procedure, the `Sollya` interpreter checks beforehand for such a statement. If one is found, the application of the procedure to its arguments evaluates to **error**. A warning is displayed. Remark that in contrast to other type or semantic correctness checks, this check is really performed before interpreting any other statement in the body of the procedure.
- Through the **var** keyword it is possible to declare local variables and thus to have full support of recursive procedures. This means a procedure defined using **proc** may contain in its *procedure body* an application of itself to some actual parameters: it suffices to assign the procedure (object) to an identifier with an appropriate name.
- `Sollya` procedures defined using **proc** may return other procedures. Further *procedure body* may contain assignments of locally defined procedure objects to identifiers. See **var** for the particular behaviour of local and global variables.

- The expression *expression* returned by a procedure is evaluated with regard to `Sollya` commands, procedures and external procedures. Simplification may be performed. However, an application of a procedure defined by **proc** to actual parameters evaluates to the expression *expression* that may contain the free global variable or that may be composed.

Example 1:

```
> succ = proc(n) { return n + 1; };
> succ(5);
6
> 3 + succ(0);
4
> succ;
proc(n)
{
nop;
return (n) + (1);
}
```

Example 2:

```
> add = proc(m,n) { var res; res := m + n; return res; };
> add(5,6);
11
> add;
proc(m, n)
{
var res;
res := (m) + (n);
return res;
}
> verbosity = 1!;
> add(3);
Warning: at least one of the given expressions or a subexpression is not correct
ly typed
or its evaluation has failed because of some error on a side-effect.
error
> add(true,false);
Warning: at least one of the given expressions or a subexpression is not correct
ly typed
or its evaluation has failed because of some error on a side-effect.
Warning: the given expression or command could not be handled.
error
```

Example 3:

```
> succ = proc(n) { return n + 1; };
> succ(5);
6
> succ(x);
1 + x
```

Example 4:

```
> hey = proc() { print("Hello world."); };
> hey();
Hello world.
> print(hey());
Hello world.
void
> hey;
proc()
{
print("Hello world.");
return void;
}
```

Example 5:

```
> fac = proc(n) { var res; if (n == 0) then res := 1 else res := n * fac(n - 1);
 return res; };
> fac(5);
120
> fac(11);
39916800
> fac;
proc(n)
{
var res;
if (n) == (0) then
res := 1
else
res := (n) * (fac((n) - (1)));
return res;
}
```

Example 6:

```
> myprocs = [| proc(m,n) { return m + n; }, proc(m,n) { return m - n; } |];
> (myprocs[0])(5,6);
11
> (myprocs[1])(5,6);
-1
> succ = proc(n) { return n + 1; };
> pred = proc(n) { return n - 1; };
> applier = proc(p,n) { return p(n); };
> applier(succ,5);
6
> applier(pred,5);
4
```

Example 7:

```
> verbosity = 1!;
> myquit = proc(n) { print(n); quit; };
> myquit;
proc(n)
{
print(n);
quit;
return void;
}
> myquit(5);
Warning: a quit or restart command may not be part of a procedure body.
The procedure will not be executed.
Warning: an error occurred while executing a procedure.
Warning: the given expression or command could not be handled.
error
```

Example 8:

```
> printsucc = proc(n) { var succ; succ = proc(n) { return n + 1; }; print("Succe
ssor of",n,"is",succ(n)); };
> printsucc(5);
Successor of 5 is 6
```

Example 9:

```
> makeadd = proc(n) { var add; print("n =",n); add = proc(m,n) { return n + m; }
; return add; };
> makeadd(4);
n = 4
proc(m, n)
{
nop;
return (n) + (m);
}
> (makeadd(4))(5,6);
n = 4
11
```

Example 10:

```
> sumall = proc(L = ...) { var acc, i; acc = 0; for i in L do acc = acc + i; ret
urn acc; };
> sumall;
proc(L = ...)
{
var acc, i;
acc = 0;
for i in L do
acc = (acc) + (i);
return acc;
}
> sumall();
0
> sumall(2);
2
> sumall(2,5);
7
> sumall(2,5,7,9,16);
39
> sumall @ [|1,...,10|];
55
```

See also: **return** (8.161), **externalproc** (8.66), **void** (8.199), **quit** (8.150), **restart** (8.160), **var** (8.197), **@** (8.29), **bind** (8.21), **getbacktrace** (8.78), **error** (8.57)

8.147 procedure

Name: **procedure**
defines and assigns a `Sollya` procedure

Usage:

procedure *identifier*(*formal parameter1*, *formal parameter2*,..., *formal parameter n*) { *procedure body* } : void → void
procedure *identifier*(*formal parameter1*, *formal parameter2*,..., *formal parameter n*) { *procedure body* **return** *expression*; } : void → void
procedure *identifier*(*formal list parameter* = ...) { *procedure body* } : void → void
procedure *identifier*(*formal list parameter* = ...) { *procedure body* **return** *expression*; } : void → void

Parameters:

- *identifier* represents the name of the procedure to be defined and assigned
- *formal parameter1*, *formal parameter2* through *formal parameter n* represent identifiers used as formal parameters
- *formal list parameter* represents an identifier used as a formal parameter for the list of an arbitrary number of parameters
- *procedure body* represents the imperative statements in the body of the procedure
- *expression* represents the expression **procedure** shall evaluate to

Description:

- The **procedure** keyword allows for defining and assigning procedures in the `Sollya` language. It is an abbreviation to a procedure definition using **proc** with the same formal parameters, procedure body and return-expression followed by an assignment of the procedure (object) to the identifier *identifier*. In particular, all rules concerning local variables declared using the **var** keyword apply for **procedure**.

Example 1:

```
> procedure succ(n) { return n + 1; };
> succ(5);
6
> 3 + succ(0);
4
> succ;
proc(n)
{
nop;
return (n) + (1);
}
```

Example 2:

```
> procedure myprint(L = ...) { var i; for i in L do i; };
> myprint("Lyon","Nancy","Beaverton","Coye-la-Foret","Amberg","Nizhny Novgorod",
"Cluj-Napoca");
Lyon
Nancy
Beaverton
Coye-la-Foret
Amberg
Nizhny Novgorod
Cluj-Napoca
```

See also: **proc** (8.146), **var** (8.197), **bind** (8.21), **getbacktrace** (8.78)

8.148 QD

Name: **QD**
short form for **quad**

See also: **quad** (8.149)

8.149 quad

Names: **quad**, **QD**
rounding to the nearest IEEE 754 quad (binary128).

Library names:

```
sollya_obj_t sollya_lib_quad(sollya_obj_t)
sollya_obj_t sollya_lib_quad_obj()
int sollya_lib_is_quad_obj(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_quad(sollya_obj_t)
#define SOLLYA_QD(x) sollya_lib_build_function_quad(x)
```

Description:

- **quad** is both a function and a constant.
- As a function, it rounds its argument to the nearest IEEE 754 quad precision (i.e. IEEE754-2008 binary128) number. Subnormal numbers are supported as well as standard numbers: it is the real rounding described in the standard.
- As a constant, it symbolizes the quad precision format. It is used in contexts when a precision format is necessary, e.g. in the commands **round** and **roundcoefficients**. It is not supported for **implementpoly**. See the corresponding help pages for examples.

Example 1:

```
> display=binary!;
> QD(0.1);
1.10011001100110011001100110011001100110011001100110011001100110011001100110011001100110
01100110011001100110011001100110011001101_2 * 2^(-4)
> QD(4.17);
1.00001010111000010100011110101110000101000111101011100001010001111010111000010100011110
00011110101110000101000111101011 1_2 * 2^(2)
> QD(1.011_2 * 2^(-16493));
1.1_2 * 2^(-16493)
```

See also: **halfprecision** (8.82), **single** (8.175), **double** (8.50), **doubleextended** (8.52), **doubledouble** (8.51), **tripledouble** (8.194), **roundcoefficients** (8.165), **implementpoly** (8.90), **fpminimax** (8.73), **round** (8.164), **printsingle** (8.144)

8.150 quit

Name: **quit**
quits `Sollya`

Usage:

quit : void → void

Description:

- The command **quit**, when executed, stops the execution of a `Sollya` script and leaves the `Sollya` interpreter unless the **quit** command is executed in a `Sollya` script read into a main `Sollya` script by **execute** or #include.

  Upon exiting the `Sollya` interpreter, all state is thrown away, all memory is deallocated, all bound libraries are unbound and the temporary files produced by **plot** and **externalplot** are deleted.

  If the **quit** command does not lead to a halt of the `Sollya` interpreter, a warning is displayed.

Example 1:

```
> quit;
```

See also: **restart** (8.160), **execute** (8.59), **plot** (8.131), **externalplot** (8.65), **return** (8.161)

8.151 range

Name: **range**
keyword representing a range type

Library name:
`SOLLYA_EXTERNALPROC_TYPE_RANGE`

Usage:

range : type type

Description:

- **range** represents the range type for declarations of external procedures by means of **externalproc**.

  Remark that in contrast to other indicators, type indicators like **range** cannot be handled outside the **externalproc** context. In particular, they cannot be assigned to variables.

See also: **externalproc** (8.66), **boolean** (8.22), **constant** (8.30), **function** (8.75), **integer** (8.94), **list of** (8.103), **string** (8.179), **object** (8.123)

8.152 rationalapprox

Name: **rationalapprox**
returns a fraction close to a given number.

Library name:
`sollya_obj_t sollya_lib_rationalapprox(sollya_obj_t, sollya_obj_t)`

Usage:

rationalapprox(x,n) : (constant, integer) → function

Parameters:

- x is a number to approximate.
- n is a integer (representing a format).

Description:

- **rationalapprox**(x,n) returns a constant function of the form a/b where a and b are integers. The value a/b is an approximation of x. The quality of this approximation is determined by the parameter n that indicates the number of correct bits that a/b should have.
- The command is not safe in the sense that it is not ensured that the error between a/b and x is less than 2^{-n}.
- The following algorithm is used: x is first rounded downwards and upwards to a format of n bits, thus obtaining an interval $[x_l, x_u]$. This interval is then developed into a continued fraction as far as the representation is the same for every elements of $[x_l, x_u]$. The corresponding fraction is returned.
- Since rational numbers are not a primitive object of `Sollya`, the fraction is returned as a constant function. This can be quite amazing, because `Sollya` immediately simplifies a constant function by evaluating it when the constant has to be displayed. To avoid this, you can use **print** (that displays the expression representing the constant and not the constant itself) or the commands **numerator** and **denominator**.

Example 1:

```
> pi10 = rationalapprox(Pi,10);
> pi50 = rationalapprox(Pi,50);
> pi100 = rationalapprox(Pi,100);
> print( pi10, ": ", dirtysimplify(floor(-log2(abs(pi10-Pi)/Pi))), "bits." );
3.140625 :  11 bits.
> print( pi50, ": ", dirtysimplify(floor(-log2(abs(pi50-Pi)/Pi))), "bits." );
85563208 / 27235615 :  51 bits.
> print( pi100, ": ", dirtysimplify(floor(-log2(abs(pi100-Pi)/Pi))), "bits." );
4422001152019829 / 1407566683404023 :  100 bits.
```

Example 2:

```
> a=0.1;
> b=rationalapprox(a,4);
> numerator(b); denominator(b);
1
10
> print(dirtysimplify(floor(-log2(abs((b-a)/a)))), "bits.");
166 bits.
```

See also: **print** (8.141), **numerator** (8.122), **denominator** (8.39), **rationalmode** (8.153)

8.153 rationalmode

Name: **rationalmode**
global variable controlling if rational arithmetic is used or not.

Library names:
```
void sollya_lib_set_rationalmode_and_print(sollya_obj_t)
void sollya_lib_set_rationalmode(sollya_obj_t)
sollya_obj_t sollya_lib_get_rationalmode()
```

Usage:

rationalmode = *activation value* : on|off → void
rationalmode = *activation value* ! : on|off → void
rationalmode : on|off

Parameters:

- *activation value* controls if rational arithmetic should be used or not

Description:

- **rationalmode** is a global variable. When its value is **off**, which is the default, `Sollya` will not use rational arithmetic to simplify expressions. All computations, including the evaluation of constant expressions given on the `Sollya` prompt, will be performed using floating-point and interval arithmetic. Constant expressions will be approximated by floating-point numbers, which are in most cases faithful roundings of the expressions, when shown at the prompt.
- When the value of the global variable **rationalmode** is **on**, `Sollya` will use rational arithmetic when simplifying expressions. Constant expressions, given at the `Sollya` prompt, will be simplified to rational numbers and displayed as such when they are in the set of the rational numbers. Otherwise, floating-point and interval arithmetic will be used to compute a floating-point approximation, which is in most cases a faithful rounding of the constant expression.
- When a decimal value is parsed, the behavior of `Sollya` is different depending on the value of the global variable **rationalmode**. If it is **off**, the value gets rounded as a floating-point at precision **prec**. But if **rationalmode** is set to **on**, the decimal value is interpreted exactly and converted as a rational number of the form $M/10^N$ where M and N are integers. Accordingly, when **rationalmode** is set to **on** and **display** is set to **decimal**, all floating-point values are displayed exactly: indeed, any floating-point number with radix 2 has a finite decimal expansion. Therefore, any rational number of the form $M/(2^P \cdot 5^Q)$ gets displayed as an exact decimal value (while other fractions get displayed as fractions).

Example 1:

```
> rationalmode=off!;
> 19/17 + 3/94;
1.14956195244055068836045056320400500625782222778473
> rationalmode=on!;
> 19/17 + 3/94;
1837 / 1598
```

Example 2:

```
> rationalmode=off!;
> exp(19/17 + 3/94);
3.15680977395514136754709208944824276340328162814417965749192184822794309468933
> rationalmode=on!;
> exp(19/17 + 3/94);
3.15680977395514136754709208944824276340328162814417965749192184822794309468933455522164268460461069584760073436423014140708373509447426386032020673155784606933
59375
```

Example 3:

```
> prec = 12!;
> rationalmode=off!;
> r = 0.1;
> r == 1/10;
false
> rationalmode=on!;
> s = 0.1;
> s == 1/10;
true
> r == s;
false
> r;
0.100006103515625
> s;
0.1
```

See also: **on** (8.126), **off** (8.125), **numerator** (8.122), **denominator** (8.39), **simplify** (8.173), **rationalapprox** (8.152), **autosimplify** (8.16)

8.154 RD

Name: **RD**
constant representing rounding-downwards mode.

Library names:
`sollya_obj_t sollya_lib_round_down()`
`int sollya_lib_is_round_down(sollya_obj_t)`

Description:

- **RD** is used in command **round** to specify that the value x must be rounded to the greatest floating-point number y such that $y \leq x$.

Example 1:

```
> display=binary!;
> round(Pi,20,RD);
1.10010010000111111101_2 * 2^(1)
```

See also: **RZ** (8.169), **RU** (8.168), **RN** (8.163), **round** (8.164), **floor** (8.72)

8.155 readfile

Name: **readfile**
reads the content of a file into a string variable

Usage:

readfile(*filename*) : string → string

Parameters:

- *filename* represents a character sequence indicating a file name

Description:

- **readfile** opens the file indicated by *filename*, reads it and puts its contents in a character sequence of type string that is returned.

  If the file indicated by *filename* cannot be opened for reading, a warning is displayed and **readfile** evaluates to an **error** variable of type error.

Example 1:

```
> print("Hello world") > "myfile.txt";
> t = readfile("myfile.txt");
> t;
Hello world
```

Example 2:

```
> verbosity=1!;
> readfile("afile.txt");
Warning: the file "afile.txt" could not be opened for reading.
Warning: at least one of the given expressions or a subexpression is not correct
ly typed
or its evaluation has failed because of some error on a side-effect.
error
```

See also: **parse** (8.128), **execute** (8.59), **write** (8.201), **print** (8.141), **bashexecute** (8.18), **error** (8.57)

8.156 readxml

Name: **readxml**
reads an expression written as a MathML-Content-Tree in a file

Library name:
`sollya_obj_t sollya_lib_readxml(sollya_obj_t)`

Usage:

readxml(*filename*) : string → function | error

Parameters:

- *filename* represents a character sequence indicating a file name

Description:

- The command **readxml** is now considered DEPRECATED in `Sollya`.
- **readxml**(*filename*) reads the first occurrence of a lambda application with one bounded variable on applications of the supported basic functions in file *filename* and returns it as a `Sollya` functional expression.

  If the file *filename* does not contain a valid MathML-Content tree, **readxml** tries to find an "annotation encoding" markup of type "sollya/text". If this annotation contains a character sequence that can be parsed by **parse**, **readxml** returns that expression. Otherwise **readxml** displays a warning and returns an **error** variable of type error.

Example 1:

```
> readxml("readxmlexample.xml");
2 + x + exp(sin(x))
```

See also: **printxml** (8.145), **readfile** (8.155), **parse** (8.128), **error** (8.57)

8.157 relative

Name: **relative**
indicates a relative error for **externalplot**, **fpminimax** or **supnorm**

Library names:
```
sollya_obj_t sollya_lib_relative()
int sollya_lib_is_relative(sollya_obj_t)
```

Usage:

relative : absolute|relative

Description:

- The use of **relative** in the command **externalplot** indicates that during plotting in **externalplot** a relative error is to be considered.

  See **externalplot** for details.

- Used with **fpminimax**, **relative** indicates that **fpminimax** must try to minimize the relative error.

  See **fpminimax** for details.

- When given in argument to **supnorm**, **relative** indicates that a relative error is to be considered for supremum norm computation.

  See **supnorm** for details.

Example 1:

```
> bashexecute("gcc -fPIC -c externalplotexample.c");
> bashexecute("gcc -shared -o externalplotexample externalplotexample.o -lgmp -l
mpfr");
> externalplot("./externalplotexample",relative,exp(x),[-1/2;1/2],12,perturb);
```

See also: **externalplot** (8.65), **fpminimax** (8.73), **absolute** (8.2), **bashexecute** (8.18), **supnorm** (8.183)

8.158 remez

Name: **remez**
computes the minimax of a function on an interval.

Library names:
```
sollya_obj_t sollya_lib_remez(sollya_obj_t, sollya_obj_t, sollya_obj_t, ...)
sollya_obj_t sollya_lib_v_remez(sollya_obj_t, sollya_obj_t, sollya_obj_t,
                                va_list)
```

Usage:

remez(*f*, *n*, *range*, *w*, *quality*, *bounds*) : (function, integer, range, function, constant, range) → function
remez(*f*, *L*, *range*, *w*, *quality*, *bounds*) : (function, list, range, function, constant, range) → function

Parameters:

- *f* is the function to be approximated
- *n* is the degree of the polynomial that must approximate *f*
- *L* is a list of integers or a list of functions and indicates the basis for the approximation of *f*
- *range* is the interval where the function must be approximated

- w (optional) is a weight function. Default is 1.
- *quality* (optional) is a parameter that controls the quality of the returned polynomial p, with respect to the exact minimax $p^\star$. Default is 1e-5.
- *bounds* (optional) is a parameter that allows the user to make the algorithm stop earlier, whenever a given accuracy is reached or a given accuracy is proved unreachable. Default is [0, $+\infty$].

Description:

- **remez** computes an approximation of the function f with respect to the weight function w on the interval *range*. More precisely, it searches p such that $\|pw - f\|_\infty$ is (almost) minimal among all p of a certain form. The norm is the infinity norm, e.g. $\|g\|_\infty = \max\{|g(x)|, x \in \text{range}\}$.
- If $w = 1$ (the default case), it consists in searching the best polynomial approximation of f with respect to the absolute error. If $f = 1$ and w is of the form $1/g$, it consists in searching the best polynomial approximation of g with respect to the relative error.
- If n is given, p is searched among the polynomials with degree not greater than n. If L is given and is a list of integers, p is searched as a linear combination of monomials X^k where k belongs to L. In the case when L is a list of integers, it may contain ellipses but cannot be end-elliptic. If L is given and is a list of functions g_k, p is searched as a linear combination of the g_k. In that case L cannot contain ellipses. It is the user responsibility to check that the g_k are linearly independent over the interval *range*. Moreover, the functions $w \cdot g_k$ must be at least twice differentiable over *range*. If these conditions are not fulfilled, the algorithm might fail or even silently return a result as if it successfully found the minimax, though the returned p is not optimal.
- The polynomial is obtained by a convergent iteration called Remez' algorithm (and an extension of this algorithm, due to Stiefel). The algorithm computes a sequence $p_1, \ldots, p_k, \ldots$ such that $e_k = \|p_k w - f\|_\infty$ converges towards the optimal value e. The algorithm is stopped when the relative error between e_k and e is less than *quality*.
  For this reason, the returned polynomial usually has dyadic coefficients. However, one should not rely on that property. Especially, a noticeable exception to that statement is when we manage to detect that f/w exactly simplifies to a polynomial of the required degree, in which case we return it as it is without further rounding.
- The optional argument *bounds* is an interval [ε_ℓ, ε_u] with the following behavior:
  - if, during the algorithm, we manage to prove that ε_u is unreachable, we stop the algorithm returning the last computed polynomial.
  - if, during the algorithm, we obtain a polynomial with an error smaller than ε_ℓ, we stop the algorithm returning that polynomial.
  - otherwise we loop until we find an optimal polynomial with the required quality, as usual.

  Examples of use:
  [0, $+\infty$] (compute the optimal polynomial with the required quality)
  [ε_u] (stops as soon as a polynomial achieving ε_u is obtained or as soon as such a polynomial is proved not to exist).
  [0, ε_u] (finds the optimal polynomial, but provided that its error is smaller than ε_u).
  [ε_ℓ, $+\infty$] (stops as soon as a polynomial achieving ε_ℓ is obtained. If such a polynomial does not exist, returns the optimal polynomial).

Example 1:

```
> p = remez(exp(x),5,[0;1]);
> degree(p);
5
> dirtyinfnorm(p-exp(x),[0;1]);
1.12956981510961487071711938292660776072222634589363e-6
```

Example 2:

```
> p = remez(1,[|0,2,4,6,8|],[0,Pi/4],1/cos(x));
> canonical=on!;
> p;
0.99999999994393732180959690352543887130348096061124 - 0.49999999571556857768772
05306372154467094946722225 9 * x^2 + 4.16666132334736330099410594805702758701132 2
0089059e-2 * x^4 - 1.38865291471456936513555238803197140510476356950 61e-3 * x^6
+ 2.43726791772241799348003285110092052181142842201 26e-5 * x^8
```

Example 3:

```
> p1 = remez(exp(x),5,[0;1],default,1e-5);
> p2 = remez(exp(x),5,[0;1],default,1e-10);
> p3 = remez(exp(x),5,[0;1],default,1e-15);
> dirtyinfnorm(p1-exp(x),[0;1]);
1.12956981510961487071711938292660776072226345893 63e-6
> dirtyinfnorm(p2-exp(x),[0;1]);
1.12956980227478675612619255125474525171079325793124e-6
> dirtyinfnorm(p3-exp(x),[0;1]);
1.12956980227478675612619255125474525171079325793124e-6
```

Example 4:

```
> L = [|exp(x), sin(x), cos(x)-1, sin(x^3)|];
> g = (2^x-1)/x;
> p1 = remez(g, L, [-1/16;1/16]);
> p2 = remez(g, 3, [-1/16;1/16]);
> dirtyinfnorm(p1 - g, [-1/16;1/16]);
9.88413238292710381376856467779516876202884621947 46e-8
> dirtyinfnorm(p2 - g, [-1/16;1/16]);
2.54337800593461418356437401152248866818783932027105e-9
```

Example 5:

```
> f = sin(x);
> I = [-3b-5;-1b-1074];
> time(popt = remez(1, [|1, 3, 4, 5, 7, 8, 9|], I, 1/f));
0.18153754500000000000000000000000000000951234855126
> time(p1 = remez(1, [|1, 3, 4, 5, 7, 8, 9|], I, 1/f, default, [0, 1b-73]));
0.13429872799999999999999999999999999998420186148156
> time(p2 = remez(1, [|1, 3, 4, 5, 7, 8, 9|], I, 1/f, default, [3b-72, +@Inf@]))
;
0.15097598299999999999999999999999999997436992893015
> dirtyinfnorm(popt/f-1, I);
2.06750931454112835098093903810531156576504665659064e-22
> dirtyinfnorm(p1/f-1, I);
2.49711266837493110470637913808914046704452778960875e-22
> dirtyinfnorm(p2/f-1, I);
5.45672475536154352463769772312538342652487569969 47e-22
> 1b-73;
1.05879118406787542383540312584955245256423950195312e-22
> 3b-72;
6.35274710440725254301241875509731471538543701171 87e-22
```

See also: **dirtyinfnorm** (8.44), **infnorm** (8.93), **fpminimax** (8.73), **guessdegree** (8.81), **taylorform** (8.189), **taylor** (8.188), **interpolate** (8.96)

8.159 rename

Name: **rename**
rename the free variable.

Library name:
`void sollya_lib_name_free_variable(const char *)`

Usage:

rename(*ident1*,*ident2*) : void

Parameters:

- *ident1* is the current name of the free variable.
- *ident2* is a fresh name.

Description:

- **rename** permits a change of the name of the free variable. `Sollya` can handle only one free variable at a time. The first time in a session that an unbound name is used in a context where it can be interpreted as a free variable, the name is used to represent the free variable of `Sollya`. In the following, this name can be changed using **rename**.
- Be careful: if *ident2* has been set before, its value will be lost. Use the command **isbound** to know if *ident2* is already used or not.
- If *ident1* is not the current name of the free variable, an error occurs.
- If **rename** is used at a time when the name of the free variable has not been defined, *ident1* is just ignored and the name of the free variable is set to *ident2*.
- It is always possible to use the special keyword `_x_` to denote the free variable. Hence *ident1* can be `_x_`.
- Notice that *ident2* is an identifier, not a string. In particular, it is not directly possible to use the content of a variable to provide the new name of the free variable. This can however be obtained using a level of indirection: first create a string that contains the call to **rename**, and then evaluate it thanks to **parse** (see example below).

Example 1:

```
> f=sin(x);
> f;
sin(x)
> rename(x,y);
> f;
sin(y)
```

Example 2:

```
> a=1;
> f=sin(x);
> rename(x,a);
> a;
a
> f;
sin(a)
```

Example 3:

```
> verbosity=1!;
> f=sin(x);
> rename(y, z);
Warning: the current free variable is named "x" and not "y". Can only rename the
 free variable.
The last command will have no effect.
> rename(_x_, z);
Information: the free variable has been renamed from "x" to "z".
```

Example 4:

```
> verbosity=1!;
> rename(x,y);
Information: the free variable has been named "y".
> isbound(x);
false
> isbound(y);
true
```

Example 5:

```
> verbosity=1!;
> f = sin(x);
> new_varname = "foo";
> rename(_x_, new_varname);
Information: the free variable has been renamed from "x" to "new_varname".
> f;
sin(new_varname)
> rename(_x_, x);
Information: the free variable has been renamed from "new_varname" to "x".
> new_varname = "foo";
> str = "rename(_x_, " @ new_varname @ ");";
> (parse("proc () { " @ str @ "}"))();
Information: the free variable has been renamed from "x" to "foo".
> f;
sin(foo)
```

Example 6:

```
> verbosity=1!;
> f = sin(x);
> f;
sin(x)
> rename(x, _x_);
Information: the free variable has been renamed from "x" to "_x_".
> f;
sin(_x_)
> g = cos(y);
> f;
sin(y)
> g;
cos(y)
> rename(_x_, _x_);
Information: the free variable has been renamed from "y" to "_x_".
> f;
sin(_x_)
> g;
cos(_x_)
```

See also: **isbound** (8.97), **parse** (8.128)

8.160 restart

Name: **restart**
brings `Sollya` back to its initial state

Usage:

restart : void → void

Description:

- The command **restart** brings `Sollya` back to its initial state. All current state is abandoned, all libraries unbound and all memory freed.

  The **restart** command has no effect when executed inside a `Sollya` script read into a main `Sollya` script using **execute**. It is executed in a `Sollya` script included by a #include macro.

  Using the **restart** command in nested elements of imperative programming like for or while loops is possible. Since in most cases abandoning the current state of `Sollya` means altering a loop invariant, warnings for the impossibility of continuing a loop may follow unless the state is rebuilt.

Example 1:

```
> print(exp(x));
exp(x)
> a = 3;
> restart;
The tool has been restarted.
> print(x);
x
> a;
Warning: the identifier "a" is neither assigned to, nor bound to a library funct
ion nor external procedure, nor equal to the current free variable.
Will interpret "a" as "x".
x
```

Example 2:

```
> print(exp(x));
exp(x)
> for i from 1 to 10 do {
      print(i);
      if (i == 5) then restart;
  };
1
2
3
4
5
The tool has been restarted.
Warning: the tool has been restarted inside a for loop.
The for loop will no longer be executed.
```

Example 3:

```
> print(exp(x));
exp(x)
> a = 3;
> for i from 1 to 10 do {
      print(i);
      if (i == 5) then {
          restart;
          i = 7;
      };
  };
1
2
3
4
5
The tool has been restarted.
8
9
10
> print(x);
x
> a;
Warning: the identifier "a" is neither assigned to, nor bound to a library funct
ion nor external procedure, nor equal to the current free variable.
Will interpret "a" as "x".
x
```

See also: **quit** (8.150), **execute** (8.59)

8.161 return

Name: **return**
indicates an expression to be returned in a procedure

Usage:

return *expression* : void

Parameters:

- *expression* represents the expression to be returned

Description:

- The keyword **return** allows for returning the (evaluated) expression *expression* at the end of a begin-end-block (-block) used as a `Sollya` procedure body. See **proc** for further details concerning `Sollya` procedure definitions.

  Statements for returning expressions using **return** are only possible at the end of a begin-end-block used as a `Sollya` procedure body. Only one **return** statement can be given per begin-end-block.

- If at the end of a procedure definition using **proc** no **return** statement is given, a **return void** statement is implicitly added. Procedures, i.e. procedure objects, when printed out in `Sollya` defined with an implicit **return void** statement are displayed with this statement explicitly given.

Example 1:

```
> succ = proc(n) { var res; res := n + 1; return res; };
> succ(5);
6
> succ;
proc(n)
{
var res;
res := (n) + (1);
return res;
}
```

Example 2:

```
> hey = proc(s) { print("Hello",s); };
> hey("world");
Hello world
> hey;
proc(s)
{
print("Hello", s);
return void;
}
```

See also: **proc** (8.146), **void** (8.199)

8.162 revert

Name: **revert**
reverts a list.

Library name:
`sollya_obj_t sollya_lib_revert(sollya_obj_t)`

Usage:

$$\mathbf{revert}(L) : \mathsf{list} \to \mathsf{list}$$

Parameters:

- L is a list.

Description:

- **revert**(L) returns the same list, but with its elements in reverse order.
- If L is an end-elliptic list, **revert** will fail with an error.

Example 1:

```
> revert([| |]);
[| |]
```

Example 2:

```
> revert([|2,3,5,2,1,4|]);
[|4, 1, 2, 5, 3, 2|]
```

See also: **sort** (8.177), **head** (8.83), **tail** (8.185)

8.163 RN

Name: **RN**
constant representing rounding-to-nearest mode.

Library names:
`sollya_obj_t sollya_lib_round_to_nearest()`
`int sollya_lib_is_round_to_nearest(sollya_obj_t)`

Description:

- **RN** is used in command **round** to specify that the value must be rounded to the nearest representable floating-point number.

Example 1:

```
> display=binary!;
> round(Pi,20,RN);
1.10010010000111111_2 * 2^(1)
```

See also: **RD** (8.154), **RU** (8.168), **RZ** (8.169), **round** (8.164), **nearestint** (8.117)

8.164 round

Name: **round**
rounds a number to a floating-point format.

Library name:
`sollya_obj_t sollya_lib_round(sollya_obj_t, sollya_obj_t, sollya_obj_t)`

Usage:

round(*x*,*n*,*mode*) : (constant, integer, RN|RZ|RU|RD) → constant
round(*x*,*format*,*mode*) : (constant, HP|halfprecision|SG|single|D|double|DE|doubleextended|DD|doubledouble|QD|quad|TD|tripledouble, RN|RZ|RU|RD) → constant

Parameters:

- *x* is a constant to be rounded.
- *n* is the precision of the target format.
- *format* is the name of a supported floating-point format.
- *mode* is the desired rounding mode.

Description:

- If used with an integer parameter *n*, **round**(*x*,*n*,*mode*) rounds *x* to a floating-point number with precision *n*, according to rounding-mode *mode*.
- If used with a format parameter *format*, **round**(*x*,*format*,*mode*) rounds *x* to a floating-point number in the floating-point format *format*, according to rounding-mode *mode*.
- Subnormal numbers are handled for the case when *format* is one of **halfprecision**, **single**, **double**, **doubleextended**, **doubledouble**, **quad** or **tripledouble**. Otherwise, when *format* is an integer, **round** does not take any exponent range into consideration, i.e. typically uses the full exponent range of the underlying MPFR library.
- It is worth mentioning that the result of **round** does not depend on the current global precision of `Sollya`, unless a warning is displayed. As a matter of fact, **round** rounds the given constant or constant expression *x* applying all rules of IEEE 754 correct rounding, unless a warning is displayed. The result of **round** is hence the floating-point value of the given precision *n* or format *format* that is nearest to *x* (resp. just below or just above, depending on *mode*), computed as if infinite precision were used for evaluating the constant *x*, unless a warning is displayed.

Example 1:

```
> display=binary!;
> round(Pi,20,RN);
1.10010010000111111_2 * 2^(1)
```

Example 2:

```
> printdouble(round(exp(17),53,RU));
0x417709348c0ea4f9
> printdouble(D(exp(17)));
0x417709348c0ea4f9
```

Example 3:

```
> display=binary!;
> a=2^(-1100);
> round(a,53,RN);
1_2 * 2^(-1100)
> round(a,D,RN);
0
> double(a);
0
```

See also: **RN** (8.163), **RD** (8.154), **RU** (8.168), **RZ** (8.169), **halfprecision** (8.82), **single** (8.175), **double** (8.50), **doubleextended** (8.52), **doubledouble** (8.51), **quad** (8.149), **tripledouble** (8.194), **roundcoefficients** (8.165), **roundcorrectly** (8.166), **printdouble** (8.142), **printsingle** (8.144), **ceil** (8.24), **floor** (8.72), **nearestint** (8.117)

8.165 roundcoefficients

Name: **roundcoefficients**
rounds the coefficients of a polynomial to classical formats.

Library name:
`sollya_obj_t sollya_lib_roundcoefficients(sollya_obj_t, sollya_obj_t)`

Usage:

roundcoefficients(p,L) : (function, list) → function

Parameters:

- p is a function. Usually a polynomial.
- L is a list of formats.

Description:

- If p is a polynomial and L a list of floating-point formats, **roundcoefficients**(p,L) rounds each coefficient of p to the corresponding format in L.
- If p is not a polynomial, **roundcoefficients** does not do anything.
- If L contains other elements than **HP**, **halfprecision**, **SG**, **single**, **D**, **double**, **DE**, **doubleextended**, **DD**, **doubledouble**, **QD**, **quad**, **TD** and **tripledouble**, an error occurs.
- The coefficients in p corresponding to X^i is rounded to the format L[i]. If L does not contain enough elements (e.g. if **length**(L) < **degree**(p)+1), a warning is displayed. However, the coefficients corresponding to an element of L are rounded. The trailing coefficients (that do not have a corresponding element in L) are kept with their own precision. If L contains too much elements, the trailing useless elements are ignored. In particular L may be end-elliptic in which case **roundcoefficients** has the natural behavior.

Example 1:

```
> p=exp(1) + x*(exp(2) + x*exp(3));
> display=binary!;
> roundcoefficients(p,[|DD,D,D|]);
1.01011011111100001010100010110001010001010111011010010101001101010101111110111
00101011000100000001001110 1_2 * 2^(1) + x * (1.11011000111001100100101110001101
10011011101101011 1_2 * 2^(2) + x * (1.01000001010111100101101111111011011110110
010000011_2 * 2^(4)))
> roundcoefficients(p,[|DD,D...|]);
1.01011011111100001010100010110001010001010111011010010101001101010101111110111
00101011000100000001001110 1_2 * 2^(1) + x * (1.11011000111001100100101110001101
10011011101101011 1_2 * 2^(2) + x * (1.01000001010111100101101111111011011110110
010000011_2 * 2^(4)))
```

Example 2:

```
> f=sin(exp(1)*x);
> display=binary!;
> f;
sin(x * (1.01011011111100001010100010110001010001010111011010010101001101010101
11111011100010101100010000000100111001111010011110011110000111011000101110011100
0101100000111101_2 * 2^(1)))
> roundcoefficients(f,[|D...|]);
sin(x * (1.01011011111100001010100010110001010001010111011010010101001101010101
11111011100010101100010000000100111001111010011110011110000111011000101110011100
0101100000111101_2 * 2^(1)))
```

Example 3:

```
> p=exp(1) + x*(exp(2) + x*exp(3));
> verbosity=1!;
> display=binary!;
> roundcoefficients(p,[|DD,D|]);
Warning: the number of the given formats does not correspond to the degree of th
e given polynomial.
Warning: the 0th coefficient of the given polynomial does not evaluate to a floa
ting-point constant without any rounding.
Will evaluate the coefficient in the current precision in floating-point before
rounding to the target format.
Warning: the 1th coefficient of the given polynomial does not evaluate to a floa
ting-point constant without any rounding.
Will evaluate the coefficient in the current precision in floating-point before
rounding to the target format.
Warning: rounding may have happened.
1.01011011111100001010100010110001010001010111011010010101001101010101111110111
00101011000100000001001110 1_2 * 2^(1) + x * (1.11011000111001100100101110001101
10011011101101011 1_2 * 2^(2) + x * (1.01000001010111100101101111111011011110110
010000010111110010110101001011110111111100010100110111010001001100001110100011110
010000010110000010110000001110010111001010 01_2 * 2^(4)))
```

See also: **halfprecision** (8.82), **single** (8.175), **double** (8.50), **doubleextended** (8.52), **doubledouble** (8.51), **quad** (8.149), **tripledouble** (8.194), **fpminimax** (8.73), **remez** (8.158), **implementpoly** (8.90), **subpoly** (8.180)

8.166 roundcorrectly

Name: **roundcorrectly**
rounds an approximation range correctly to some precision

Library name:
`sollya_obj_t sollya_lib_roundcorrectly(sollya_obj_t)`

Usage:

roundcorrectly(*range*) : range → constant

Parameters:

- *range* represents a range in which an exact value lies

Description:

- Let *range* be a range of values, determined by some approximation process, safely bounding an unknown value v. The command **roundcorrectly**(*range*) determines a precision such that for this precision, rounding to the nearest any value in *range* yields to the same result, i.e. to the correct rounding of v.

  If no such precision exists, a warning is displayed and **roundcorrectly** evaluates to NaN.

Example 1:

```
> printbinary(roundcorrectly([1.010001_2; 1.0101_2]));
1.01_2
> printbinary(roundcorrectly([1.00001_2; 1.001_2]));
1_2
```

Example 2:

```
> roundcorrectly([-1; 1]);
NaN
```

See also: **round** (8.164), **mantissa** (8.109), **exponent** (8.63), **precision** (8.139)

8.167 roundingwarnings

Name: **roundingwarnings**
global variable controlling whether or not a warning is displayed when roundings occur.

Library names:
`void sollya_lib_set_roundingwarnings_and_print(sollya_obj_t)`
`void sollya_lib_set_roundingwarnings(sollya_obj_t)`
`sollya_obj_t sollya_lib_get_roundingwarnings()`

Usage:

roundingwarnings = *activation value* : on|off → void
roundingwarnings = *activation value* ! : on|off → void
roundingwarnings : on|off

Parameters:

- *activation value* controls if warnings should be shown or not

Description:

- **roundingwarnings** is a global variable. When its value is **on**, warnings are emitted in appropriate verbosity modes (see **verbosity**) when roundings occur. When its value is **off**, these warnings are suppressed.

- This mode depends on a verbosity of at least 1. See **verbosity** for more details.
- Default is **on** when the standard input is a terminal and **off** when `Sollya` input is read from a file.

Example 1:

```
> verbosity=1!;
> roundingwarnings = on;
Rounding warning mode has been activated.
> exp(0.1);
Warning: Rounding occurred when converting the constant "0.1" to floating-point
with 165 bits.
If safe computation is needed, try to increase the precision.
Warning: rounding has happened. The value displayed is a faithful rounding to 16
5 bits of the true result.
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
1.10517091807564762481170782649024666822454719473 75
> roundingwarnings = off;
Rounding warning mode has been deactivated.
> exp(0.1);
1.10517091807564762481170782649024666822454719473 75
```

See also: **on** (8.126), **off** (8.125), **verbosity** (8.198), **midpointmode** (8.112), **rationalmode** (8.153), **suppressmessage** (8.184), **unsuppressmessage** (8.196), **showmessagenumbers** (8.172), **getsuppressedmessages** (8.79)

8.168 RU

Name: **RU**
constant representing rounding-upwards mode.

Library names:
`sollya_obj_t sollya_lib_round_up()`
`int sollya_lib_is_round_up(sollya_obj_t)`

Description:

- **RU** is used in command **round** to specify that the value x must be rounded to the smallest floating-point number y such that $x \leq y$.

Example 1:

```
> display=binary!;
> round(Pi,20,RU);
1.100100100001111111_2 * 2^(1)
```

See also: **RZ** (8.169), **RD** (8.154), **RN** (8.163), **round** (8.164), **ceil** (8.24)

8.169 RZ

Name: **RZ**
constant representing rounding-to-zero mode.

Library names:
`sollya_obj_t sollya_lib_round_towards_zero()`
`int sollya_lib_is_round_towards_zero(sollya_obj_t)`

Description:

- **RZ** is used in command **round** to specify that the value must be rounded to the closest floating-point number towards zero. It just consists in truncating the value to the desired format.

Example 1:

```
> display=binary!;
> round(Pi,20,RZ);
1.10010010000111111101_2 * 2^(1)
```

See also: **RD** (8.154), **RU** (8.168), **RN** (8.163), **round** (8.164), **floor** (8.72), **ceil** (8.24)

8.170 searchgal

Name: **searchgal**

searches for a preimage of a function such that the rounding the image yields an error smaller than a constant

Library name:

```
sollya_obj_t sollya_lib_searchgal(sollya_obj_t, sollya_obj_t, sollya_obj_t,
                                  sollya_obj_t, sollya_obj_t, sollya_obj_t)
```

Usage:

searchgal(*function*, *start*, *preimage precision*, *steps*, *format*, *error bound*) : (function, constant, integer, integer, HP|halfprecision|SG|single|D|double|DE|doubleextended|DD|doubledouble|QD|quad|TD|tripledouble, constant) → list

searchgal(*list of functions*, *start*, *preimage precision*, *steps*, *list of format*, *list of error bounds*) : (list, constant, integer, integer, list, list) → list

Parameters:

- *function* represents the function to be considered
- *start* represents a value around which the search is to be performed
- *preimage precision* represents the precision (discretization) for the eligible preimage values
- *steps* represents the binary logarithm ($\log_2$) of the number of search steps to be performed
- *format* represents the format the image of the function is to be rounded to
- *error bound* represents a upper bound on the relative rounding error when rounding the image
- *list of functions* represents the functions to be considered
- *list of formats* represents the respective formats the images of the functions are to be rounded to
- *list of error bounds* represents a upper bound on the relative rounding error when rounding the image

Description:

- The command **searchgal** searches for a preimage z of function *function* or a list of functions *list of functions* such that z is a floating-point number with *preimage precision* significant mantissa bits and the image y of the function, respectively each image y_i of the functions, rounds to format *format* respectively to the corresponding format in *list of format* with a relative rounding error less than *error bound* respectively the corresponding value in *list of error bounds*. During this search, at most 2^{steps} attempts are made. The search starts with a preimage value equal to *start*. This value is then increased and decreased by 1 ulp in precision *preimage precision* until a value is found or the step limit is reached.

  If the search finds an appropriate preimage z, **searchgal** evaluates to a list containing this value. Otherwise, **searchgal** evaluates to an empty list.

Example 1:

```
> searchgal(log(x),2,53,15,DD,1b-112);
[| |]
> searchgal(log(x),2,53,18,DD,1b-112);
[|2.000000000038497205423482228070497512817382812 5|]
```

Example 2:

```
> f = exp(x);
> s = searchgal(f,2,53,18,DD,1b-112);
> if (s != [||]) then {
    v = s[0];
    print("The rounding error is 2^(",evaluate(log2(abs(DD(f)/f - 1)),v),")");
  } else print("No value found");
The rounding error is 2^( -112.106878438809380148206984258358542322113874177832
)
```

Example 3:

```
> searchgal([|sin(x),cos(x)|],1,53,15,[|D,D|],[|1b-62,1b-60|]);
[|1.000000000001594946397176499885972589254379272460 94|]
```

See also: **round** (8.164), **double** (8.50), **doubledouble** (8.51), **tripledouble** (8.194), **evaluate** (8.58), **worstcase** (8.200)

8.171 SG

Name: **SG**
short form for **single**

See also: **single** (8.175)

8.172 showmessagenumbers

Name: **showmessagenumbers**
activates, deactivates or inspects the state variable controlling the displaying of numbers for messages

Library names:
`void sollya_lib_set_showmessagenumbers_and_print(sollya_obj_t)`
`void sollya_lib_set_showmessagenumbers(sollya_obj_t)`
`sollya_obj_t sollya_lib_get_showmessagenumbers()`

Usage:

showmessagenumbers = *activation value* : on|off → void
showmessagenumbers = *activation value* ! : on|off → void
showmessagenumbers : on|off

Parameters:

- *activation value* represents **on** or **off**, i.e. activation or deactivation

Description:

- An assignment **showmessagenumbers** = *activation value*, where *activation value* is one of **on** or **off**, activates respectively deactivates the displaying of numbers for warning and information messages. Every `Sollya` warning or information message (that is not fatal to the tool's execution) has a message number. By default, these numbers are not displayed when a message is output. When message number displaying is activated, the message numbers are displayed together with the message. This allows the user to recover the number of a particular message in order to suppress resp. unsuppress the displaying of this particular message (see **suppressmessage** and **unsuppressmessage**).

- The user should be aware of the fact that message number display activation resp. deactivation through **showmessagenumbers** does not affect message displaying in general. For instance, even with message number displaying activated, messages with only displayed when general verbosity and rounding warning mode are set accordingly.
- If the assignment **showmessagenumbers** = *activation value* is followed by an exclamation mark, no message indicating the new state is displayed. Otherwise the user is informed of the new state of the global mode by an indication.

Example 1:

```
> verbosity = 1;
The verbosity level has been set to 1.
> 0.1;
Warning: Rounding occurred when converting the constant "0.1" to floating-point
with 165 bits.
If safe computation is needed, try to increase the precision.
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
0.1
> showmessagenumbers = on;
Displaying of message numbers has been activated.
> 0.1;
Warning (174): Rounding occurred when converting the constant "0.1" to floating-
point with 165 bits.
If safe computation is needed, try to increase the precision.
Warning (478): For at least 1 of the constants displayed in decimal, rounding ha
s happened.
0.1
> showmessagenumbers;
on
> showmessagenumbers = off!;
> 0.1;
Warning: Rounding occurred when converting the constant "0.1" to floating-point
with 165 bits.
If safe computation is needed, try to increase the precision.
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
0.1
```

Example 2:

```
> showmessagenumbers = on;
Displaying of message numbers has been activated.
> verbosity = 1;
The verbosity level has been set to 1.
> diff(0.1 * x + 1.5 * x^2);
Warning (174): Rounding occurred when converting the constant "0.1" to floating-
point with 165 bits.
If safe computation is needed, try to increase the precision.
Warning (478): For at least 1 of the constants displayed in decimal, rounding ha
s happened.
0.1 + x * 3
> verbosity = 0;
The verbosity level has been set to 0.
> diff(0.1 * x + 1.5 * x^2);
0.1 + x * 3
> verbosity = 12;
The verbosity level has been set to 12.
> diff(0.1 * x + 1.5 * x^2);
Warning (174): Rounding occurred when converting the constant "0.1" to floating-
point with 165 bits.
If safe computation is needed, try to increase the precision.
Information (196): formally differentiating a function.
Information (197): differentiating the expression '0.1 * x + 1.5 * x^2'
Information (195): expression '0.1 + 2 * 1.5 * x' has been simplified to express
ion '0.1 + 3 * x'.
Warning (478): For at least 3 of the constants previously displayed in decimal,
rounding has happened.
Warning (478): For at least 1 of the constants displayed in decimal, rounding ha
s happened.
0.1 + x * 3
```

See also: **getsuppressedmessages** (8.79), **suppressmessage** (8.184), **unsuppressmessage** (8.196), **verbosity** (8.198), **roundingwarnings** (8.167)

8.173 simplify

Name: **simplify**
simplifies an expression representing a function

Library name:
`sollya_obj_t sollya_lib_simplify(sollya_obj_t)`

Usage:

simplify(*function*) : function → function

Parameters:

- *function* represents the expression to be simplified

Description:

- The command **simplify** simplifies the expression given in argument representing the function *function*. The command **simplify** does not endanger the safety of computations even in `Sollya`'s floating-point environment: the function returned is mathematically equal to the function *function*.

  Remark that the simplification provided by **simplify** is not perfect: they may exist simpler equivalent expressions for expressions returned by **simplify**.

Example 1:

```
> print(simplify((6 + 2) + (5 + exp(0)) * x));
8 + x * 6
```

Example 2:

```
> print(simplify((log(x - x + 1) + asin(1))));
(pi) / 2
```

Example 3:

```
> print(simplify((log(x - x + 1) + asin(1)) - (atan(1) * 2)));
(pi) / 2 - (pi) / 4 * 2
```

See also: **dirtysimplify** (8.46), **autosimplify** (8.16), **rationalmode** (8.153), **horner** (8.87)

8.174 sin

Name: **sin**
the sine function.

Library names:
```
sollya_obj_t sollya_lib_sin(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_sin(sollya_obj_t)
#define SOLLYA_SIN(x) sollya_lib_build_function_sin(x)
```

Description:

- **sin** is the usual sine function.
- It is defined for every real number x.

See also: **asin** (8.11), **cos** (8.31), **tan** (8.186)

8.175 single

Names: **single**, **SG**
rounding to the nearest IEEE 754 single (binary32).

Library names:
```
sollya_obj_t sollya_lib_single(sollya_obj_t)
sollya_obj_t sollya_lib_single_obj()
int sollya_lib_is_single_obj(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_single(sollya_obj_t)
#define SOLLYA_SG(x) sollya_lib_build_function_single(x)
```

Description:

- **single** is both a function and a constant.
- As a function, it rounds its argument to the nearest IEEE 754 single precision (i.e. IEEE754-2008 binary32) number. Subnormal numbers are supported as well as standard numbers: it is the real rounding described in the standard.
- As a constant, it symbolizes the single precision format. It is used in contexts when a precision format is necessary, e.g. in the commands **round** and **roundcoefficients**. In is not supported for **implementpoly**. See the corresponding help pages for examples.

Example 1:

```
> display=binary!;
> SG(0.1);
1.10011001100110011001101_2 * 2^(-4)
> SG(4.17);
1.00001010111000010100 1_2 * 2^(2)
> SG(1.011_2 * 2^(-1073));
0
```

See also: **halfprecision** (8.82), **double** (8.50), **doubleextended** (8.52), **doubledouble** (8.51), **quad** (8.149), **tripledouble** (8.194), **roundcoefficients** (8.165), **implementpoly** (8.90), **round** (8.164), **printsingle** (8.144)

8.176 sinh

Name: **sinh**
the hyperbolic sine function.

Library names:
`sollya_obj_t sollya_lib_sinh(sollya_obj_t)`
`sollya_obj_t sollya_lib_build_function_sinh(sollya_obj_t)`
`#define SOLLYA_SINH(x) sollya_lib_build_function_sinh(x)`

Description:

- **sinh** is the usual hyperbolic sine function: $\sinh(x) = \frac{e^x - e^{-x}}{2}$.
- It is defined for every real number x.

See also: **asinh** (8.12), **cosh** (8.32), **tanh** (8.187)

8.177 sort

Name: **sort**
sorts a list of real numbers.

Library name:
`sollya_obj_t sollya_lib_sort(sollya_obj_t)`

Usage:

$$\textbf{sort}(L) : \mathsf{list} \to \mathsf{list}$$

Parameters:

- L is a list.

Description:

- If L contains only constant values, **sort**(L) returns the same list, but sorted in increasing order.
- If L contains at least one element that is not a constant, the command fails with a type error.
- If L is an end-elliptic list, **sort** will fail with an error.

Example 1:

```
> sort([| |]);
[| |]
> sort([|2,3,5,2,1,4|]);
[|1, 2, 2, 3, 4, 5|]
```

See also: **revert** (8.162)

8.178 sqrt

Name: **sqrt**
square root.

Library names:
```
sollya_obj_t sollya_lib_sqrt(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_sqrt(sollya_obj_t)
#define SOLLYA_SQRT(x) sollya_lib_build_function_sqrt(x)
```

Description:

- **sqrt** is the square root, e.g. the inverse of the function square: $\sqrt{y}$ is the unique positive x such that $x^2 = y$.
- It is defined only for x in $[0; +\infty]$.

8.179 string

Name: **string**
keyword representing a string type

Library name:
```
SOLLYA_EXTERNALPROC_TYPE_STRING
```

Usage:

string : type type

Description:

- **string** represents the string type for declarations of external procedures by means of **externalproc**.

  Remark that in contrast to other indicators, type indicators like **string** cannot be handled outside the **externalproc** context. In particular, they cannot be assigned to variables.

See also: **externalproc** (8.66), **boolean** (8.22), **constant** (8.30), **function** (8.75), **integer** (8.94), **list of** (8.103), **range** (8.151), **object** (8.123)

8.180 subpoly

Name: **subpoly**
restricts the monomial basis of a polynomial to a list of monomials

Library name:
```
sollya_obj_t sollya_lib_subpoly(sollya_obj_t, sollya_obj_t)
```

Usage:

subpoly(*polynomial*, *list*) : (function, list) → function

Parameters:

- *polynomial* represents the polynomial the coefficients are taken from
- *list* represents the list of monomials to be taken

Description:

- **subpoly** extracts the coefficients of a polynomial *polynomial* and builds up a new polynomial out of those coefficients associated to monomial degrees figuring in the list *list*.

  If *polynomial* represents a function that is not a polynomial, subpoly returns 0.

  If *list* is a list that is end-elliptic, let be j the last value explicitly specified in the list. All coefficients of the polynomial associated to monomials greater or equal to j are taken.

Example 1:

```
> p = taylor(exp(x),5,0);
> s = subpoly(p,[|1,3,5|]);
> print(p);
1 + x * (1 + x * (0.5 + x * (1 / 6 + x * (1 / 24 + x * 1 / 120))))
> print(s);
x * (1 + x^2 * (1 / 6 + x^2 / 120))
```

Example 2:

```
> p = remez(atan(x),10,[-1,1]);
> subpoly(p,[|1,3,5...|]);
x * (0.99986632941452949026018468446163586361700915018232 + x^2 * (-0.3303047850
24559363626677940599884431309264334217 39 + x^2 * (0.18015929317818 75646289423703
782473512913009557 4422 + x * (2.28455841154247882851125015653585766424298569630 7
2e-9 + x * (-8.5156349064111377895500552996061844977507560037485e-2 + x * (-2.71
756340962775019916818769239340943524383018921 8e-9 + x * (2.084511343071147293732
399105491698724546869558 95e-2 + x * 1.10889861181129057657199664386826630081793 4
00489512e-9)))))))
```

Example 3:

```
> subpoly(exp(x),[|1,2,3|]);
0
```

See also: **roundcoefficients** (8.165), **taylor** (8.188), **remez** (8.158), **fpminimax** (8.73), **implementpoly** (8.90)

8.181 substitute

Name: **substitute**
replace the occurrences of the free variable in an expression.

Library name:
`sollya_obj_t sollya_lib_substitute(sollya_obj_t, sollya_obj_t)`

Usage:

substitute(*f*,*g*) : (function, function) → function
substitute(*f*,*t*) : (function, constant) → constant

Parameters:

- f is a function.
- g is a function.
- t is a real number.

Description:

- **substitute**(f, g) produces the function $(f \circ g) : x \mapsto f(g(x))$.
- **substitute**(f, t) is the constant $f(t)$. Note that the constant is represented by its expression until it has been evaluated (exactly the same way as if you type the expression f replacing instances of the free variable by t).
- If f is stored in a variable F, the effect of the commands **substitute**(F,g) or **substitute**(F,t) is absolutely equivalent to writing *F(g)* resp. *F(t)*.

Example 1:

```
> f=sin(x);
> g=cos(x);
> substitute(f,g);
sin(cos(x))
> f(g);
sin(cos(x))
```

Example 2:

```
> a=1;
> f=sin(x);
> substitute(f,a);
0.84147098480789650665250232163029899962256306079837
> f(a);
0.84147098480789650665250232163029899962256306079837
```

See also: **evaluate** (8.58), **composepolynomials** (8.28)

8.182 sup

Name: **sup**
gives the upper bound of an interval.

Library name:
`sollya_obj_t sollya_lib_sup(sollya_obj_t)`

Usage:

$$\textbf{sup}(I) : \text{range} \rightarrow \text{constant}$$
$$\textbf{sup}(x) : \text{constant} \rightarrow \text{constant}$$

Parameters:

- I is an interval.
- x is a real number.

Description:

- Returns the upper bound of the interval I. Each bound of an interval has its own precision, so this command is exact, even if the current precision is too small to represent the bound.
- When called on a real number x, **sup** behaves like the identity.

Example 1:

```
> sup([1;3]);
3
> sup(5);
5
```

Example 2:

```
> display=binary!;
> I=[0; 0.111110000011111_2];
> sup(I);
1.11110000011111_2 * 2^(-1)
> prec=12!;
> sup(I);
1.11110000011111_2 * 2^(-1)
```

See also: **inf** (8.92), **mid** (8.111), **max** (8.110), **min** (8.113)

8.183 supnorm

Name: **supnorm**

computes an interval bounding the supremum norm of an approximation error (absolute or relative) between a given polynomial and a function.

Library name:

```
sollya_obj_t sollya_lib_supnorm(sollya_obj_t, sollya_obj_t, sollya_obj_t,
                                sollya_obj_t, sollya_obj_t)
```

Usage:

supnorm(*p*, *f*, *I*, *errorType*, *accuracy*) : (function, function, range, absolute|relative, constant) → range

Parameters:

- *p* is a polynomial.
- *f* is a function.
- *I* is an interval.
- *errorType* is the type of error to be considered: **absolute** or **relative** (see details below).
- *accuracy* is a constant that controls the relative tightness of the interval returned.

Description:

- **supnorm**(*p*, *f*, *I*, *errorType*, *accuracy*) tries to compute an interval bound $r = [\ell, u]$ for the supremum norm of the error function $\varepsilon_{\text{absolute}} = p - f$ (when *errorType* evaluates to **absolute**) or $\varepsilon_{\text{relative}} = p/f - 1$ (when *errorType* evaluates to **relative**), over the interval I, such that $\sup_{x \in I}\{|\varepsilon(x)|\} \subseteq r$ and $0 \leq \left|\frac{u}{\ell} - 1\right| \leq$ *accuracy*. If **supnorm** succeeds in computing a suitable interval r, which it returns, that interval is guaranteed to contain the supremum norm value and to satisfy the required quality. Otherwise, **supnorm** evaluates to **error**, displaying a corresponding error message. These failure cases are rare and basically happen only for functions which are too complicated.
- Roughly speaking, **supnorm** is based on **taylorform** to obtain a higher degree polynomial approximation for *f*. This process is coupled with an a posteriori validation of a potential supremum norm upper bound. The validation is based on showing a certain polynomial the problem gets reduced to does not vanish. In cases when this process alone does not succeed, for instance because **taylorform** is unable to compute a sufficiently good approximation to *f*, **supnorm** falls back to bisecting the working interval until safe supremum norm bounds can be computed with the required accuracy or until the width of the subintervals becomes less than **diam** times the original interval *I*, in which case **supnorm** fails.
- The algorithm used for **supnorm** is quite complex, which makes it impossible to explain it here in further detail. Please find a complete description in the following article:

  Sylvain Chevillard, John Harrison, Mioara Joldes, Christoph Lauter
  Efficient and accurate computation of upper bounds of approximation errors
  Journal of Theoretical Computer Science (TCS), 2011, 412(16), p.1523-1543
  https://hal.science/ensl-00445343

- In practical cases, **supnorm** should be able to automatically handle removable discontinuities that relative errors might have. This means that usually, if f vanishes at a point x_0 in the interval considered, the approximation polynomial p is designed such that it also vanishes at the same point with a multiplicity large enough. Hence, although f vanishes, $\varepsilon_{\text{relative}} = p/f - 1$ may be defined by continuous extension at such points x_0, called removable discontinuities (see Example 3).

Example 1:

```
> p = remez(exp(x), 5, [-1;1]);
> midpointmode=on!;
> supnorm(p, exp(x), [-1;1], absolute, 2^(-40));
0.45205513967~0/2~e-4
```

Example 2:

```
> prec=200!;
> midpointmode=on!;
> d = [1;2];
> f = exp(cos(x)^2 + 1);
> p = remez(1,15,d,1/f,1e-40);
> theta=1b-60;
> prec=default!;
> mode=relative;
> supnorm(p,f,d,mode,theta);
0.30893006200251428~5/6~e-13
```

Example 3:

```
> midpointmode=on!;
> mode=relative;
> theta=1b-135;
> d = [-1b-2;1b-2];
> f = expm1(x);
> p = x * (1 +  x * ( 2097145 * 2^(-22) + x * ( 349527 * 2^(-21) + x * (87609 *
2^(-21) + x * 4369 * 2^(-19)))));
> theta=1b-40;
> supnorm(p,f,d,mode,theta);
0.98349131972~2/3~e-7
```

See also: **dirtyinfnorm** (8.44), **infnorm** (8.93), **checkinfnorm** (8.26), **absolute** (8.2), **relative** (8.157), **taylorform** (8.189), **autodiff** (8.15), **numberroots** (8.121), **diam** (8.40)

8.184 suppressmessage

Name: **suppressmessage**
suppresses the displaying of messages with a certain number

Library names:
`void sollya_lib_suppressmessage(sollya_obj_t, ...);`
`void sollya_lib_v_suppressmessage(sollya_obj_t, va_list);`

Usage:

suppressmessage(*msg num 1*, ..., *msg num n*) : (integer, ..., integer) → void
suppressmessage(*msg list*) : list → void

Parameters:

- *msg num 1* thru *msg num n* represent the numbers of n messages to be suppressed
- *msg list* represents a list with numbers of messages to be suppressed

Description:

- The **suppressmessage** command allows particular warning and information messages to be suppressed from message output, while maintaining global verbosity levels (see **verbosity**) high. Every `Sollya` warning or information message (that is not fatal to the tool's execution) has a message number. When these message numbers *msg num 1* thru *msg num n* are given to **suppressmessage**, the corresponding message are no longer displayed. The **unsuppressmessage** command reverts this suppression from output for a particular message.

- Instead of giving **suppressmessage** several message numbers *msg num 1* thru *msg num n* or calling **suppressmessage** several times, it is possible to give a whole list *msg list* of message numbers to **suppressmessage**.
- The user should be aware that **suppressmessage** presents sticky behavior for the warning and information messages suppressed from output. This means that even if subsequent calls to **suppressmessage** occur, a message suppressed by a call to **suppressmessage** stays suppressed until it is unsuppressed using **unsuppressmessage** or the tool is restarted. This behavior distinguishes message suppression from other global states of the `Sollya` tool. The user may use **getsuppressedmessages** to obtain a list of currently suppressed messages.
- When **suppressmessage** is used on message numbers that do not exist in the current version of the tool, a warning is displayed. The call has no other effect though.

Example 1:

```
> verbosity = 1;
The verbosity level has been set to 1.
> 0.1;
Warning: Rounding occurred when converting the constant "0.1" to floating-point
with 165 bits.
If safe computation is needed, try to increase the precision.
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
0.1
> suppressmessage(174);
> 0.1;
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
0.1
> suppressmessage(407);
> 0.1;
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
0.1
> verbosity = 12;
The verbosity level has been set to 12.
> showmessagenumbers = on;
Displaying of message numbers has been activated.
> diff(exp(x * 0.1));
Information (196): formally differentiating a function.
Information (197): differentiating the expression 'exp(x * 0.1)'
Information (207): no Horner simplification will be performed because the given
tree is already in Horner form.
Warning (478): For at least 1 of the constants previously displayed in decimal,
rounding has happened.
Warning (478): For at least 2 of the constants displayed in decimal, rounding ha
s happened.
exp(x * 0.1) * 0.1
> suppressmessage(207, 196);
> diff(exp(x * 0.1));
Information (197): differentiating the expression 'exp(x * 0.1)'
Warning (478): For at least 1 of the constants previously displayed in decimal,
rounding has happened.
Warning (478): For at least 2 of the constants displayed in decimal, rounding ha
s happened.
exp(x * 0.1) * 0.1
> unsuppressmessage(174);
> 0.1;
Warning (174): Rounding occurred when converting the constant "0.1" to floating-
point with 165 bits.
If safe computation is needed, try to increase the precision.
Warning (478): For at least 1 of the constants displayed in decimal, rounding ha
s happened.
0.1
```

Example 2:

```
> verbosity = 12;
The verbosity level has been set to 12.
> showmessagenumbers = on;
Displaying of message numbers has been activated.
> diff(exp(x * 0.1));
Warning (174): Rounding occurred when converting the constant "0.1" to floating-
point with 165 bits.
If safe computation is needed, try to increase the precision.
Information (196): formally differentiating a function.
Information (197): differentiating the expression 'exp(x * 0.1)'
Information (207): no Horner simplification will be performed because the given
tree is already in Horner form.
Warning (478): For at least 1 of the constants previously displayed in decimal,
rounding has happened.
Warning (478): For at least 2 of the constants displayed in decimal, rounding ha
s happened.
exp(x * 0.1) * 0.1
> suppressmessage([| 174, 207, 196 |]);
> diff(exp(x * 0.1));
Information (197): differentiating the expression 'exp(x * 0.1)'
Warning (478): For at least 1 of the constants previously displayed in decimal,
rounding has happened.
Warning (478): For at least 2 of the constants displayed in decimal, rounding ha
s happened.
exp(x * 0.1) * 0.1
```

See also: **getsuppressedmessages** (8.79), **suppressmessage** (8.184), **unsuppressmessage** (8.196), **verbosity** (8.198), **roundingwarnings** (8.167)

8.185 tail

Name: **tail**
gives the tail of a list.

Library name:
`sollya_obj_t sollya_lib_tail(sollya_obj_t)`

Usage:

$$\textbf{tail}(L) : \mathsf{list} \to \mathsf{list}$$

Parameters:

- L is a list.

Description:

- **tail**(L) returns the list L without its first element.
- If L is empty, the command will fail with an error.
- **tail** can also be used with end-elliptic lists. In this case, the result of **tail** is also an end-elliptic list.

Example 1:

```
> tail([|1,2,3|]);
[|2, 3|]
> tail([|1,2...|]);
[|2...|]
```

See also: **head** (8.83), **revert** (8.162)

8.186 tan

Name: **tan**
the tangent function.

Library names:
```
sollya_obj_t sollya_lib_tan(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_tan(sollya_obj_t)
#define SOLLYA_TAN(x) sollya_lib_build_function_tan(x)
```

Description:

- **tan** is the tangent function, defined by $\tan(x) = \sin(x)/\cos(x)$.
- It is defined for every real number x that is not of the form $n\pi + \pi/2$ where n is an integer.

See also: **atan** (8.13), **cos** (8.31), **sin** (8.174)

8.187 tanh

Name: **tanh**
the hyperbolic tangent function.

Library names:
```
sollya_obj_t sollya_lib_tanh(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_tanh(sollya_obj_t)
#define SOLLYA_TANH(x) sollya_lib_build_function_tanh(x)
```

Description:

- **tanh** is the hyperbolic tangent function, defined by $\tanh(x) = \sinh(x)/\cosh(x)$.
- It is defined for every real number x.

See also: **atanh** (8.14), **cosh** (8.32), **sinh** (8.176)

8.188 taylor

Name: **taylor**
computes a Taylor expansion of a function in a point

Library name:
```
sollya_obj_t sollya_lib_taylor(sollya_obj_t, sollya_obj_t, sollya_obj_t)
```

Usage:

taylor(*function*, *degree*, *point*) : (function, integer, constant) → function

Parameters:

- *function* represents the function to be expanded
- *degree* represents the degree of the expansion to be delivered
- *point* represents the point in which the function is to be developed

Description:

- The command **taylor** returns an expression that is a Taylor expansion of function *function* in point *point* having the degree *degree*.

  Let f be the function *function*, t be the point *point* and n be the degree *degree*. Then, **taylor**(*function*,*degree*,*point*) evaluates to an expression mathematically equal to

$$\sum_{i=0}^{n} \frac{f^{(i)}(t)}{i!} x^i.$$

In other words, if $p(x)$ denotes the polynomial returned by **taylor**, $p(x-t)$ is the Taylor polynomial of degree n of f developed at point t.

Remark that **taylor** evaluates to 0 if the degree *degree* is negative.

Example 1:

```
> print(taylor(exp(x),3,1));
exp(1) + x * (exp(1) + x * (0.5 * exp(1) + x * exp(1) / 6))
```

Example 2:

```
> print(taylor(asin(x),7,0));
x * (1 + x^2 * (1 / 6 + x^2 * (3 / 40 + x^2 * 5 / 112)))
```

Example 3:

```
> print(taylor(erf(x),6,0));
x * (2 / sqrt(pi) + x^2 * ((2 / sqrt(pi) * (-2)) / 6 + x^2 * (2 / sqrt(pi) * 12)
 / 120))
```

See also: **remez** (8.158), **fpminimax** (8.73), **taylorform** (8.189)

8.189 taylorform

Name: **taylorform**

computes a rigorous polynomial approximation (polynomial, interval error bound) for a function, based on Taylor expansions.

Library names:

```
sollya_obj_t sollya_lib_taylorform(sollya_obj_t, sollya_obj_t,
                                   sollya_obj_t, ...)
sollya_obj_t sollya_lib_v_taylorform(sollya_obj_t, sollya_obj_t,
                                     sollya_obj_t, va_list)
```

Usage:

taylorform(*f*, *n*, x_0, *I*, *errorType*) : (function, integer, constant, range, absolute|relative) → list
taylorform(*f*, *n*, x_0, *I*, *errorType*) : (function, integer, range, range, absolute|relative) → list
taylorform(*f*, *n*, x_0, *errorType*) : (function, integer, constant, absolute|relative) → list
taylorform(*f*, *n*, x_0, *errorType*) : (function, integer, range, absolute|relative) → list

Parameters:

- *f* is the function to be approximated.
- *n* is the degree of the polynomial that must approximate *f*.
- x_0 is the point (it can be a real number or an interval) where the Taylor exansion of the function is to be considered.
- *I* is the interval over which the function is to be approximated. If this parameter is omitted, the behavior is changed (see detailed description below).
- *errorType* (optional) is the type of error to be considered. See the detailed description below. Default is **absolute**.

Description:

- **WARNING: taylorform** is a certified command, not difficult to use but not completely straightforward to use either. In order to be sure to use it correctly, the reader is invited to carefully read this documentation entirely.

- **taylorform** computes an approximation polynomial and an interval error bound for function f. More precisely, it returns a list $L = [p, \text{coeffErrors}, \Delta]$ where:
  - p is an approximation polynomial of degree n such that $p(x - x_0)$ is roughly speaking a numerical Taylor expansion of f at the point x_0.
  - coeffsErrors is a list of $n + 1$ intervals. Each interval coeffsErrors$[i]$ contains an enclosure of all the errors accumulated when computing the i-th coefficient of p.
  - Δ is an interval that provides a bound for the approximation error between p and f. Its significance depends on the *errorType* considered.
- The polynomial p and the bound Δ are obtained using Taylor Models principles.
- Please note that x_0 can be an interval. In general, it is meant to be a small interval approximating a non representable value. If x_0 is given as a constant expression, it is first numerically evaluated (leading to a faithful rounding $\tilde{x_0}$ at precision **prec**), and it is then replaced by the (exactly representable) point-interval $[\tilde{x_0}, \tilde{x_0}]$. In particular, it is not the same to call **taylorform** with $x_0 =$ **pi** and with $x_0 =$ [**pi**], for instance. In general, if the point around which one desires to compute the polynomial is not exactly representable, one should preferably use a small interval for x_0.
- More formally, the mathematical property ensured by the algorithm may be stated as follows. For all ξ_0 in x_0, there exist (small) values $\varepsilon_i \in$ coeffsErrors$[i]$ such that:
  If *errorType* is **absolute**, $\forall x \in I, \exists \delta \in \Delta,\ f(x) - p(x - \xi_0) = \sum_{i=0}^{n} \varepsilon_i \, (x - \xi_0)^i + \delta$.
  If *errorType* is **relative**, $\forall x \in I, \exists \delta \in \Delta,\ f(x) - p(x - \xi_0) = \sum_{i=0}^{n} \varepsilon_i \, (x - \xi_0)^i + \delta \, (x - \xi_0)^{n+1}$.
- It is also possible to use a large interval for x_0, though it is not obvious to give an intuitive sense to the result of **taylorform** in that case. A particular case that might be interesting is when $x_0 = I$ in relative mode. In that case, denoting by p_i the coefficient of p of order i, the interval $p_i +$ coeffsError$[i]$ gives an enclosure of $f^{(i)}(I)/i!$. However, the command **autodiff** is more convenient for computing such enclosures.
- When the interval I is not given, the approximated Taylor polynomial is computed but no remainder is produced. In that case the returned list is $L = [p, \text{coeffErrors}]$.
- The relative case is especially useful when functions with removable singularities are considered. In such a case, this routine is able to compute a finite remainder bound, provided that the expansion point given is the problematic removable singularity point.
- The algorithm does not guarantee that by increasing the degree of the approximation, the remainder bound will become smaller. Moreover, it may even become larger due to the dependency phenomenon present with interval arithmetic. In order to reduce this phenomenon, a possible solution is to split the definition domain I into several smaller intervals.
- The command **taylor** also computes a Taylor polynomial of a function. However it does not provide a bound on the remainder. Besides, **taylor** is a somehow symbolic command: each coefficient of the Taylor polynomial is computed exactly and returned as an expression tree exactly equal to theoretical value. It is henceforth much more inefficient than **taylorform** and **taylorform** should be preferred if only numercial (yet safe) computations are required. The same difference exists between commands **diff** and **autodiff**.

Example 1:

```
> TL=taylorform(sin(x)/x, 10, 0, [-1,1], relative);
> p=TL[0];
> Delta=TL[2];
> errors=TL[1];
> for epsi in errors do epsi;
[0;0]
[0;0]
[0;5.3455294201843912922810729343029637576303937602le-51]
[0;0]
[-3.3409558876152445576756705839393523485189961001313e-52;3.3409558876152445576756705839393523485189961001313e-52]
[0;0]
[-1.0440487148797639242736470574810476089121862812910 3e-53;1.0440487148797639242736470574810476089121862812910 3e-53]
[0;0]
[-1.6313261169996311316775735273141368889252910645172 4e-55;1.6313261169996311316775735273141368889252910645172 4e-55]
[0;0]
[-1.9117102933589427324346564773212541667093254662311 4e-57;1.9117102933589427324346564773212541667093254662311 4e-57]
> p; Delta;
1 + x^2 * (-0.16666666666666666666666666666666666666666666666667 + x^2 * (8.3333333333333333333333333333333333333333333333333e-3 + x^2 * (-1.9841269841269841269841269841269841269841269841270e-4 + x^2 * (2.7557319223985890652557319223985890652557319223986e-6 + x^2 * (-2.5052108385441718775052108385441718775052108385442e-8)))))
[-1.6135797443886066084999806203254010793747502812764e-10;1.6135797443886066084999806203254010793747502812764e-10]
```

Example 2:

```
> TL=taylorform(exp(x), 10, 0, [-1,1], absolute);
> p=TL[0];
> Delta=TL[2];
> p; Delta;
1 + x * (1 + x * (0.5 + x * (0.16666666666666666666666666666666666666666666666667 + x * (4.1666666666666666666666666666666666666666666666668e-2 + x * (8.3333333333333333333333333333333333333333333333333e-3 + x * (1.3888888888888888888888888888888888888888888888889e-3 + x * (1.9841269841269841269841269841269841269841269841270e-4 + x * (2.4801587301587301587301587301587301587301587301587e-5 + x * (2.7557319223985890652557319223985890652557319223986e-6 + x * 2.7557319223985890652557319223985890652557319223986e-7)))))))))
[-2.3114271964118761944124253418268474583253955510297e-8;2.7312660755642474420206278018039434042553645532164e-8]
```

Example 3:

```
> TL1 = taylorform(exp(x), 10, log2(10), [-1,1], absolute);
> TL2 = taylorform(exp(x), 10, [log2(10)], [-1,1], absolute);
> TL1==TL2;
false
```

Example 4:

```
> TL1 = taylorform(exp(x), 3, 0, [0,1], relative);
> TL2 = taylorform(exp(x), 3, 0, relative);
> TL1[0]==TL2[0];
true
> TL1[1]==TL2[1];
true
> length(TL1);
3
> length(TL2);
2
```

Example 5:

```
> f = exp(cos(x)); x0 = 0;
> TL = taylorform(f, 3, x0);
> T1 = TL[0];
> T2 = taylor(f, 3, x0);
> print(coeff(T1, 2));
-1.3591409142295226176801437356763312488786235468 5
> print(coeff(T2, 2));
-(0.5 * exp(1))
```

See also: **diff** (8.42), **autodiff** (8.15), **taylor** (8.188), **remez** (8.158), **chebyshevform** (8.25)

8.190 taylorrecursions

Name: **taylorrecursions**
controls the number of recursion steps when applying Taylor's rule.

Library names:
`void sollya_lib_set_taylorrecursions_and_print(sollya_obj_t)`
`void sollya_lib_set_taylorrecursions(sollya_obj_t)`
`sollya_obj_t sollya_lib_get_taylorrecursions()`

Usage:

taylorrecursions = n : integer → void
taylorrecursions = n ! : integer → void
taylorrecursions : integer

Parameters:

- n represents the number of recursions

Description:

- **taylorrecursions** is a global variable. Its value represents the number of steps of recursion that are used when applying Taylor's rule. This rule is applied by the interval evaluator present in the core of `Sollya` (and particularly visible in commands like **infnorm**).
- To improve the quality of an interval evaluation of a function f, in particular when there are problems of decorrelation), the evaluator of `Sollya` uses Taylor's rule: $f([a,b]) \subseteq f(m) + [a-m, b-m] \cdot f'([a,b])$ where $m = \frac{a+b}{2}$. This rule can be applied recursively. The number of step in this recursion process is controlled by **taylorrecursions**.
- Setting **taylorrecursions** to 0 makes `Sollya` use this rule only once; setting it to 1 makes `Sollya` use the rule twice, and so on. In particular: the rule is always applied at least once.

Example 1:

```
> f=exp(x);
> p=remez(f,3,[0;1]);
> taylorrecursions=0;
The number of recursions for Taylor evaluation has been set to 0.
> evaluate(f-p, [0;1]);
[-0.468393648162683687751746578141124602432490796710 39;0.469477817546468206472930197284029347469746525846 71]
> taylorrecursions=1;
The number of recursions for Taylor evaluation has been set to 1.
> evaluate(f-p, [0;1]);
[-0.138131114954063839905475752120786856031651747712954;0.139215284337848358626659371263691601068907476849 27]
```

See also: **hopitalrecursions** (8.86), **evaluate** (8.58), **infnorm** (8.93)

8.191 TD

Name: **TD**
short form for **tripledouble**

See also: **tripledouble** (8.194)

8.192 time

Name: **time**
procedure for timing `Sollya` code.

Usage:

$$\textbf{time}(\textit{code}) : \mathsf{code} \rightarrow \mathsf{constant}$$

Parameters:

- *code* is the code to be timed.

Description:

- **time** permits timing a `Sollya` instruction, resp. a begin-end block of `Sollya` instructions. The timing value, measured in seconds, is returned as a `Sollya` constant (and not merely displayed as for **timing**). This permits performing computations of the timing measurement value inside `Sollya`.
- The extended **nop** command permits executing a defined number of useless instructions. Taking the ratio of the time needed to execute a certain `Sollya` instruction and the time for executing a **nop** therefore gives a way to abstract from the speed of a particular machine when evaluating an algorithm's performance.

Example 1:

```
> t = time(p=remez(sin(x),10,[-1;1]));
> write(t,"s were spent computing p = ",p,"\n");
4.26705779999999999999999999999999997491079024385e-2s were spent computing p =
 9.04868987499779909869088513577591917113547770146 02e-17 * x^10 + 2.687625951151235962999593209591416400126834067365 86e-6 * x^9 + -2.42479784925213133490732322892462057278562686980 01e-16 * x^8 + -1.98344863020965929701245606503586461226130937598776e-4 * x^7 + 2.27482147577535443491624262818579101625754921262 67e-16 * x^6 + 8.33330371865609805676978214208137995472764814097 02e-3 * x^5 + -8.57471519897206697417069613035495313121105112188 7e-17 * x^4 + -0.16666666138601323707621656649395384777156455274417 3 * x^3 + 1.05699558969863875841493332282097022580493449058156e-17 * x^2 + 0.99999999973628365676559825181776417246038944720795 * x + (-3.12065309566018830243163208536426045628106466008 78e-19)
```

Example 2:

```
> write(time({ p=remez(sin(x),10,[-1;1]); write("The error is 2^(", log2(dirtyin
fnorm(p-sin(x),[-1;1])), ")\n"); }), "s were spent\n");
The error is 2^(log2(2.396019794465244866066495282899334820702948080 74097e-11))
7.040149199999999999999999999999999998019608367904e-2s were spent
```

Example 3:

```
> t = time(bashexecute("sleep 10"));
> write(~(t-10),"s of execution overhead.\n");
2.124573999999999999999999999999999400245385646393 1e-3s of execution overhead.
```

Example 4:

```
> ratio := time(p=remez(sin(x),10,[-1;1]))/time(nop(10));
> write("This ratio = ", ratio, " should somehow be independent of the type of m
achine.\n");
This ratio = 1.39045076529209293074742092014152229759491194488164 should somehow
 be independent of the type of machine.
```

See also: **timing** (8.193), **nop** (8.119)

8.193 timing

Name: **timing**
global variable controlling timing measures in `Sollya`.

Library names:
`void sollya_lib_set_timing_and_print(sollya_obj_t)`
`void sollya_lib_set_timing(sollya_obj_t)`
`sollya_obj_t sollya_lib_get_timing()`

Usage:

timing = *activation value* : on|off → void
timing = *activation value* ! : on|off → void
timing : on|off

Parameters:

- *activation value* controls if timing should be performed or not

Description:

- **timing** is a global variable. When its value is **on**, the time spent in each command is measured and displayed (for **verbosity** levels higher than 1).

Example 1:

```
> verbosity=1!;
> timing=on;
Timing has been activated.
> p=remez(sin(x),10,[-1;1]);
Information: Remez: computing the quality of approximation spent 5 ms
Information: Remez: computing the quality of approximation spent 4 ms
Information: Remez: computing the quality of approximation spent 4 ms
Information: computing a minimax approximation spent 28 ms
Information: assignment spent 33 ms
Information: full execution of the last parse chunk spent 39 ms
```

See also: **on** (8.126), **off** (8.125), **time** (8.192)

8.194 tripledouble

Names: **tripledouble**, **TD**
represents a number as the sum of three IEEE doubles.

Library names:
```
sollya_obj_t sollya_lib_triple_double(sollya_obj_t)
sollya_obj_t sollya_lib_triple_double_obj()
int sollya_lib_is_triple_double_obj(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_triple_double(sollya_obj_t)
#define SOLLYA_TD(x) sollya_lib_build_function_triple_double(x)
```

Description:

- **tripledouble** is both a function and a constant.
- As a function, it rounds its argument to the nearest number that can be written as the sum of three double precision numbers.
- The algorithm used to compute **tripledouble**(x) is the following: let $x_h =$ **double**(x), let $x_m =$ **double**($x - x_h$) and let $x_l =$ **double**($x - x_h - x_m$). Return the number $x_h + x_m + x_l$. Note that if the current precision is not sufficient to represent exactly $x_h + x_m + x_l$, a rounding will occur and the result of **tripledouble**(x) will be useless.
- As a constant, it symbolizes the triple-double precision format. It is used in contexts when a precision format is necessary, e.g. in the commands **roundcoefficients** and **implementpoly**. See the corresponding help pages for examples.

Example 1:

```
> verbosity=1!;
> a = 1+ 2^(-55)+2^(-115);
> TD(a);
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
1.00000000000000002775557561562891353466491600711096
> prec=110!;
> TD(a);
1.000000000000000027755575615628913534664916007110955975699724282311591481726270
602692352440499234944581985473632812 5
```

See also: **halfprecision** (8.82), **single** (8.175), **double** (8.50), **doubleextended** (8.52), **doubledouble** (8.51), **quad** (8.149), **roundcoefficients** (8.165), **implementpoly** (8.90), **fpminimax** (8.73), **printexpansion** (8.143)

8.195 true

Name: **true**
the boolean value representing the truth.

Library names:
```
sollya_obj_t sollya_lib_true()
int sollya_lib_is_true(sollya_obj_t)
```

Description:

- **true** is the usual boolean value.

Example 1:

```
> true && false;
false
> 2>1;
true
```

See also: **false** (8.67), && (8.6), || (8.127)

8.196 unsuppressmessage

Name: **unsuppressmessage**
unsuppresses the displaying of messages with a certain number

Library names:
```
void sollya_lib_unsuppressmessage(sollya_obj_t, ...);
void sollya_lib_v_unsuppressmessage(sollya_obj_t, va_list);
```

Usage:

unsuppressmessage(*msg num 1*, ..., *msg num n*) : (integer, ..., integer) → void
unsuppressmessage(*msg list*) : list → void

Parameters:

- *msg num 1* thru *msg num n* represent the numbers of n messages to be suppressed
- *msg list* represents a list with numbers of messages to be suppressed

Description:

- The **unsuppressmessage** command allows particular warning and information messages that have been suppressed from message output to be unsuppressed, i.e. activated for display again. Every `Sollya` warning or information message (that is not fatal to the tool's execution) has a message number. When these message numbers *msg num 1* thru *msg num n* are given to **unsuppressmessage**, the corresponding message are displayed again, as they are by default at according verbosity levels. Actually, the **unsuppressmessage** command just reverts the effects of the **suppressmessage** command.
- Instead of giving **unsuppressmessage** several message numbers *msg num 1* thru *msg num n* or calling **unsuppressmessage** several times, it is possible to give a whole list *msg list* of message numbers to **unsuppressmessage**.
- The user should be aware that **unsuppressmessage** presents sticky behavior for the warning and information messages suppressed from output. In fact, **unsuppressmessage** just unsuppresses the warning or information messages given in argument. All other suppressed messages stay suppressed until they get unsuppressed by subsequent calls to **unsuppressmessage** or the `Sollya` tool is restarted. This behavior distinguishes message suppression from other global states of the `Sollya` tool. The user may use **getsuppressedmessages** to obtain a list of currently suppressed messages. In particular, in order to unsuppressed all currently suppressed warning or information messages, the user may feed the output of **getsuppressedmessages** (a list) into **unsuppressmessage**.
- The user should also note that unsuppressing warning or information messages with **unsuppressmessage** just reverts the effects of the **suppressmessage** command but that other conditions exist that affect the actual displaying of a message, such as global verbosity (see **verbosity**) and modes like rounding warnings (see **roundingwarnings**). A message will not just get displayed because it was unsuppressed with **unsuppressmessage**.
- When **unsuppressmessage** is used on message numbers that do not exist in the current version of the tool, a warning is displayed. The call has no other effect though.

Example 1:

```
> verbosity = 1;
The verbosity level has been set to 1.
> 0.1;
Warning: Rounding occurred when converting the constant "0.1" to floating-point
with 165 bits.
If safe computation is needed, try to increase the precision.
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
0.1
> suppressmessage(174);
> 0.1;
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
0.1
> suppressmessage(174);
> 0.1;
Warning: For at least 1 of the constants displayed in decimal, rounding has happ
ened.
0.1
```

Example 2:

```
> verbosity = 12;
The verbosity level has been set to 12.
> showmessagenumbers = on;
Displaying of message numbers has been activated.
> diff(exp(x * 0.1));
Warning (174): Rounding occurred when converting the constant "0.1" to floating-
point with 165 bits.
If safe computation is needed, try to increase the precision.
Information (196): formally differentiating a function.
Information (197): differentiating the expression 'exp(x * 0.1)'
Information (207): no Horner simplification will be performed because the given
tree is already in Horner form.
Warning (478): For at least 1 of the constants previously displayed in decimal,
rounding has happened.
Warning (478): For at least 2 of the constants displayed in decimal, rounding ha
s happened.
exp(x * 0.1) * 0.1
> suppressmessage([| 174, 207, 196 |]);
> diff(exp(x * 0.1));
Information (197): differentiating the expression 'exp(x * 0.1)'
Warning (478): For at least 1 of the constants previously displayed in decimal,
rounding has happened.
Warning (478): For at least 2 of the constants displayed in decimal, rounding ha
s happened.
exp(x * 0.1) * 0.1
> unsuppressmessage([| 174, 196 |]);
```

Example 3:

```
> verbosity = 12;
The verbosity level has been set to 12.
> showmessagenumbers = on;
Displaying of message numbers has been activated.
> suppressmessage(207, 387, 390, 388, 391, 196, 195, 197, 205);
> getsuppressedmessages();
[|195, 196, 197, 205, 207, 387, 388, 390, 391|]
> evaluate(x/sin(x) - 1, [-1;1]);
Warning (478): For at least 1 of the constants displayed in decimal, rounding ha
s happened.
[0;0.85081571768092561791175324139865019347039665509 41]
> unsuppressmessage(getsuppressedmessages());
> getsuppressedmessages();
[| |]
```

See also: **getsuppressedmessages** (8.79), **suppressmessage** (8.184), **unsuppressmessage** (8.196), **verbosity** (8.198), **roundingwarnings** (8.167)

8.197 var

Name: **var**
declaration of a local variable in a scope

Usage:

var *identifier1*, *identifier2*,... , *identifiern* : void

Parameters:

- *identifier1*, *identifier2*,... , *identifiern* represent variable identifiers

Description:

- The keyword **var** allows for the declaration of local variables *identifier1* through *identifiern* in a begin-end-block ({}-block). Once declared as a local variable, an identifier will shadow identifiers declared in higher scopes and undeclared identifiers available at top-level.

  Variable declarations using **var** are only possible in the beginning of a begin-end-block. Several **var** statements can be given. Once another statement is given in a begin-end-block, no more **var** statements can be given.

  Variables declared by **var** statements are dereferenced as **error** until they are assigned a value.

Example 1:

```
> exp(x);
exp(x)
> a = 3;
> {var a, b; a=5; b=3; {var a; var b; b = true; a = 1; a; b;}; a; b; };
1
true
5
3
> a;
3
```

See also: **error** (8.57), **proc** (8.146)

8.198 verbosity

Name: **verbosity**
global variable controlling the amount of information displayed by commands.

Library names:
`void sollya_lib_set_verbosity_and_print(sollya_obj_t)`
`void sollya_lib_set_verbosity(sollya_obj_t)`
`sollya_obj_t sollya_lib_get_verbosity()`

Usage:

verbosity = n : integer → void
verbosity = n ! : integer → void
verbosity : integer

Parameters:

- n controls the amount of information to be displayed

Description:

- **verbosity** accepts any integer value. At level 0, commands do not display anything on standard output. Note that very critical information may however be displayed on standard error.
- Default level is 1. It displays important information such as warnings when roundings happen.
- For higher levels more information is displayed depending on the command.

Example 1:

```
> verbosity=0!;
> 1.2+"toto";
error
> verbosity=1!;
> 1.2+"toto";
Warning: Rounding occurred when converting the constant "1.2" to floating-point
with 165 bits.
If safe computation is needed, try to increase the precision.
Warning: at least one of the given expressions or a subexpression is not correct
ly typed
or its evaluation has failed because of some error on a side-effect.
error
> verbosity=2!;
> 1.2+"toto";
Warning: Rounding occurred when converting the constant "1.2" to floating-point
with 165 bits.
If safe computation is needed, try to increase the precision.
Warning: at least one of the given expressions or a subexpression is not correct
ly typed
or its evaluation has failed because of some error on a side-effect.
Information: the expression or a partial evaluation of it has been the following
:
(1.2) + ("toto")
Warning: For at least 1 of the constants previously displayed in decimal, roundi
ng has happened.
error
```

See also: **roundingwarnings** (8.167), **suppressmessage** (8.184), **unsuppressmessage** (8.196), **showmessagenumbers** (8.172), **getsuppressedmessages** (8.79)

8.199 void

Name: **void**
the functional result of a side-effect or empty argument resp. the corresponding type

Library names:
`sollya_obj_t sollya_lib_void()`
`int sollya_lib_is_void(sollya_obj_t)`
`SOLLYA_EXTERNALPROC_TYPE_VOID`

Usage:

void : void | type type

Description:

- The variable **void** represents the functional result of a side-effect or an empty argument. It is used only in combination with the applications of procedures or identifiers bound through **externalproc** to external procedures.

  The **void** result produced by a procedure or an external procedure is not printed at the prompt. However, it is possible to print it out in a print statement or in complex data types such as lists.

  The **void** argument is implicit when giving no argument to a procedure or an external procedure when applied. It can nevertheless be given explicitly. For example, suppose that foo is a procedure or an external procedure with a void argument. Then foo() and foo(void) are correct calls to foo. Here, a distinction must be made for procedures having an arbitrary number of arguments. In this case, an implicit **void** as the only parameter to a call of such a procedure gets converted into an empty list of arguments, an explicit **void** gets passed as-is in the formal list of parameters the procedure receives.

- **void** is used also as a type identifier for **externalproc**. Typically, an external procedure taking **void** as an argument or returning **void** is bound with a signature **void** $->$ some type or some type $->$ **void**. See **externalproc** for more details.

Example 1:

```
> print(void);
void
> void;
```

Example 2:

```
> hey = proc() { print("Hello world."); };
> hey;
proc()
{
print("Hello world.");
return void;
}
> hey();
Hello world.
> hey(void);
Hello world.
> print(hey());
Hello world.
void
```

Example 3:

```
> bashexecute("gcc -fPIC -Wall -c externalprocvoidexample.c");
> bashexecute("gcc -fPIC -shared -o externalprocvoidexample externalprocvoidexam
ple.o");
> externalproc(foo, "./externalprocvoidexample", void -> void);
> foo;
foo
> foo();
Hello from the external world.
> foo(void);
Hello from the external world.
> print(foo());
Hello from the external world.
void
```

Example 4:

```
> procedure blub(L = ...) { print("Argument list:", L); };
> blub(1);
Argument list: [|1|]
> blub();
Argument list: [| |]
> blub(void);
Argument list: [|void|]
```

See also: **error** (8.57), **proc** (8.146), **externalproc** (8.66)

8.200 worstcase

Name: **worstcase**
searches for hard-to-round cases of a function

Library names:

```
void sollya_lib_worstcase(sollya_obj_t, sollya_obj_t, sollya_obj_t,
                          sollya_obj_t, sollya_obj_t, ...)
void sollya_lib_v_worstcase(sollya_obj_t, sollya_obj_t, sollya_obj_t,
                            sollya_obj_t, sollya_obj_t, va_list)
```

Usage:

worstcase(*function*, *preimage precision*, *preimage exponent range*, *image precision*, *error bound*) : (function, integer, range, integer, constant) → void
worstcase(*function*, *preimage precision*, *preimage exponent range*, *image precision*, *error bound*, *filename*) : (function, integer, range, integer, constant, string) → void

Parameters:

- *function* represents the function to be considered
- *preimage precision* represents the precision of the preimages
- *preimage exponent range* represents the exponents in the preimage format
- *image precision* represents the precision of the format the images are to be rounded to
- *error bound* represents the upper bound for the search w.r.t. the relative rounding error
- *filename* represents a character sequence containing a filename

Description:

- The **worstcase** command is deprecated. It searches for hard-to-round cases of a function. The command **searchgal** has a comparable functionality.

Example 1:

```
> worstcase(exp(x),24,[1,2],24,1b-26);
prec = 165
x = 1.9999998807907104492187 5        f(x) = 7.3890552520751953125        eps = 4
.5998601423446695596184695493764120138001954979037e-9 = 2^(-27.695763)
x = 2        f(x) = 7.38905620574951171875        eps = 1.4456360874967301812222
837939553341787812515058707 2e-8 = 2^(-26.043720)
```

See also: **round** (8.164), **searchgal** (8.170), **evaluate** (8.58)

8.201 write

Name: **write**
prints an expression without separators

Usage:

write(*expr1*,...,*exprn*) : (any type,..., any type) → void
write(*expr1*,...,*exprn*) > *filename* : (any type,..., any type, string) → void
write(*expr1*,...,*exprn*) >> *filename* : (any type,...,any type, string) → void

Parameters:

- *expr* represents an expression
- *filename* represents a character sequence indicating a file name

Description:

- **write**(*expr1*,...,*exprn*) prints the expressions *expr1* through *exprn*. The character sequences corresponding to the expressions are concatenated without any separator. No newline is displayed at the end. In contrast to **print**, **write** expects the user to give all separators and newlines explicitly.

  If a second argument *filename* is given after a single ">", the displaying is not output on the standard output of `Sollya` but if in the file *filename* that get newly created or overwritten. If a double ">>" is given, the output will be appended to the file *filename*.

  The global variables **display**, **midpointmode** and **fullparentheses** have some influence on the formatting of the output (see **display**, **midpointmode** and **fullparentheses**).

  Remark that if one of the expressions *expri* given in argument is of type string, the character sequence *expri* evaluates to is displayed. However, if *expri* is of type list and this list contains a variable of type string, the expression for the list is displayed, i.e. all character sequences get displayed surrounded by quotes ("). Nevertheless, escape sequences used upon defining character sequences are interpreted immediately.

Example 1:

```
> write(x + 2 + exp(sin(x)));
> write("Hello\n");
x + 2 + exp(sin(x))Hello
> write("Hello","world\n");
Helloworld
> write("Hello","you", 4 + 3, "other persons.\n");
Helloyou7other persons.
```

Example 2:

```
> write("Hello","\n");
Hello
> write([|"Hello"|],"\n");
[|"Hello"|]
> s = "Hello";
> write(s,[|s|],"\n");
Hello[|"Hello"|]
> t = "Hello\tyou";
> write(t,[|t|],"\n");
Hello    you[|"Hello\tyou"|]
```

Example 3:

```
> write(x + 2 + exp(sin(x))) > "foo.sol";
> readfile("foo.sol");
x + 2 + exp(sin(x))
```

Example 4:

```
> write(x + 2 + exp(sin(x))) >> "foo.sol";
```

See also: **print** (8.141), **printexpansion** (8.143), **printdouble** (8.142), **printsingle** (8.144), **printxml** (8.145), **readfile** (8.155), **autosimplify** (8.16), **display** (8.47), **midpointmode** (8.112), **fullparentheses** (8.74), **evaluate** (8.58), **roundingwarnings** (8.167), **autosimplify** (8.16)

8.202 __x__

Name: **__x__**
universal name for the mathematical free variable.

Library names:
```
sollya_obj_t sollya_lib_free_variable()
sollya_obj_t sollya_lib_build_function_free_variable()
#define SOLLYA_X_ (sollya_lib_build_function_free_variable())
```

Description:

- **__x__** is an identifier that always denotes the mathematical free variable. It cannot be assigned.
- `Sollya` manipulates mathematical functions of a single variable. The first time that a variable name is used without having been assigned before, this variable name is automatically considered by `Sollya` as the name of the free variable. Subsequently, any other unassigned variable name will be considered as the free variable with a warning making this conversion explicit. This is convenient for an every-day use of the interactive tool, but it has the drawback that the free variable name can change from a session to another. There are contexts (*e.g.*, within a procedure, or for doing pattern matching) when one might want to refer to the free variable regardless of its name in the current session. For this purpose **__x__** is a universal identifier, always available and always denoting the free variable, whatever its name is in the current context.
- **__x__** can be used to retrieve the name of the current free variable as a string by simply concatenating it to "".

Example 1:

```
> verbosity=1!;
> sin(a);
sin(a)
> b;
Warning: the identifier "b" is neither assigned to, nor bound to a library funct
ion nor external procedure, nor equal to the current free variable.
Will interpret "b" as "a".
a
> _x_;
a
```

Example 2:

```
> verbosity=1!;
> sin(y);
sin(y)
> f = proc(a) {
    return sin(a + _x_);
  };
> rename(y,z);
Information: the free variable has been renamed from "y" to "z".
> f(1);
sin(1 + z)
```

Example 3:

```
> f = sin(y);
> match f with
    sin(a) : { print("sin of a with a =", a);
               match a with
                 _x_ : { print("a turns out to be the free variable"); }
             default : { print("a is some expression"); };
             }
    _x_ : { print("Free variable") ; }
    default: { print("Something else"); };
sin of a with a = y
a turns out to be the free variable
```

Example 4:

```
> sin(wurst);
sin(wurst)
> varname = _x_ @ "";
> print("The current name of the free variable is " @ varname);
The current name of the free variable is wurst
```

See also: **rename** (8.159), **isbound** (8.97), **proc** (8.146)

9 Appendix: interval arithmetic philosophy in `Sollya`

Although it is currently based on the MPFI library, `Sollya` has its own way of interpreting interval arithmetic when infinities or NaN occur, or when a function is evaluated on an interval containing points out of its domain, etc. This philosophy may differ from the one applied in MPFI. It is also possible that the behavior of `Sollya` does not correspond to the behavior that one would expect, *e.g.*, as a natural consequence of the IEEE-754 standard.

The topology that we consider is always the usual topology of $\overline{\mathbb{R}} = \mathbb{R} \cup \{-\infty, +\infty\}$. For any function, if one of its arguments is empty (respectively NaN), we return empty (respectively NaN).

9.1 Univariate functions

Let f be a univariate basic function and I an interval. We denote by J the result of the interval evaluation of f over I in `Sollya`. If I is completely included in the domain of f, J will usually be the smallest interval (at the current precision) containing the exact image $f(I)$. However, in some cases, it may happen that J is not as small as possible. It is guaranteed however, that J tends to $f(I)$ when the precision of the tool tends to infinity.

When f is not defined at some point x but is defined on a neighborhood of x, we consider that the "value" of f at x is the convex hull of the limit points of f around x. For instance, consider the evaluation of $f = \tan$ on $[0, \pi]$. It is not defined at $\pi/2$ (and only at this point). The limit points of f around $\pi/2$ are $-\infty$ and $+\infty$, so, we return $[-\infty, \infty]$. Another example: $f = \sin$ on $[+\infty]$. The function has no limit at this point, but all points of $[-1, 1]$ are limit points. So, we return $[-1, 1]$.

Finally, if I contains a subinterval on which f is not defined, we return [NaN, NaN] (example: $\sqrt{[-1, 2]}$).

9.2 Bivariate functions

Let f be a bivariate function and I_1 and I_2 be intervals. If $I_1 = [x]$ and $I_2 = [y]$ are both point-intervals, we return the convex hull of the limit points of f around (x, y) if it exists. In particular, if f is defined at (x, y) we return its value (or a small interval around it, if it is not exactly representable). As an example $[1]/[+\infty]$ returns $[0]$. Also, $[1]/[0]$ returns $[-\infty, +\infty]$ (note that `Sollya` does not consider signed zeros). If it is not possible to give a meaning to the expression $f(I_1, I_2)$, we return NaN: for instance $[0]/[0]$ or $[0] \times [+\infty]$.

In the general case, we return the convex hull of the union of all $f([x], [y])$ for $x \in I_1$ and $y \in I_2$. In particular, notice that if there is a point $(x, y) \in I_1 \times I_2$ such that $f([x], [y])$ would return NaN, we return NaN for $f(I_1, I_2)$ also (indeed, I_1 might be the overestimated result of a computation whose exact value is x, and accordingly for I_2 and y). Therefore, please note that $[0]/[-1, 1]$ returns NaN even though one could be tempted to see it as the function $x \mapsto 0/x$ evaluated on $[-1, 1]$ (which would return $[0]$ according to the rule described in the previous paragraph).

As a special exception to these rules, $[0]^{[0]}$ returns $[1]$.

10 Appendix: the Sollya library

10.1 Introduction

The header file of the Sollya library is sollya.h. Its inclusion may provoke the inclusion of other header files, such as gmp.h, mpfr.h or mpfi.h.

The library provides a virtual Sollya session that is perfectly similar to an interactive session: global variables such as verbosity, prec, display, midpointmode, etc. are maintained and affect the behavior of the library, warning messages are displayed when something is not exact, etc. Please notice that the Sollya library currently is **not** re-entrant and can only be opened once. A process using the library must hence not be multi-threaded and is limited to one single virtual Sollya session.

In order to get started with the Sollya library, the first thing to do is hence to initialize this virtual session. This is performed with the sollya_lib_init function. Accordingly, one should close the session at the end of the program (which has the effect of releasing all the memory used by Sollya). Please notice that Sollya uses its own allocation functions and registers them to GMP using the custom allocation functions provided by GMP. Particular precautions should hence be taken when using the Sollya library in a program that also registers its own functions to GMP: in that case sollya_lib_init_with_custom_memory_functions should be used instead of sollya_lib_init for initializing the library. This is discussed in Section 10.19.

In the usual case when Sollya is used in a program that does not register allocation functions to GMP, a minimal file using the library is hence the following.

```
#include <sollya.h>

int main(void) {
  sollya_lib_init();

    /* Functions of the library can be called here */

  sollya_lib_close();
  return 0;
}
```

Suppose that this code is saved in a file called foo.c. The compilation is performed as usual without forgetting to link against libsollya (since the libraries libgmp, libmpfr and libmpfi are dependencies of Sollya, it might also be necessary to explicitly link against them):

```
~/% cc foo.c -c
~/% cc foo.o -o foo -lsollya -lmpfi -lmpfr -lgmp
```

10.2 Sollya object data-type

The library provides a single data type called sollya_obj_t that can contain any Sollya object (a Sollya object is anything that can be stored in a variable within the interactive tool. See Section 5 of the present documentation for details). Please notice that sollya_obj_t is in fact a pointer type; this has two consequences:

- NULL is a placeholder that can be used as a sollya_obj_t in some contexts. This placeholder is particularly useful as an end marker for functions with a variable number of arguments (see Sections 10.5.4 and 10.17).
- An assignment with the "=" sign does not copy an object but only copies the reference to it. In order to perform a (deep) copy, the sollya_lib_copy_obj() function is available.

Except for a few functions for which the contrary is explicitly specified, the following conventions are used:

- A function does not touch its arguments. Hence if `sollya_lib_foo` is a function of the library, a call to `sollya_lib_foo(a)` leaves the object referenced by `a` unchanged (the notable exceptions to that rule are the functions containing `build` in their name, *e.g.*, `sollya_lib_build_foo`).
- A function that returns a `sollya_obj_t` creates a new object (this means that memory is dynamically allocated for that object). The memory allocated for that object should manually be cleared when the object is no longer used and all references to it (on the stack) get out of reach, *e.g.*, on a function return: this is performed by the `sollya_lib_clear_obj()` function. By convenience `sollya_lib_clear_obj(NULL)` is a valid call, which just does nothing.

In general, except if the user perfectly knows what they are doing, the following rules should be applied (here `a` and `b` are C variables of type `sollya_obj_t`, and `sollya_lib_foo` and `sollya_lib_bar` are functions of the library):

- One should never write `a = b`. Instead, use `a = sollya_lib_copy_obj(b)`.
- One should never write `a = sollya_lib_foo(a)` because one loses the reference to the object initially referenced by the variable `a` (which is hence not cleared).
- One should never chain function calls such as, *e.g.*, `a = sollya_lib_foo(sollya_lib_bar(b))` (the reference to the object created by `sollya_lib_bar(b)` would be lost and hence not cleared).
- A variable `a` should never be used twice at the left-hand side of the "=" sign (or as an lvalue in general) without performing `sollya_lib_clear_obj(a)` in-between.
- In an assignment of the form "`a = ...`", the right-hand side should always be a function call (*i.e.*, something like `a = sollya_lib_foo(...);`).

Please notice that `sollya_lib_close()` clears the memory allocated by the virtual `Sollya` session but not the objects that have been created and stored in C variables. All the `sollya_obj_t` created by function calls should be cleared manually.

We can now write a simple Hello World program using the `Sollya` library:

```
#include <sollya.h>

int main(void) {
  sollya_obj_t s1, s2, s;
  sollya_lib_init();

  s1 = sollya_lib_string("Hello ");
  s2 = sollya_lib_string("World!");
  s = sollya_lib_concat(s1, s2);
  sollya_lib_clear_obj(s1);
  sollya_lib_clear_obj(s2);

  sollya_lib_printf("%b\n", s);
  sollya_lib_clear_obj(s);
  sollya_lib_close();
  return 0;
}
```

A universal function allows the user to execute any expression, as if it were given at the prompt of the `Sollya` tool, and to get a `sollya_obj_t` containing the result of the evaluation: this function is `sollya_lib_parse_string("some expression here")`. This is very convenient, and indeed, any script written in the `Sollya` language, could easily be converted into a C program by intensively using `sollya_lib_parse_string`. However, this should not be the preferred way if efficiency is targeted because (as its name suggests) this function uses a parser to decompose its argument, then constructs intermediate data structures to store the abstract interpretation of the expression, etc. Low-level functions are provided for efficiently creating `Sollya` objects; they are detailed in the next Section.

10.3 Conventions in use in the library

The library follows some conventions that are useful to remember:

- When a function is a direct transposition of a command or function available in the interactive tool, it returns a `sollya_obj_t`. This is true, even when it would sound natural to return, *e.g.*, an `int`. For instance `sollya_lib_get_verbosity()` returns a `sollya_obj_t`, whose integer value must then be recovered with `sollya_lib_get_constant_as_int`. This forces the user to declare (and clear afterwards) a temporary `sollya_obj_t` to store the value, but this is the price of homogeneity in the library.
- When a function returns an integer, this integer generally is a boolean in the usual C meaning, *i.e.*, 0 represents false and any non-zero value represents true. In many cases, the integer returned by the function indicates a status of success or failure: the convention is "false means failure" and "true means success". In case of failure, the convention is that the function did not touch any of its arguments.
- When a function would need to return several things, or when a function would need to return something together with a status of failure or success, the convention is that pointers are given as the first arguments of the function. These pointers shall point to valid addresses where the function will store the results. This can sometimes give obscure signatures, when the function would in principle returns a pointer and actually takes as argument a pointer to a pointer (this typically happens when the function allocates a segment of memory and should return a pointer to that segment of memory).

10.4 Displaying `Sollya` objects and numerical values

Within the interactive tool, the most simple way of displaying the content of a variable or the value of an expression is to write the name of the variable or the expression, followed by the character ";". As a result, `Sollya` evaluates the expression or the variable and displays the result. Alternatively, a set of objects can be displayed the same way, separating the objects with commas. In library mode, the same behavior can be reproduced using the function `void sollya_lib_autoprint(sollya_obj_t, ...)`. Please notice that this function has a variable number of arguments: they are all displayed, until an argument equal to `NULL` is found. The `NULL` argument is mandatory, even if only one object shall be displayed (the function has no other way to know if other arguments follow or not). So, if only one argument should be displayed, the correct function call is `sollya_lib_autoprint(arg, NULL)`. Accordingly, if two arguments should be displayed, the function call is `sollya_lib_autoprint(arg1, arg2, NULL)`, etc. The function `void sollya_lib_v_autoprint(sollya_obj_t, va_list)` is the same, but it takes a `va_list` argument instead of a variable number of arguments.

Further, there is another way of printing formatted strings containing `Sollya` objects, using a `printf`-like syntax. Eight functions are provided, namely:

- `sollya_lib_printf`, `sollya_lib_v_printf`,
- `sollya_lib_fprintf`, `sollya_lib_v_fprintf`,
- `sollya_lib_sprintf`, `sollya_lib_v_sprintf`,
- `sollya_lib_snprintf` and `sollya_lib_v_snprintf`.

Each one of these functions overloads the usual function (respectively, `printf`, `vprintf`, `fprintf`, `vfprintf`, `sprintf`, `vsprintf`, `snprintf` and `vsnprintf`). The full syntax of conversions specifiers supported with the usual functions is handled (please note that the style using '`$`' – as in `%3$` or `%*3$` – is not handled though. It is not included in the C99 standard anyway). Additionally, the following conversion specifiers are provided:

- `%b`: corresponds to a `sollya_obj_t` argument. There is no precision modifier support.
- `%v`: corresponds to a `mpfr_t` argument. An optional precision modifier can be applied (*e.g.*, `%.5v`).
- `%w`: corresponds to a `mpfi_t` argument. An optional precision modifier can be applied (*e.g.*, `%.5w`).

- `%r`: corresponds to a `mpq_t` argument. There is no precision modifier support.
- `%k`: corresponds to a `mpz_t` argument. There is no precision modifier support.

When one of the above conversion specifiers is used, the corresponding argument is displayed as it would be within the interactive tool: *i.e.*, the way the argument is displayed depends on `Sollya` environment variables, such as `prec`, `display`, `midpointmode`, etc. When a precision modifier n is used, the argument is first rounded to a binary precision of roughly $\log_2(10) \times n$ bits (*i.e.*, roughly equivalent to n decimal digits) before being displayed. As with traditional `printf`, the precision modifier can be replaced with `*` which causes the precision to be determined by an additional `int` argument.

Flag characters (*e.g.*, '`#`', '`0`', etc.) are allowed but have no effect, except flag character '`-`' that is supported with its usual meaning of left-aligning the converted value. The full syntax for minimum field width is supported: it can be given directly as an integer in the format string (*e.g.*, `%22b`) or it can be replaced with `*`, which causes the field width to be determined by an additional `int` argument. As usual, a negative field width is taken as a '`-`' flag followed by a positive width.

As a special (and sometimes convenient) case, `%b` accepts that its corresponding `sollya_obj_t` argument be `NULL`: in this particular case, the string "NULL" is used in the displayed string. Please notice that, except for the particular case of `NULL`, the behavior of `sollya_lib_printf` is completely undefined if the argument of `%b` is not a valid `Sollya` object.

The `sollya_lib_printf` functions return an integer with the same meaning as the traditional `printf` functions. It indicates the number of characters that have been output (excluding the final `\0` character). Similarly, the conversion specifier `%n` can be used, even together with the `Sollya` conversion specifiers `%b`, `%v`, `%w`, `%r` and `%k`. The functions `sollya_lib_snprintf` and `sollya_lib_v_snprintf` will never write more characters than indicated by their size argument (including the final `\0` character). If the output gets truncated due to this limit, they will return the number of characters (excluding the final `\0` character) that would have been output if there had not been any truncation. In case of error, all `sollya_lib_printf` functions return a negative value.

10.5 Creating `Sollya` objects

`Sollya` objects conceptually fall into one of five categories: numerical constants (*e.g.*, 1 or 1.5), functional expressions (they might contain numerical constants, *e.g.*, $\sin(\cos(x + 1.5))$), other simple objects (intervals, strings, built-in constants such as `dyadic`, etc.), lists of objects (*e.g.*, `[|1, "Hello"|]`) and structures (*e.g.*, `{.a = 1; .b = "Hello"}`).

10.5.1 Numerical constants

Table 1 lists the different functions available to construct numerical constants. A `Sollya` constant is always created without rounding (whatever the value of global variable `prec` is at the moment of the function call): a sufficient precision is always allocated so that the constant is stored exactly. All these functions return a constant floating-point number except `sollya_lib_constant_from_mpq` that may return a constant *expression* if the value of the rational number given as argument is not exactly representable as a floating-point number at some precision. The returned expression is of the form p/q in this case.

The objects returned by these functions are newly allocated and copies of the argument. For instance, after the instruction `a = sollya_lib_constant(b)`, the user will eventually have to clear `a` (with `sollya_lib_clear(a)`) and `b` (with `mpfr_clear(b)`).

The function `sollya_lib_constant_from_double` (or more conveniently its shortcut `SOLLYA_CONST`) is probably the preferred way for constructing numerical constants. As the name indicates it, its argument is a `double`; however, due to implicit casting in C, it is perfectly possible to give an `int` as argument: it will be converted into a `double` (without rounding if the integer fits on 53 bits) before being passed to `SOLLYA_CONST`. On the contrary, the user should be aware of the fact that if decimal non-integer constants are given, C rules of rounding (to double) are applied, regardless of the setting of the `Sollya` precision variable `prec`.

Table 1: Creating numerical constants (Creates a fresh `sollya_obj_t`. Conversion is always exact)

Type of the argument	Name of the function	Shortcut macro
`double`	`sollya_lib_constant_from_double(x)`	`SOLLYA_CONST(x)`
`uint64_t`	`sollya_lib_constant_from_uint64(x)`	`SOLLYA_CONST_UI64(x)`
`int64_t`	`sollya_lib_constant_from_int64(x)`	`SOLLYA_CONST_SI64(x)`
`int`	`sollya_lib_constant_from_int(x)`	N/A
`mpq_t`	`sollya_lib_constant_from_mpq(x)`	N/A
`mpz_t`	`sollya_lib_constant_from_mpz(x)`	N/A
`mpfr_t`	`sollya_lib_constant(x)`	N/A

10.5.2 Functional expressions

Functional expressions are built by composition of basic functions with constants and the free mathematical variable. Since it is convenient to build such expressions by chaining function calls, the library provides functions that "eat up" their arguments (actually embedding them in a bigger expression). The convention is that functions that eat up their arguments are prefixed by `sollya_lib_build_`. For the purpose of building expressions, shortcut macros for the corresponding functions exist. They are all listed in Table 2.

It is worth mentioning that, although `SOLLYA_X_` and `SOLLYA_PI` are used without parentheses (as if they denoted constants), they are in fact function calls that create a new object each time they are used. The absence of parentheses is just more convenient for constructing expressions, such as, *e.g.*, `SOLLYA_COS(SOLLYA_X_)`.

For each function of the form `sollya_lib_build_function_foo`, there exists a function called `sollya_lib_foo`. There are two differences between them:

- First, `sollya_lib_foo` does not "eat up" its argument. This can sometimes be useful, *e.g.*, if one has an expression stored in a variable `a` and one wants to build the expression `exp(a)` without loosing the reference to the expression represented by `a`.
- Second, while `sollya_lib_build_function_foo` mechanically constructs an expression, function `sollya_lib_foo` also evaluates it, as far as this is possible without rounding.
  For instance, after the instructions `a = SOLLYA_CONST(0); b = sollya_lib_exp(a);` the variable `b` contains the number 1, whereas it would have contained the expression `"exp(0)"` if it had been created by `b = sollya_lib_build_function_exp(a)`.

Actually, `sollya_lib_foo` has exactly the same behavior as writing an expression at the prompt within the interactive tool. In particular, it is possible to give a range as an argument to `sollya_lib_foo`: the returned object will be the result of the evaluation of function `foo` on that range by interval arithmetic. In contrast, trying to use `sollya_lib_build_function_foo` on a range would result in a typing error.

Alternatively, one may create functional expressions with the functions
`int sollya_lib_construct_function(sollya_obj_t *res, sollya_base_function_t type, ...)`
`int sollya_lib_v_construct_function(sollya_obj_t *, sollya_base_function_t, va_list)`.
The advantage of these functions with respect to the others presented above lies in the fact that they offer a way to create any functional expression, the basic function that one wants to construct being provided with the argument `type`. Since these functions are indeed doing the exact contrary of `sollya_lib_decompose_function`, they are described in details in the corresponding Section 10.12.

10.5.3 Other simple objects

Other simple objects are created with functions listed in Table 3. The functions with a name of the form `sollya_lib_something` follow the same convention as `sollya_lib_constant`: they build a new object from a copy of their argument, and the conversion is always exact, whatever the value of `prec` is.

Please note that in the interactive tool, `D` either denotes the discrete mathematical function that maps a real number to its closest `double` number, or is used as a symbolic constant to indicate that the

Table 2: Building functional expressions (Eats up arguments, embedding them in the returned object.)

Name in the interactive tool	Function to build it	Shortcut macro
_x_	sollya_lib_build_function_free_variable()	SOLLYA_X_
pi	sollya_lib_build_function_pi()	SOLLYA_PI
e1 + e2	sollya_lib_build_function_add(e1, e2)	SOLLYA_ADD(e1, e2)
e1 - e2	sollya_lib_build_function_sub(e1, e2)	SOLLYA_SUB(e1, e2)
e1 * e2	sollya_lib_build_function_mul(e1, e2)	SOLLYA_MUL(e1, e2)
e1 / e2	sollya_lib_build_function_div(e1, e2)	SOLLYA_DIV(e1, e2)
pow(e1, e2)	sollya_lib_build_function_pow(e1, e2)	SOLLYA_POW(e1, e2)
-e	sollya_lib_build_function_neg(e)	SOLLYA_NEG(e)
sqrt(e)	sollya_lib_build_function_sqrt(e)	SOLLYA_SQRT(e)
abs(e)	sollya_lib_build_function_abs(e)	SOLLYA_ABS(e)
erf(e)	sollya_lib_build_function_erf(e)	SOLLYA_ERF(e)
erfc(e)	sollya_lib_build_function_erfc(e)	SOLLYA_ERFC(e)
exp(e)	sollya_lib_build_function_exp(e)	SOLLYA_EXP(e)
expm1(e)	sollya_lib_build_function_expm1(e)	SOLLYA_EXPM1(e)
log(e)	sollya_lib_build_function_log(e)	SOLLYA_LOG(e)
log2(e)	sollya_lib_build_function_log2(e)	SOLLYA_LOG2(e)
log10(e)	sollya_lib_build_function_log10(e)	SOLLYA_LOG10(e)
log1p(e)	sollya_lib_build_function_log1p(e)	SOLLYA_LOG1P(e)
sin(e)	sollya_lib_build_function_sin(e)	SOLLYA_SIN(e)
cos(e)	sollya_lib_build_function_cos(e)	SOLLYA_COS(e)
tan(e)	sollya_lib_build_function_tan(e)	SOLLYA_TAN(e)
asin(e)	sollya_lib_build_function_asin(e)	SOLLYA_ASIN(e)
acos(e)	sollya_lib_build_function_acos(e)	SOLLYA_ACOS(e)
atan(e)	sollya_lib_build_function_atan(e)	SOLLYA_ATAN(e)
sinh(e)	sollya_lib_build_function_sinh(e)	SOLLYA_SINH(e)
cosh(e)	sollya_lib_build_function_cosh(e)	SOLLYA_COSH(e)
tanh(e)	sollya_lib_build_function_tanh(e)	SOLLYA_TANH(e)
asinh(e)	sollya_lib_build_function_asinh(e)	SOLLYA_ASINH(e)
acosh(e)	sollya_lib_build_function_acosh(e)	SOLLYA_ACOSH(e)
atanh(e)	sollya_lib_build_function_atanh(e)	SOLLYA_ATANH(e)
D(e), double(e)	sollya_lib_build_function_double(e)	SOLLYA_D(e)
SG(e), single(e)	sollya_lib_build_function_single(e)	SOLLYA_SG(e)
QD(e), quad(e)	sollya_lib_build_function_quad(e)	SOLLYA_QD(e)
HP(e), halfprecision(e)	sollya_lib_build_function_halfprecision(e)	SOLLYA_HP(e)
DD(e), doubledouble(e)	sollya_lib_build_function_double_double(e)	SOLLYA_DD(e)
TD(e), tripledouble(e)	sollya_lib_build_function_triple_double(e)	SOLLYA_TD(e)
DE(e), doubleextended(e)	sollya_lib_build_function_doubleextended(e)	SOLLYA_DE(e)
ceil(e)	sollya_lib_build_function_ceil(e)	SOLLYA_CEIL(e)
floor(e)	sollya_lib_build_function_floor(e)	SOLLYA_FLOOR(e)
nearestint(e)	sollya_lib_build_function_nearestint(e)	SOLLYA_NEARESTINT(e)

`double` format must be used (as an argument of `round` for instance). In the library, they are completely distinct objects, the mathematical function being obtained with `sollya_lib_build_function_double` and the symbolic constant with `sollya_lib_double_obj`. The same holds for other formats (`DD`, `SG`, etc.)

Table 3: Creating `Sollya` objects from scratch (Returns a new `sollya_obj_t`)

Name in the interactive tool	Function to create it
on	`sollya_lib_on();`
off	`sollya_lib_off();`
dyadic	`sollya_lib_dyadic();`
powers	`sollya_lib_powers();`
binary	`sollya_lib_binary();`
hexadecimal	`sollya_lib_hexadecimal();`
file	`sollya_lib_file();`
postscript	`sollya_lib_postscript();`
postscriptfile	`sollya_lib_postscriptfile();`
perturb	`sollya_lib_perturb();`
RD	`sollya_lib_round_down();`
RU	`sollya_lib_round_up();`
RZ	`sollya_lib_round_towards_zero();`
RN	`sollya_lib_round_to_nearest();`
honorcoeffprec	`sollya_lib_honorcoeffprec();`
true	`sollya_lib_true();`
false	`sollya_lib_false();`
void	`sollya_lib_void();`
default	`sollya_lib_default();`
decimal	`sollya_lib_decimal();`
absolute	`sollya_lib_absolute();`
relative	`sollya_lib_relative();`
fixed	`sollya_lib_fixed();`
floating	`sollya_lib_floating();`
error	`sollya_lib_error();`
D, double	`sollya_lib_double_obj();`
SG, single	`sollya_lib_single_obj();`
QD, quad	`sollya_lib_quad_obj();`
HP, halfprecision	`sollya_lib_halfprecision_obj();`
DE, doubleextended	`sollya_lib_doubleextended_obj();`
DD, doubledouble	`sollya_lib_double_double_obj();`
TD, tripledouble	`sollya_lib_triple_double_obj();`
"Hello"	`sollya_lib_string("Hello")`
[1, 3.5]	`sollya_lib_range_from_interval(a);`[a]
[1, 3.5]	`sollya_lib_range_from_bounds(b, c);`[b]
[1, 3.5]	`sollya_lib_range(d, e);`[c]

[a] `a` is a `mpfi_t` containing the interval $[1, 3.5]$. Conversion is always exact.
[b] `b` and `c` are `mpfr_t` respectively containing the numbers 1 and 3.5. Conversion is always exact.
[c] `d` and `e` are `sollya_obj_t` respectively containing the numbers 1 and 3.5. Conversion is always exact.

10.5.4 Lists

There are actually two kinds of lists: regular lists (such as, *e.g.*, `[|1, 2, 3|]`) and semi-infinite lists (such as, *e.g.*, `[|1, 2...|]`). Withing the interactive tool, the ellipsis "`...`" can sometimes be used as a shortcut to define regular lists, *e.g.*, `[|1, 2, ..., 10|]`.

In the library, there is no symbol for the ellipsis, and there are two distinct types: one for regular lists

and one for semi-infinite lists (called end-elliptic). Defining a regular list with an ellipsis is not possible in the library (except of course with `sollya_lib_parse_string`).

Constructing regular lists is achieved through three functions:

- `sollya_obj_t sollya_lib_list(sollya_obj_t[] L, int n)`: this function returns a new object that is a list the elements of which are copies of `L[0]`, ..., `L[n-1]`.
- `sollya_obj_t sollya_lib_build_list(sollya_obj_t obj1, ...)`: this function accepts a variable number of arguments. The last one **must** be `NULL`. It "eats up" its arguments and returns a list containing the objects given as arguments. Since arguments are eaten up, they may be directly produced by function calls, without being stored in variables. A typical use could be

  ```
  sollya_lib_build_list(SOLLYA_CONST(1), SOLLYA_CONST(2), SOLLYA_CONST(3), NULL);
  ```

- `sollya_obj_t sollya_lib_v_build_list(va_list)`: the same as the previous functions, but with a `va_list`.

Following the same conventions, end-elliptic lists can be constructed with the following functions:

- `sollya_obj_t sollya_lib_end_elliptic_list(sollya_obj_t[] L, int n)`.
- `sollya_obj_t sollya_lib_build_end_elliptic_list(sollya_obj_t obj1, ...)`.
- `sollya_obj_t sollya_lib_v_build_end_elliptic_list(va_list)`.

10.5.5 Structures

`Sollya` structures are also available in library mode as any other `Sollya` object. The support for `Sollya` structures is however minimal and creating them might seem cumbersome[2]. The only function available to create structures is

```
int sollya_lib_create_structure(sollya_obj_t *res, sollya_obj_t s, char *name,
                                sollya_obj_t val).
```

This function returns a boolean integer: false means failure, and true means success. Three cases of success are possible. In all cases, the function creates a new object and stores it at the address referred to by `res`.

- If `s` is `NULL`: `*res` is filled with a structure with only one field. This field is named after the string `name` and contains a copy of the object `val`.
- If `s` is an already existing structure that has a field named after the string `name`: `*res` is filled with a newly created structure. This structure is the same as `s` except that the field corresponding to `name` contains a copy of `val`.
- If `s` is an already existing structure that does **not** have a field named after the string `name`: `*res` is filled with a newly created structure. This structure is the same as `s` except that it has been augmented with a field corresponding to `name` and that contains a copy of `val`.

Please notice that `s` is not changed by this function: the structure stored in `*res` is a new one that does not refer to any of the components of `s`. As a consequence, one should not forget to explicitly clear `s` as well as `*res` when they become useless.

[2] Users are encouraged to make well-founded feature requests if they feel the need for better support of structures.

10.5.6 Library functions, library constants and procedure functions

In addition to the mathematical base functions and constants provided by `Sollya` and listed in the Section above, the user may bind other mathematical functions and constants to `Sollya` objects under the condition that they can provide code to evaluate these functions or constants. The mechanism behind is similar to the one available in interactive `Sollya` through the `library`, `libraryconstant` and `function` commands (see Sections 8.101, 8.102 and 8.75).

With the `Sollya` library, this binding is done through one of the following functions:

- Binding of a (non-constant) mathematical function for which evaluation code is available through a `C` pointer to a function:

```
sollya_obj_t sollya_lib_libraryfunction(sollya_obj_t e,
                                        char *name,
                                        int (*f)(mpfi_t, mpfi_t, int));
sollya_obj_t sollya_lib_build_function_libraryfunction(sollya_obj_t e,
                                        char *name,
                                        int (*f)(mpfi_t, mpfi_t, int));
sollya_obj_t sollya_lib_libraryfunction_with_data(
                                        sollya_obj_t e,
                                        char *name,
                                        int (*f)(mpfi_t, mpfi_t, int, void *),
                                        void *data,
                                        void (*dealloc_func)(void *));
sollya_obj_t sollya_lib_build_function_libraryfunction_with_data(
                                        sollya_obj_t e,
                                        char *name,
                                        int (*f)(mpfi_t, mpfi_t, int, void *),
                                        void *data,
                                        void (*dealloc_func)(void *));
```

  These four functions construct a `Sollya` object representing $f(e)$ where e is given as the `Sollya` object `e` and f is given as the pointer to a function `f(mpfi_t y, mpfi_t x, int n)` (resp. `f(mpfi_t y, mpfi_t x, int n, void *data)`). This code must evaluate the n-th derivative of f over the interval x, yielding y.

  As usual, the functions whose name contains `_build_function_` "eat up" the object `e`, while the corresponding functions (without `_build_function_` in their name) do not.

  The `name` argument of the function is taken as a suggestion to the name the `Sollya` object representing the function should be printed as when displayed. The user may choose to provide `NULL` instead. In any case, upon the binding, the `Sollya` library will determine a unique displaying name for the function. If it is not yet taken as a name (for some other `Sollya` object or `Sollya` keyword), the suggested name will be used. If no suggested name is provided, the name of the dynamic object behind the pointer to the function will be used if it can be determined. Otherwise, a more-or-less random name is used. If the (suggested) base name is already taken, the name is unified appending an underscore and a unique number to it. The `name` argument is never "eaten up", *i.e.*, it is up to the user to free any memory allocated to that pointer.

  The functions whose name contains `_with_data` allow for the same binding of an external function to a `Sollya` object as the corresponding functions (without `_with_data` in their name), but additionally permit an opaque data pointer `data` to be registered together with the function pointer `f`. The data pointer `data` will be represented to the function `f` on each call, in an additional (last) argument of type `void *` that the function `f` is supposed to take.

  Such opaque data pointers may be used, *e.g.*, to distinguish between several different external procedure objects when only unique function pointer is available and the actual procedural code is contained in a closure represented thru the data pointer.

  As the data field the `data` pointer points to may require deallocation once the `Sollya` object built thru invocation of the functions described inhere and all of its copies eventually get deallocated, a data-field-deallocation function `dealloc_func` may be registered together with the data

field. That function will be called with the `data` pointer in argument when the `Sollya` object is deallocated. When the user does not need such a deallocation function, they may provide `NULL` as the `dealloc_func` argument to the `sollya_lib_libraryfunction_with_data` function or `sollya_lib_build_function_libraryfunction_with_data` function, in which case the argument is ignored and no deallocation function gets called for the `Sollya` object built.

- Binding of a mathematical constant for which evaluation code is available through a `C` pointer to a function:
```
sollya_obj_t sollya_lib_libraryconstant(char *name,
                                        void (*c)(mpfr_t, mp_prec_t));
sollya_obj_t sollya_lib_build_function_libraryconstant(char *name,
                                        void (*c)(mpfr_t, mp_prec_t));
sollya_obj_t sollya_lib_libraryconstant_with_data(
                                        char *name,
                                        void (*c)(mpfr_t, mp_prec_t, void *),
                                        void *data,
                                        void (*dealloc_func)(void *));
sollya_obj_t sollya_lib_build_function_libraryconstant_with_data(
                                        char *name,
                                        void (*c)(mpfr_t, mp_prec_t, void *),
                                        void *data,
                                        void (*dealloc_func)(void *));
```
  These four functions construct a `Sollya` object representing the mathematical constant c for which a pointer to a function `c(mpfr_t rop, mp_prec_t prec)` (resp. `c(mpfr_t rop, mp_prec_t prec, void *data)`) is provided. This code must evaluate the constant to precision `prec` and affect the result to `rop`. See Section 8.102 for details with respect to `prec`.

  The same remark as above concerning the suggested displaying name of the `Sollya` object applies for the `name` argument.

  In the same manner, the same remarks as above concerning the functions taking a data field pointer `data` apply.

- Binding of a mathematical function for which evaluation code is available through a `Sollya` object representing a `Sollya` procedure:
```
sollya_obj_t sollya_lib_procedurefunction(sollya_obj_t e, sollya_obj_t f);
sollya_obj_t sollya_lib_build_function_procedurefunction(sollya_obj_t e,
                                                         sollya_obj_t f);
```
  These two functions construct a `Sollya` library object representing $f(e)$ where e corresponds to the mathematical function (or constant) given with argument `e` and where f is given as a `Sollya` procedure `f(x, n, p)` evaluating the n-th derivative of f over the interval x with precision p. See Section 8.75 concerning details of the arguments of that `Sollya` procedure.

  As usual, `sollya_lib_build_function_procedurefunction` "eats up" its arguments `e` and `f` while `sollya_obj_t sollya_lib_procedurefunction` does not.

  Currently, the only way of constructing a `Sollya` library object representing a `Sollya` procedure is to use `sollya_lib_parse_string`.

10.5.7 External procedures and external data symbols

Similarly to library functions or library constants, the binding of which is discussed in Section 10.5.6, `Sollya` allows external procedural code to be bound and then used inside `Sollya` in a procedure-like manner. This is provided in the interactive tool with the `externalproc` command, described in Section 8.66. The same mechanism is available in the `Sollya` library thanks to the following functions:

- To bind a function pointer `p` as a procedure named `name`, having arity `arity`, returning a result of type `res_type` and accepting arguments of types `arg_types[0]` thru `arg_types[arity - 1]`,

the function

```
sollya_obj_t
    sollya_lib_externalprocedure(sollya_externalprocedure_type_t res_type,
                                 sollya_externalprocedure_type_t *arg_types,
                                 int arity,
                                 char *name,
                                 void *p);
```

may be used.

The `name` argument of the function is only taken as a suggestion to the name the `Sollya` object representing the function should be printed as when displayed. The user may choose to provide `NULL` instead. In any case, upon the binding, the `Sollya` library will determine a unique displaying name for the procedure. If it is not yet taken as a name (for some other `Sollya` object or `Sollya` keyword), the suggested name will be used. If no suggested name is provided, the name of the dynamic object behind the pointer to the function will be used if it can be determined. Otherwise, a more-or-less random name is used. If the (suggested) base name is already taken, the name is unified appending an underscore and a unique number to it. The `name` argument is never "eaten up", *i.e.*, it is up to the user to free any memory allocated for that pointer.

The result type `res_type` as well as the argument types `arg_types` take one of the values defined by the enumeration type `sollya_externalprocedure_type_t`, detailed in Table 4. The array (resp. pointer) to the argument types `arg_types` provided to the `sollya_lib_externalprocedure` function is not "eaten up" by the function, *i.e.*, it is up to the user to free any memory allocated for that pointer (where applicable). When the external procedure does not take any argument, its arity is to be set to zero. In this case, the argument type pointer `arg_types` is ignored by the `sollya_lib_externalprocedure` function; it hence may be invalid or `NULL` in this case.

The actual `C` function to be bound is supposed to have a function type corresponding to the result and argument types indicated. It is supposed to be provided to the `sollya_lib_externalprocedure` function as a `void *` function pointer, though, for the sake of unification of the `Sollya` library interface. A detailed description of the actual type of the `C` function is given in Section 8.66.

- In addition to the basic binding function described above, the

```
sollya_obj_t
    sollya_lib_externalprocedure_with_data(
                              sollya_externalprocedure_type_t res_type,
                              sollya_externalprocedure_type_t *arg_types,
                              int arity,
                              char *name,
                              void *p,
                              void *data,
                              void (*dealloc_func)(void *));
```

function allows for the same binding of an external procedure to a `Sollya` object but additionally permits an opaque data pointer `data` to be registered together with the function pointer `p`. The data pointer `data` will be represented to the function `p` on each call, in an additional (last) argument of type `void *` that the function `p` is supposed to take.

Such opaque data pointers may be used, *e.g.*, to distinguish between several different external procedure objects when only unique function pointer is available and the actual procedural code is contained in a closure represented thru the data pointer.

As the data field the `data` pointer points to may require deallocation once the `Sollya` object built thru invocation of the `sollya_lib_externalprocedure_with_data` and all of its copies eventually get deallocated, a data-field-deallocation function `dealloc_func` may be registered together with the data field. That function will be called with the `data` pointer in argument when the `Sollya` object is deallocated. When the user does not need such a deallocation function, they may provide `NULL` as the `dealloc_func` argument to the `sollya_lib_externalprocedure_with_data` function, in which case the argument is ignored and no deallocation function gets called for the `Sollya` object built.

Table 4: Possible return and argument types for external procedures

SOLLYA_EXTERNALPROC_TYPE_VOID
SOLLYA_EXTERNALPROC_TYPE_CONSTANT
SOLLYA_EXTERNALPROC_TYPE_FUNCTION
SOLLYA_EXTERNALPROC_TYPE_RANGE
SOLLYA_EXTERNALPROC_TYPE_INTEGER
SOLLYA_EXTERNALPROC_TYPE_STRING
SOLLYA_EXTERNALPROC_TYPE_BOOLEAN
SOLLYA_EXTERNALPROC_TYPE_OBJECT
SOLLYA_EXTERNALPROC_TYPE_CONSTANT_LIST
SOLLYA_EXTERNALPROC_TYPE_FUNCTION_LIST
SOLLYA_EXTERNALPROC_TYPE_RANGE_LIST
SOLLYA_EXTERNALPROC_TYPE_INTEGER_LIST
SOLLYA_EXTERNALPROC_TYPE_STRING_LIST
SOLLYA_EXTERNALPROC_TYPE_BOOLEAN_LIST
SOLLYA_EXTERNALPROC_TYPE_OBJECT_LIST

In addition to binding external procedures to `Sollya` and letting `Sollya` know the signature of these external procedures, `Sollya` supports the binding of opaque external symbols. The external symbol gets wrapped into a `Sollya` object which `Sollya` can assign to variables, print, compare for equality, transmit as parameters to procedures and external procedures and, in some limited manner, match. This is provided in the interactive tool with the `externaldata` command, described in Section 8.64. The same mechanism is available in the `Sollya` library thanks to the following function: to bind an external object referenced by the pointer `data` as a `Sollya` object with `name` as a name, the function

```
sollya_obj_t sollya_lib_externaldata(char *name,
                                     void *data,
                                     void (*dealloc)(void *));
```

may be used.

When called, the function `sollya_lib_externaldata` will return a `Sollya` object that wraps the external object referenced by `data`. When the `Sollya` object is printed, `name` will be used for the displaying inside `Sollya`. As with library functions, library constants and external procedures, the `name` argument of the function is only taken as a suggestion and may be `NULL`, in which case `Sollya` determines a name for the object. If `dealloc` is not `NULL`, it must point to a function that `Sollya` calls when the object encapsulating `data` is deallocated, typically because `sollya_lib_clear_obj` is called. Writers of external bindings for the `Sollya` library may use this function pointer argument to have their `data` object's reference counter decremented when appropriate. They may also use the value of this function pointer to ensure that they can distinguish between their `Sollya` external data objects and `Sollya` external data objects bound to `Sollya` by other means, such as other libraries using the `Sollya` library or the interactive command `externaldata`.

The external-data-functionnality seems to be disappointing at first sight, as `Sollya` stays agnostic to the type and meaning of the external symbol. Together with the functionnality that allows external procedures to be bound to `Sollya`, it however becomes possible to have external objects passed to `Sollya` code and passed back into external code for manipulation. Bindings of the `Sollya` library may use this external-data-functionnality to wrap their own objects as `Sollya` objects in cases when translation of these objects to native `Sollya` objects is not possible as `Sollya` does not allow objects of these types to be represented natively.

10.6 Getting the type of an object

Functions are provided that allow the user to test the type of a `Sollya` object. They are listed in Table 5. They all return an `int` interpreted as the boolean result of the test. Please note that from a typing point of view, a mathematical constant and a non-constant functional expression are both functions.

Table 5: Testing the type of a `Sollya` object (Returns non-zero if true, 0 otherwise)

`sollya_lib_obj_is_function(obj)`
`sollya_lib_obj_is_range(obj)`
`sollya_lib_obj_is_string(obj)`
`sollya_lib_obj_is_list(obj)`
`sollya_lib_obj_is_end_elliptic_list(obj)`
`sollya_lib_obj_is_structure(obj)`
`sollya_lib_obj_is_procedure(obj)`
`sollya_lib_obj_is_externalprocedure(obj)`
`sollya_lib_obj_is_externaldata(obj)`
`sollya_lib_obj_is_error(obj)`
`sollya_lib_is_on(obj)`
`sollya_lib_is_off(obj)`
`sollya_lib_is_dyadic(obj)`
`sollya_lib_is_powers(obj)`
`sollya_lib_is_binary(obj)`
`sollya_lib_is_hexadecimal(obj)`
`sollya_lib_is_file(obj)`
`sollya_lib_is_postscript(obj)`
`sollya_lib_is_postscriptfile(obj)`
`sollya_lib_is_perturb(obj)`
`sollya_lib_is_round_down(obj)`
`sollya_lib_is_round_up(obj)`
`sollya_lib_is_round_towards_zero(obj)`
`sollya_lib_is_round_to_nearest(obj)`
`sollya_lib_is_honorcoeffprec(obj)`
`sollya_lib_is_true(obj)`
`sollya_lib_is_false(obj)`
`sollya_lib_is_void(obj)`
`sollya_lib_is_default(obj)`
`sollya_lib_is_decimal(obj)`
`sollya_lib_is_absolute(obj)`
`sollya_lib_is_relative(obj)`
`sollya_lib_is_fixed(obj)`
`sollya_lib_is_floating(obj)`
`sollya_lib_is_double_obj(obj)`
`sollya_lib_is_single_obj(obj)`
`sollya_lib_is_quad_obj(obj)`
`sollya_lib_is_halfprecision_obj(obj)`
`sollya_lib_is_doubleextended_obj(obj)`
`sollya_lib_is_double_double_obj(obj)`
`sollya_lib_is_triple_double_obj(obj)`
`sollya_lib_is_pi(obj)`

10.7 Recovering the value of a range

If a `sollya_obj_t` is a range, it is possible to recover the values corresponding to the bounds of the range. The range can be recovered either as a `mpfi_t` or as two `mpfr_t` (one per bound). This is achieved with the following conversion functions:

- `int sollya_lib_get_interval_from_range(mpfi_t res, sollya_obj_t arg)`,
- `int sollya_lib_get_bounds_from_range(mpfr_t res_left, mpfr_t res_right, sollya_obj_t arg)`.

They return a boolean integer: false means failure (*i.e.*, if the `sollya_obj_t` is not a range) and true means success. These functions follow the same conventions as those of the `MPFR` and `MPFI` libraries: the variables `res`, `res_left` and `res_right` must be initialized beforehand, and are used to store the result of the conversion. Also, the functions `sollya_lib_get_something_from_range` **do not change the internal precision** of `res`, `res_left` and `res_right`. If the internal precision is sufficient to perform the conversion without rounding, then it is guaranteed to be exact. If, on the contrary, the internal precision is not sufficient, the actual bounds of the range stored in `arg` will be rounded at the target precision using a rounding mode that ensures that the inclusion property remains valid, *i.e.*, `arg` $\subseteq$ `res` (resp. `arg` $\subseteq$ [`res_left`, `res_right`]).

Function `int sollya_lib_get_prec_of_range(mp_prec_t *prec, sollya_obj_t arg)` stores at `*prec` a precision that is guaranteed to be sufficient to represent the range stored in `arg` without rounding. The returned value of this function is a boolean that follows the same convention as above. In conclusion, this is an example of a completely safe conversion:

```
...
mp_prec_t prec;
mpfr_t a, b;

if (!sollya_lib_get_prec_of_range(&prec, arg)) {
  sollya_lib_printf("Unexpected error: %b is not a range\n", arg);
}
else {
  mpfr_init2(a, prec);
  mpfr_init2(b, prec);
  sollya_lib_get_bounds_from_range(a, b, arg);

  /* Now [a, b] = arg exactly */
}
...
```

10.8 Recovering the value of a numerical constant or a constant expression

From a conceptual point of view, a numerical constant is nothing but a very simple constant functional expression. Hence there is no difference in `Sollya` between the way constants and constant expressions are handled. The functions presented in this section allow one to recover the value of such constants or constant expressions into usual C data types.

A constant expression being given, three cases are possible:

- When naively evaluated at the current global precision, the expression always leads to provably exact computations (*i.e.*, at each step of the evaluation, no rounding happens). For instance numerical constants or simple expressions such as $(\exp(0) + 5)/16$ fall in this category.
- The constant expressions would be exactly representable at some precision but this is not straightforward from a naive evaluation at the current global precision. An example would be $\sin(\pi/3)/\sqrt{3}$ or even $1 + 2^{-\mathrm{prec}-10}$.
- Finally, a third possibility is that the value of the expression is not exactly representable at any precision on a binary floating-point number. Possible examples are π or $1/10$.

From now on, we suppose that `arg` is a `sollya_obj_t` that contains a constant expression (or, as a particular case, a numerical constant). The general scheme followed by the conversion functions is the following: `Sollya` chooses an initial working precision greater than the target precision. If the value of `arg` is easily proved to be exactly representable at that precision, `Sollya` first computes this exact value and then rounds it to the nearest number of the target format (ties-to-even). Otherwise, `Sollya` tries to adapt the working precision automatically in order to ensure that the result of the conversion is one of both numbers in the target format that are closest to the exact value (a faithful rounding). A warning message indicates that the conversion is not exact and that a faithful rounding has been performed. In some cases really hard to evaluate, the algorithm can even fail to find a faithful rounding. In that case, too, a warning message is emitted indicating that the result of the conversion should not be trusted. Let us remark that these messages can be caught instead of being displayed and adapted handling can be provided by the user of the library at each emission of a warning (see Section 10.18).

The conversion functions are the following. They return a boolean integer: false means failure (*i.e.*, `arg` is not a constant expression) and true means success.

- `int sollya_lib_get_constant_as_double(double *res, sollya_obj_t arg)`
- `int sollya_lib_get_constant_as_int(int *res, sollya_obj_t arg)`: any value too big to be represented (this includes $\pm$`Inf`) is converted to `INT_MIN` or `INT_MAX` and a warning is emitted. `NaN` is converted to 0 with a specific warning.
- `int sollya_lib_get_constant_as_int64(int64_t *res, sollya_obj_t arg)`: any value too big to be represented (this includes $\pm$`Inf`) is converted to `INT64_MIN` or `INT64_MAX` and a warning is emitted. `NaN` is converted to 0 with a specific warning.
- `int sollya_lib_get_constant_as_uint64(uint64_t *res, sollya_obj_t arg)`: negative values are converted to 0 with a warning. Any value too big to be represented (this includes `Inf`) is converted to `UINT64_MAX` and a warning is emitted. `NaN` is converted to 0 with a specific warning.
- `int sollya_lib_get_constant_as_mpz(mpz_t res, sollya_obj_t arg)`: the result of the conversion is stored in `res`. Please note that `res` must be initialized beforehand. Infinities and `NaN` are converted to 0 with specific warnings.
- `int sollya_lib_get_constant_as_uint64_array(int *sign, uint64_t **value, size_t *length, sollya_obj_t arg)`: the result of the conversion is equivalent to the one obtained with `sollya_lib_get_constant_as_mpz` but it is stored differently. The sign σ of the result (one of -1, 0 or 1) is put into the variable pointed to by `sign`. A `uint64_t`-array of a certain size s is allocated; the variable pointed to by `value` is set to that pointer. The variable pointed to by `length` is set to the size s of the array. The elements v_i =`(*value)[i]` are set to a value such that $\sigma \cdot \sum_{i=0}^{s-1} v_i \cdot 2^{64\,i}$ is equal to the value that would have been the result of the conversion with `sollya_lib_get_constant_as_mpz`. In case of failure, the variables pointed to by `sign`, `length` and `*value` are left unchanged and no allocation is performed. The user is in charge of deallocating the array `*value` allocated by this function when it succeeds, using `sollya_lib_free`. The size s of the array is at least 1 in all cases, even if the result of the conversion is zero. The element of the array v_{s-1} is guaranteed to be non-zero, unless the result of the whole conversion is zero. Infinities and `NaN` are converted to 0 with specific warnings. This function is provided as a convenience to wrapper libraries that cannot afford binding to the GMP library but need support for arbitrary length integers, though.
- `int sollya_lib_get_constant_as_mpq(mpq_t res, sollya_obj_t arg)`: the result of the conversion is stored in `res`. Please note that `res` must be initialized beforehand. If `arg` cannot be proved to be exactly a floating-point number or the ratio of two floating-point numbers at some precision, the function returns false and `res` is left unchanged.
- `int sollya_lib_get_constant(mpfr_t res, sollya_obj_t arg)`: the result of the conversion is stored in `res`. Please note that `res` must be initialized beforehand and that its internal precision is not modified by the algorithm.

Function `int sollya_lib_get_prec_of_constant(mp_prec_t *prec, sollya_obj_t arg)` tries to find a precision that would be sufficient to exactly represent the value of `arg` without rounding. If it manages to find such a precision, it stores it at `*prec` and returns true. If it does not manage to find such a precision, or if `arg` is not a constant expression, it returns false and `*prec` is left unchanged.

In conclusion, here is an example of use for converting a constant expression to a `mpfr_t`:

```
...
mp_prec_t prec;
mpfr_t a;
int test = 0;

test = sollya_lib_get_prec_of_constant(&prec, arg);
if (test) {
  mpfr_init2(a, prec);
  sollya_lib_get_constant(a, arg); /* Exact conversion */
}
else {
  mpfr_init2(a, 165); /* Initialization at some default precision */
  test = sollya_lib_get_constant(a, arg);
  if (!test) {
    sollya_lib_printf("Error: %b is not a constant expression\n", arg);
  }
}
...
```

10.9 Converting a string from `Sollya` to C

If `arg` is a `sollya_obj_t` that contains a string, that string can be recovered using

`int sollya_lib_get_string(char **res, sollya_obj_t arg).`

If `arg` really is a string, this function allocates enough memory on the heap to store the corresponding string, it copies the string at that newly allocated place, and sets `*res` so that it points to it. The function returns a boolean integer: false means failure (*i.e.*, `arg` is not a string) and true means success.

Since this function allocates memory on the heap, this memory should manually be cleared by the user with `sollya_lib_free` once it becomes useless.

10.10 Recovering the contents of a `Sollya` list

It is possible to recover the i-th element of a list `arg` (as one would do using `arg[i]` withing `Sollya`) with the following function:

`int sollya_lib_get_element_in_list(sollya_obj_t *res, sollya_obj_t arg, int i).`

It returns a boolean integer: false means failure (*i.e.*, `arg` is not a list or the index is out of range) and true means success. In case of success, a copy of the i-th element of `arg` is stored at the address referred to by `res`. Since it is a copy, it should be cleared with `sollya_lib_clear_obj` when it becomes useless. Please notice that this function works with regular lists as well as with end-elliptic lists, just as within the interactive tool.

Another function allows user to recover all elements of a list in a single call. This function returns a C array of `sollya_obj_t` objects and has the following signature:

```
int sollya_lib_get_list_elements(sollya_obj_t **L, int *n, int *end_ell,
                                 sollya_obj_t arg).
```

Three cases are possible:

- If `arg` is a regular list of length N, the function allocates memory on the heap for N `sollya_obj_t`, sets `*L` so that it points to that memory segment, and copies each of the elements N of `arg` to `(*L)[0]`, ..., `(*L)[N-1]`. Finally, it sets `*n` to N, `*end_ell` to zero and returns true. A particular case is when `arg` is the empty list: everything is the same except that no memory is allocated and `*L` is left unchanged.
- If `arg` is an end-elliptic list containing N elements plus the ellipsis. The function allocates memory on the heap for N `sollya_obj_t`, sets `*L` so that it points to that memory segment, and copies each of the elements N of `arg` at `(*L)[0]`, ..., `(*L)[N-1]`. Finally, it sets `*n` to N, `*end_ell` to a non-zero value and returns true. The only difference between a regular list and an end-elliptic list containing the same elements is hence that `*end_ell` is set to a non-zero value in the latter.
- If `arg` is neither a regular nor an end-elliptic list, `*L`, `*n` and `*end_ell` are left unchanged and the function returns false.

In case of success, please notice that `(*L)[0]`, ..., `(*L)[N-1]` should manually be cleared with `sollya_lib_clear_obj` when they become useless. Also, the pointer `*L` itself should be cleared with `sollya_lib_free` since it points to a segment of memory allocated on the heap by `Sollya`.

10.11 Recovering the contents of a `Sollya` structure

If `arg` is a `sollya_obj_t` that contains a structure, the contents of a given field can be recovered using

```
int sollya_lib_get_element_in_structure(sollya_obj_t *res, char *name,
                                        sollya_obj_t arg).
```

If `arg` really is a structure and if that structure has a field named after the string `name`, this function copies the contents of that field into the `Sollya` object `*res`. The function returns a boolean integer: false means failure (*i.e.*, if `arg` is not a structure or if it does not have a field named after `name`) and true means success.

It is also possible to get all the field names and their contents. This is achieved through the function

```
int sollya_lib_get_structure_elements(char ***names, sollya_obj_t **objs, int *n,
                                      sollya_obj_t arg).
```

If `arg` really is a structure, say with N fields called "fieldA", ..., "fieldZ", this functions sets `*n` to N, allocates and fills an array of N strings and sets `*names` so that it points to that segment of memory (hence `(*names)[0]` is the string "fieldA", ..., `(*names)[N-1]` is the string "fieldZ"). Moreover, it allocates memory for N `sollya_obj_t`, sets `*objs` so that it points on that memory segment, and copies the contents of each of the N fields at `(*objs)[0]`, ..., `(*objs)[N-1]`. Finally it returns true. If `arg` is not a structure, the function simply returns false without doing anything. Please note that since `*names` and `*objs` point to memory segments that have been dynamically allocated, they should manually be cleared by the user with `sollya_lib_free` once they become useless.

10.12 Decomposing a functional expression

If a `sollya_obj_t` contains a functional expression, one can decompose the expression tree using the following functions. These functions all return a boolean integer: true in case of success (*i.e.*, if the `sollya_obj_t` argument really contains a functional expression) and false otherwise.

- `int sollya_lib_get_function_arity(int *n, sollya_obj_t f)`: it stores the arity of the head function in `f` at the address referred to by `n`. Currently, the mathematical functions handled in `Sollya` are at most dyadic. Mathematical constants are considered as 0-adic functions. The free variable is regarded as the identity function applied to the free variable: its arity is hence 1.
- `int sollya_lib_get_head_function(sollya_base_function_t *type, sollya_obj_t f)`: it stores the type of `f` at the address referred to by `type`. The `sollya_base_function_t` is an enum type listing all possible cases (see Table 6).

Table 6: List of values defined in type `sollya_base_function_t`

SOLLYA_BASE_FUNC_COS	SOLLYA_BASE_FUNC_DOUBLE	SOLLYA_BASE_FUNC_LOG
SOLLYA_BASE_FUNC_ACOS	SOLLYA_BASE_FUNC_DOUBLEDOUBLE	SOLLYA_BASE_FUNC_LOG_2
SOLLYA_BASE_FUNC_ACOSH	SOLLYA_BASE_FUNC_DOUBLEEXTENDED	SOLLYA_BASE_FUNC_LOG_10
SOLLYA_BASE_FUNC_COSH	SOLLYA_BASE_FUNC_TRIPLEDOUBLE	SOLLYA_BASE_FUNC_LOG_1P
SOLLYA_BASE_FUNC_SIN	SOLLYA_BASE_FUNC_HALFPRECISION	SOLLYA_BASE_FUNC_EXP
SOLLYA_BASE_FUNC_ASIN	SOLLYA_BASE_FUNC_SINGLE	SOLLYA_BASE_FUNC_EXP_M1
SOLLYA_BASE_FUNC_ASINH	SOLLYA_BASE_FUNC_QUAD	SOLLYA_BASE_FUNC_NEG
SOLLYA_BASE_FUNC_SINH	SOLLYA_BASE_FUNC_FLOOR	SOLLYA_BASE_FUNC_SUB
SOLLYA_BASE_FUNC_TAN	SOLLYA_BASE_FUNC_CEIL	SOLLYA_BASE_FUNC_ADD
SOLLYA_BASE_FUNC_ATAN	SOLLYA_BASE_FUNC_NEARESTINT	SOLLYA_BASE_FUNC_MUL
SOLLYA_BASE_FUNC_ATANH	SOLLYA_BASE_FUNC_LIBRARYCONSTANT	SOLLYA_BASE_FUNC_DIV
SOLLYA_BASE_FUNC_TANH	SOLLYA_BASE_FUNC_LIBRARYFUNCTION	SOLLYA_BASE_FUNC_POW
SOLLYA_BASE_FUNC_ERF	SOLLYA_BASE_FUNC_PROCEDUREFUNCTION	SOLLYA_BASE_FUNC_SQRT
SOLLYA_BASE_FUNC_ERFC	SOLLYA_BASE_FUNC_FREE_VARIABLE	SOLLYA_BASE_FUNC_PI
SOLLYA_BASE_FUNC_ABS	SOLLYA_BASE_FUNC_CONSTANT	

- `int sollya_lib_get_subfunctions(sollya_obj_t f, int *n, ...)`: let us denote by `g_1`, ..., `g_k` the arguments following the argument `n`. They must be of type `sollya_obj_t *`. The function stores the arity of `f` at the address referred to by `n` (except if `n` is `NULL`, in which case, `sollya_lib_get_subfunctions` simply ignores it and goes on). Suppose that `f` contains an expression of the form $f_0(f_1, \ldots, f_s)$ (as a particular case, if `f` is just the free variable, it is regarded in this context as the identity function applied to the free variable, so both f_0 and f_1 are the free variable). For each i from 1 to s, the expression corresponding to f_i is stored at the address referred to by `g_i`, unless one of the `g_i` is `NULL` in which case the function returns when encountering it. In practice, it means that the user should always put `NULL` as last argument, in order to prevent the case when they would not provide enough variables `g_i`. They can check afterwards that they provided enough variables by checking the value contained at the address referred to by `n`. If the user does not put `NULL` as last argument and do not provide enough variables `g_i`, the algorithm will continue storing arguments at random places in the memory (on the contrary, providing more arguments than necessary does not harm: useless arguments are simply ignored and left unchanged). In the case when f_0 is a library function, a constant (*i.e.*, represented by the `sollya_base_function_t` `SOLLYA_BASE_FUNC_CONSTANT`), a library constant, the constant π (*i.e.*, represented by the `sollya_base_function_t` `SOLLYA_BASE_FUNC_PI`) or a procedure function, and if the user provides a non-`NULL` argument `g_t` after `g_s`, additional information is returned in the remaining argument:

  - If f_0 is a library function, a `Sollya` object corresponding to the expression $f_0(x)$ is stored at the address referred to by `g_t`. This allows the user to get a `Sollya` object corresponding to function f_0. This object can further be used to evaluate f_0 at points or to build new expressions involving f_0. Please notice that a library function object is not necessarily the result of a call to the `library` command: it can also be, *e.g.*, the derivative of a function created by a call to `library`.
  - If f_0 is a procedure function, a `Sollya` object corresponding to the expression $f_0(x)$ is stored at the address referred to by `g_t`. The same remarks as above apply.
  - If f_0 is a constant, the constant π, or a library constant, f_0 itself is stored at the address referred to by `g_t`. In this particular case, $t = 1$ and the object referred to by `g_t` simply gets a copy of `f`. This (somehow useless) mechanism is made only to handle the cases of library functions, procedure functions, constants and library constants in a unified way.

  Please note that the objects that have been stored in variables `g_i` must manually be cleared once they become useless.

- `int sollya_lib_v_get_subfunctions(sollya_obj_t f, int *n, va_list va)`: the same as the previous function, but with a `va_list` argument.

- `int sollya_lib_get_nth_subfunction(sollya_obj_t *res, sollya_obj_t f, int m)`: while `sollya_lib_get_subfunctions` allows the user to retrieve all the subtrees of a functional expression (including an extra subtree in the case of a constant, the constant π, a library constant, a library function or a procedure function), this function allows the user to retrieve only one of them. More precisely (using the same notations as in the documentation of `sollya_lib_get_subfunctions` above) if `sollya_lib_get_subfunctions` would put something at the address referred to by variable `g_m`, then `sollya_lib_get_nth_subfunction(res, f, m)` would put the same thing at the address referred to by `res` and return a boolean integer representing true. In any other case, it would let `res` unchanged and return a boolean integer representing false. Notice that the subfunctions are numbered starting from 1 (as opposed to, *e.g.*, arrays in C), hence in the expression $f = e_1 + e_2$, the subexpression e_1 corresponds to $m = 1$ and the subexpression e_2 corresponds to $m = 2$. Accordingly, in an expression like $f = \sin(e)$, the subexpression e corresponds to $m = 1$.

- `int sollya_lib_decompose_function(sollya_obj_t f, sollya_base_function_t *type, int *n, ...)`:
  this function is a all-in-one function equivalent to using `sollya_lib_get_head_function` and `sollya_lib_get_subfunctions` in only one function call.

- `int sollya_lib_v_decompose_function(sollya_obj_t f, sollya_base_function_t *type, int *n, va_list va)`:
  the same as the previous function, but with a `va_list`.

To construct a functional expression, functions are provided that precisely undo what `sollya_lib_decompose_function` does. These functions are the following:

- `int sollya_lib_construct_function(sollya_obj_t *res, sollya_base_function_t type, ...)`:
  let us denote by `g_1`, ..., `g_k` the arguments following the argument `type`. They must be of type `sollya_obj_t`. The function creates a functional expression whose head function corresponds to the basic function represented by variable `type` and whose arguments are `g_1`, ..., `g_s` where s denotes the arity of the considered basic function. It is the responsibility of the user to provide enough arguments with respect to the required arity. As a particular case, when the desired type corresponds to a library function, a constant, the constant π, a library constant or a procedure function, the user **must** provide an extra argument `g_t` after `g_s` corresponding to what `sollya_lib_decompose_function` would store in this extra argument on such a case (namely, a `Sollya` object corresponding to the expression $f_0(x)$ in the case of a library function or procedure function, and f_0 itself in the case of a constant, the constant π or a library constant). As a particular case, and to make it more useful in practice, the argument `g_t` is allowed to be equal to `NULL` whenever `type` is equal to `SOLLYA_BASE_FUNC_PI`, in which case the function will succeed, even though `g_t` does not contain the constant π as it theoretically should. Notice however that any other value than `NULL` leads to a failure if it does not contain the constant π itself. If everything goes well the functional expression is created and stored at the address referred to by `res` and a boolean integer representing true is returned. Notice that the arguments `g_1`, ..., `g_k` are not eaten up by this function and the user must subsequently manually clear these objects. If something goes wrong (bad number of arguments, arguments not having the proper type, etc.) `res` is left unchanged and a boolean integer representing false is returned.

- `int sollya_lib_v_construct_function(sollya_obj_t *res, sollya_base_function_t type, va_list varlist)`:
  the same as the previous function, but with a `va_list`.

As an example of use of the functions described in the present section, the following code returns 1 if `f` denotes a functional expression made only of constants (*i.e.*, without the free variable), and returns 0 otherwise:

```
#include <sollya.h>

/* Note: we suppose that the library has already been initialized */
int is_made_of_constants(sollya_obj_t f) {
  sollya_obj_t tmp1 = NULL;
  sollya_obj_t tmp2 = NULL;
  int n, r, res;
  sollya_base_function_t type;

  r = sollya_lib_decompose_function(f, &type, &n, &tmp1, &tmp2, NULL);
  if (!r) { sollya_lib_printf("Not a mathematical function\n"); res = 0; }
  else if (n >= 3) {
    sollya_lib_printf("Unexpected error: %b has more than two arguments.\n", f);
    res = 0;
  }
  else {
    switch (type) {
      case SOLLYA_BASE_FUNC_FREE_VARIABLE: res = 0; break;
      case SOLLYA_BASE_FUNC_PI: res = 1; break;
      case SOLLYA_BASE_FUNC_CONSTANT: res = 1; break;
      case SOLLYA_BASE_FUNC_LIBRARYCONSTANT: res = 1; break;
      default:
        res = is_made_of_constants(tmp1);
        if ((res) && (n==2)) res = is_made_of_constants(tmp2);
    }
  }

  if (tmp1) sollya_lib_clear_obj(tmp1);
  if (tmp2) sollya_lib_clear_obj(tmp2);

  return res;
}
```

Functions are provided to allow the user to retrieve further information from library function, library constant, procedure function, external procedure and external data objects:

- `int sollya_lib_decompose_libraryfunction(int (**f)(mpfi_t, mpfi_t, int), int *deriv, sollya_obj_t *e, sollya_obj_t g)`:
  assume that `g` represents an expression $f_0(f_1)$ where f_0 is a library function. Then, f_0 is the n-th derivative (for some n) of a function provided within `Sollya` via an external C function `int func(mpfi_t, mpfi_t, int)`.
  As a result of a call to `sollya_lib_decompose_libraryfunction`, the value n is stored at the address referred to by `deriv`, a pointer to `func` is stored at the address referred to by `f` and a `Sollya` object representing f_1 is stored at the address referred to by `e`. Please notice that the object stored in `e` must manually be cleared once it becomes useless. Upon success, a boolean integer representing true is returned. If `g` is not a library function object, nothing happens and false is returned.

- `int sollya_lib_decompose_libraryfunction_with_data(int (**f)(mpfi_t, mpfi_t, int, void *), int *deriv, sollya_obj_t *e, void **data, void (**dealloc)(void *), sollya_obj_t g)`:
  works exactly as the previous function but additionally returns a pointer to the `void *` data field and to the pointer to the deallocation function that had been provided when the library

function was created. Notice that, in the case when `g` represents an expression $f_0(f_1)$ where f_0 is indeed a library function, but has been constructed with `sollya_lib_libraryfunction` or `sollya_lib_build_function_libraryfunction` and not with one of the `_with_data` variants, this function will fail and return false without touching any of its argument.

- `int sollya_lib_decompose_procedurefunction(sollya_obj_t *f, int *deriv, sollya_obj_t *e, sollya_obj_t g)`:
  assume that `g` represents an expression $f_0(f_1)$ where f_0 is a procedure function. Then, f_0 is the n-th derivative (for some n) of a function provided within `Sollya` via a procedure `proc(X, n, p) {...}`. As a result of a call to `sollya_lib_decompose_procedurefunction`, the value n is stored at the address referred to by `deriv`, a `Sollya` object representing the procedure is stored at the address referred to by `f`, a `Sollya` object representing f_1 is stored at the address referred to by `e`. Please notice that the objects stored in `f` and `e` must manually be cleared once they become useless. Upon success, a boolean integer representing true is returned. If `g` is not a procedure function object, nothing happens and false is returned.

- `int sollya_lib_decompose_libraryconstant(void (**f)(mpfr_t, mp_prec_t), sollya_obj_t c)`:
  assume that `c` is a constant provided via an external C function `void func(mpfr_t, mp_prec_t)`. As a result of a call to `sollya_lib_decompose_libraryconstant`, a pointer to `func` is stored at the address referred to by `f` and a boolean integer representing true is returned. Otherwise, nothing happens and false is returned.

- `int sollya_lib_decompose_libraryconstant_with_data(void (**f)(mpfr_t, mp_prec_t, void *), void **data, void (**dealloc)(void *), sollya_obj_t c)`:
  works exactly as the previous function but additionally returns a pointer to the `void *` data field and the pointer to the deallocation function that had been provided when the library constant was created. Similarly to `sollya_lib_decompose_libraryfunction_with_data`, this function fails on library constants that have been created with `sollya_lib_libraryconstant` or `sollya_lib_build_function_libraryconstant` instead of using the `_with_data` variant of these constructors.

- `int sollya_lib_decompose_externalprocedure(sollya_externalprocedure_type_t *res, sollya_externalprocedure_type_t **args, int *arity, void **f, sollya_obj_t p)`:
  assume that `p` is an external procedure provided via an external C function `func` (of appropriate type) bound to `Sollya` by one of the means provided for that purpose. As a result of a call to `sollya_lib_decompose_externalprocedure`, a pointer to `func` is stored at the address pointed to by `f`, the result type of the external procedure is stored at `res`, an array of its argument types is allocated, filled and stored at the address pointed to by `args` (unless the procedure function takes no argument in which case the `args` argument is ignored), the arity of the external procedure (and hence number of elements of the array allocated and stored at `args`) is stored at the integer pointed by `arity` and true is returned. If `p` is no external procedure object, nothing happens and false is returned.

- `int sollya_lib_decompose_externalprocedure_with_data(sollya_externalprocedure_type_t *res, sollya_externalprocedure_type_t **args, int *arity, void **f, void **data, void (**dealloc)(void *), sollya_obj_t p)`:
  works exactly as the previous function but additionally returns a pointer to the `void *` data field and the pointer to the deallocation function that had been provided when the external procedure was created. The same remark as with `sollya_lib_decompose_libraryfunction_with_data` and `sollya_lib_decompose_libraryconstant_with_data` also applies: this function fails when

used on an external procedure that has been constructed without the `_with_data` variant of the constructor.

- `int sollya_lib_decompose_externaldata(void **data,`
  `void (**dealloc)(void *),`
  `sollya_obj_t obj);`
  assume that `obj` is an external data object provided as a data pointer bound to `Sollya` by one of the means provided for that purpose. As a result of a call to `sollya_lib_decompose_externaldata`, the data pointer is stored at the address pointed to by `data` and true is returned. If `obj` is no external data object, nothing happens and false is returned. In addition, if `dealloc` is not `NULL`, the pointer to the deallocation function that had been provided when the external data object was created is stored at the memory location pointed to by `dealloc`. If no deallocation function was provided but `dealloc` is non-`NULL`, which is the case for instance when the external data object was created in the interactive tool with the `externaldata` command, the `NULL` pointer will be stored at the location pointed to by `dealloc`.

10.13 Faithfully evaluate a functional expression

Let us suppose that `f` is a functional expression and `a` is a numerical value or a constant expression. One of the very convenient features of the interactive tool is that the user can simply write `f(a)` at the prompt: the tool automatically adapts its internal precision in order to compute a value that is a faithful rounding (at the current tool precision) of the true value $f(a)$. Sometimes it does not achieve to find a faithful rounding, but in any case, if the result is not proved to be exact, a warning is displayed explaining how confident one should be with respect to the returned value. The object `a` can also be an interval, in which case `Sollya` automatically performs the evaluation using an enhanced interval arithmetic, *e.g.*, using L'Hopital's rule to produce finite (yet valid of course) enclosures even in cases when f exhibits removable singularities (for instance $\sin(x)/x$ over an interval containing 0). This behavior is reproduced in the library with the `sollya_lib_apply` function (this function is in fact to be used to reproduce any construction of the form `obj1(obj2, obj3, ...)` within the library; for instance `obj1` might also be a procedure. See Section 10.17 for a more detailed description of this function). More precisely if `f` and `a` are two `sollya_obj_t` representing respectively a univariate function and a constant or an interval, the following call returns a new `sollya_obj_t` representing the object that would be produced as a result of typing `f(a);` at the interactive prompt:

```
b = sollya_lib_apply(f, a, NULL);
```

However, when using the library, it might be interesting to have access to this feature when the argument `a` is not a `Sollya` object but rather directly a multiprecision constant of type `mpfr_t` or `mpfi_t`. Also, in this case, one may want to have a finer-grain access to the evaluation algorithm, *e.g.*, to correctly react to cases where a faithful rounding has not been achieved without having to catch warning messages emitted by `Sollya`. This is the reason why the library proposes the following functions.

To evaluate a unary function at a constant expression or constant value, the library provides the two following functions:

- `sollya_fp_result_t`
  `sollya_lib_evaluate_function_at_constant_expression(mpfr_t res, sollya_obj_t f,`
  `sollya_obj_t a,`
  `mpfr_t *cutoff),`

- `sollya_fp_result_t`
  `sollya_lib_evaluate_function_at_point(mpfr_t res, sollya_obj_t f,`
  `mpfr_t a, mpfr_t *cutoff).`

In the former, the argument `a` is any `sollya_obj_t` containing a numerical constant or a constant expression, while in the latter `a` is a constant already stored in a `mpfr_t`. These functions store the result in `res` and return a `sollya_fp_result_t` which is an enum type described in Table 7. In order to understand the role of the `cutoff` parameter and the value returned by the function, it is necessary to describe the algorithm in a nutshell:

Input: a functional expression `f`, a constant expression `a`, a target precision q, a parameter ε.

1. Choose an initial working precision p.
2. Evaluate `a` with interval arithmetic, performing the computations at precision p.
3. Replace the occurrences of the free variable in `f` by the interval obtained at step 2. Evaluate the resulting expression with interval arithmetic, performing the computations at precision p. This yields an interval $I = [x, y]$.
4. Examine the following cases successively (RN denotes rounding to nearest at precision q):
   (a) If $\mathrm{RN}(x) = \mathrm{RN}(y)$, set `res` to that value and return.
   (b) If I does not contain any floating-point number at precision q, set `res` to one of both floating-point numbers enclosing I and return.
   (c) If I contains exactly one floating-point number at precision q, set `res` to that number and return.
   (d) If all numbers in I are smaller than ε in absolute value, then set `res` to 0 and return.
   (e) If p has already been increased many times, then set `res` to some value in I and return.
   (f) Otherwise, increase p and go back to step 2.

The target precision q is chosen to be the precision of the `mpfr_t` variable `res`. The parameter ε corresponds to the parameter `cutoff`. The reason why `cutoff` is a pointer is that, most of the time, the user may not want to provide it, and using a pointer makes it possible to pass `NULL` instead. So, if `NULL` is given, ε is set to 0. If `cutoff` is not `NULL`, the absolute value of `*cutoff` is used as value for ε. Using a non-zero value for ε can be useful when one does not care about the precise value of $f(a)$ whenever its absolute value is below a given threshold. Typically, if one wants to compute the maximum of $|f(a_1)|$, $\ldots$, $|f(a_n)|$, it is not necessary to spend too much effort on the computation of $|f(a_i)|$ if one already knows that it is smaller than $\varepsilon = \max\{|f(a_1)|, \ldots, |f(a_{i-1})|\}$.

To evaluate a unary function on an interval, the following function is provided:

```
int sollya_lib_evaluate_function_over_interval(mpfi_t res, sollya_obj_t f, mpfi_t a).
```

This function returns a boolean integer: false means failure (*i.e.*, `f` is not a functional expression), in which case `res` is left unchanged, and true means success, in which case `res` contains the result of the evaluation. The function might succeed, and yet `res` might contain something useless such as an unbounded interval or even $[\mathrm{NaN}, \mathrm{NaN}]$ (this happens for instance when `a` contains points that lie in the interior of the complement of the definition domain of `f`). It is the user's responsibility to check afterwards whether the computed interval is bounded, unbounded or NaN.

10.14 Comparing objects structurally and computing hashes on `Sollya` objects

The library provides function

```
int sollya_lib_cmp_objs_structurally(sollya_obj_t obj1, sollya_obj_t obj2)
```

to allow the user to perform a structural comparison of any two `Sollya` objects. It returns an integer (interpreted as a boolean) that is true if and only if `obj1` and `obj2` are syntactically the same (as opposed to mathematically). For instance the fractions 2/3 and 4/6 are recognized as mathematically equal by `Sollya` when compared with `==` (or `sollya_lib_cmp_equal` with the library) but are syntactically different.

Certain language bindings require hashes to be available for any object represented. In order to help with such language bindings, the `Sollya` library supports a function that computes a 64 bit unsigned integer as a hash for a given `Sollya` object:

Table 7: List of values defined in type `sollya_fp_result_t`

Value	Meaning
`SOLLYA_FP_OBJ_NO_FUNCTION`	`f` is not a functional expression.
`SOLLYA_FP_EXPRESSION_NOT_CONSTANT`	`a` is not a constant expression.
`SOLLYA_FP_FAILURE`	The algorithm ended up at step (e) and I contained NaN. This typically happens when a is not in the definition domain of f.
`SOLLYA_FP_CUTOFF_IS_NAN`	`cutoff` was not `NULL` and the value of `*cutoff` is NaN.
`SOLLYA_FP_INFINITY`	The algorithm ended up at step (a) with I of the form $[+\infty, +\infty]$ or $[-\infty, -\infty]$. Hence $f(a)$ is proved to be an exact infinity.
`SOLLYA_FP_PROVEN_EXACT`	The algorithm ended up at step (a) with a finite value and $x = \mathrm{RN}(x) = \mathrm{RN}(y) = y$.
`SOLLYA_FP_CORRECTLY_ROUNDED_PROVEN_INEXACT`	The algorithm ended up at step (b) with a finite value and $\texttt{res} < x \leq y$ or $x \leq y < \texttt{res}$.
`SOLLYA_FP_CORRECTLY_ROUNDED`	The algorithm ended up at step (a) with a finite value and $x \leq \texttt{res} \leq y$. [a]
`SOLLYA_FP_FAITHFUL_PROVEN_INEXACT`	The algorithm ended up at step (b) with a finite value and $\texttt{res} < x \leq y$ or $x \leq y < \texttt{res}$.
`SOLLYA_FP_FAITHFUL`	The algorithm ended up at step (c) with a finite value. [a]
`SOLLYA_FP_BELOW_CUTOFF`	The algorithm ended up at step (d).
`SOLLYA_FP_NOT_FAITHFUL_ZERO_CONTAINED_BELOW_THRESHOLD`	The algorithm ended up at step (e) and I was of the form $[-\delta_1, \delta_2]$ where $0 < \delta_i \ll 1$ (below some threshold of the algorithm). This typically happens when $f(a)$ exactly equals zero, but the algorithm does not manage to prove this exact equality.
`SOLLYA_FP_NOT_FAITHFUL_ZERO_CONTAINED_NOT_BELOW_THRESHOLD`	The algorithm ended up at step (e) with an interval I containing 0 but too large to fall in the above case. [b]
`SOLLYA_FP_NOT_FAITHFUL_ZERO_NOT_CONTAINED`	The algorithm ended up at step (e) with an interval I that does not contain 0. [b]
`SOLLYA_FP_NOT_FAITHFUL_INFINITY_CONTAINED`	The algorithm ended up at step (e) and (at least) one of the bounds of I was infinite. This typically happens when the limit of $f(x)$ when x goes to a is infinite.

[a] Please notice that this means that the algorithm did not manage to conclude whether the result is exact or not. However, it might have been able to conclude if the working precision had been increased.

[b] In general, this should be considered as a case of failure and the value stored in `res` might be completely irrelevant.

`uint64_t sollya_lib_hash(sollya_obj_t obj).`

The `Sollya` library guarantees that any two objects that are syntactically equal (as when compared with `sollya_lib_cmp_objs_structurally`) will have the same hash value. For some particular objects (*e.g.*, polynomials) `Sollya` can normalize the expression before computing the hash value and in this case two objects that are mathematically equal (even though they are not structurally equal) will have the same hash value. However, except in such particular cases, two objects that are syntactically different are likely to have different hashes (although this is not guaranteed, of course).

Computing the hash of an object takes a time proportional to the size of the directed acyclic graph internally used to represent that object. However, `Sollya` will cache an object's hash value for further use after it has been computed, so the cost of computing the hash of a given object is paid only once.

The user should also be aware that the hash value for a given object is currently not guaranteed to be portable between platforms nor over consecutive `Sollya` versions.

10.15 Executing `Sollya` procedures

Objects representing procedures written in `Sollya` language (see also Section 7.1) can be created using the `Sollya` library functions `sollya_lib_parse_string` and `sollya_lib_parse` or through execution of a `Sollya` script using `sollya_lib_execute`.

In order to execute such procedure objects on arguments, available as `Sollya` objects, too, the functions

- `sollya_obj_t sollya_lib_execute_procedure(sollya_obj_t proc, ...)` and
- `sollya_obj_t sollya_lib_v_execute_procedure(sollya_obj_t proc, va_list arglist)`

may be used. These functions apply the given procedure `proc` on the following arguments (or the elements in the argument list `arglist`). If no argument is needed to execute the procedure, the variadic argument list shall immediately be terminated using `NULL`; otherwise the argument list shall be terminated with an extra `NULL` argument. An arity test is performed by `Sollya` before the procedure is executed: if the arity of the given procedure does not correspond to the actual number of given arguments (and the `Sollya` procedure is not variadic), an error object is returned instead of the procedure's result.

When the functions are used to execute procedures that return a `Sollya` object, the object is returned by the function. When the procedure does not use the `Sollya return` statement or returns the `Sollya void` object, a `Sollya void` object is returned. The user should not forget to deallocate that `void` object.

10.16 Name of the free variable

The default name for the free variable is the same in the library and in the interactive tool: it is `_x_`. In the interactive tool, this name is automatically changed at the first use of an undefined symbol. Accordingly in library mode, if an object is defined by `sollya_lib_parse_string` with an expression containing an undefined symbol, that symbol will become the free variable name if it has not already been changed before. But what if one does not use `sollya_lib_parse_string` (because it is not efficient) but one wants to change the name of the free variable? The name can be changed with `sollya_lib_name_free_variable("some_name")`.

It is possible to get the current name of the free variable with `sollya_lib_get_free_variable_name()`. This function returns a `char *` containing the current name of the free variable. Please note that this `char *` is dynamically allocated on the heap and should be cleared after its use with `sollya_lib_free()` (see below).

10.17 Commands and functions

Besides some exceptions, every command and every function available in the `Sollya` interactive tool has its equivalent (with a very close syntax) in the library. Section 8 of the present documentation gives the library syntax as well as the interactive tool syntax of each commands and functions. The same information is available within the interactive tool by typing `help some_command`. So if one knows the name of a command or function in the interactive tool, it is easy to recover its library name and signature.

There are some commands and functions available in interactive mode which, for syntactical reasons, have a different function name in the `Sollya` library:

- The `Sollya` language construction `(obj1)(obj2, obj3, ...)` which applies the object `obj1` to the objects `obj2`, `obj3`, etc. is expressed in the `Sollya` library through a call to `sollya_obj_t sollya_lib_apply(sollya_obj_t obj1, sollya_obj_t obj2, ...)` resp. `sollya_obj_t sollya_lib_v_apply(sollya_obj_t obj1, sollya_obj_t obj2, va_list)`.

A particular point is worth mentioning: some functions of the tool such as `remez` for instance have a variable number of arguments. For instance, one might call `remez(exp(x), 4, [0,1])` or `remez(1, 4, [0,1], 1/exp(x))`. This feature is rendered in the C library by the use of variadic functions (functions with an arbitrary number of arguments), as they are permitted by the C standard. The notable difference is that there must **always be an explicit NULL argument** at the end of the function call. Hence one can write `sollya_lib_remez(a, b, c, NULL)` or `sollya_lib_remez(a, b, c, d, NULL)`. It is very easy to forget the `NULL` argument and to use for instance `sollya_lib_remez(a, b, c)`. This is **completely wrong** because the memory will be read until a `NULL` pointer is found. In the best case, this will lead to an error or a result obviously wrong, but it could also lead to subtle, not-easy-to-debug errors. The user is advised to be particularly careful with respect to this point.

Each command or function accepting a variable number of arguments comes in a `sollya_lib_v_` version accepting a `va_list` parameter containing the list of optional arguments. For instance, one might write a function that takes as arguments a function f, an interval I, optionally a weight function w, optionally a quality parameter q. That function would display the minimax obtained when approximating f over I (possibly with weight w and quality q) by polynomials of degree $n = 2$ to 20. So, that function would get a variable number of arguments (*i.e.*, a `va_list` in fact) and pass them straight to remez. In that case, one needs to use the `v_remez` version, as the following code shows:

```
#include <sollya.h>
#include <stdarg.h>

/* Note: we suppose that the library has already been initialized */
void my_function(sollya_obj_t f, sollya_obj_t I, ...) {
  sollya_obj_t n, res;
  int i;
  va_list va;

  for(i=2;i<=20;i++) {
    n = SOLLYA_CONST(i);
    va_start(va, I);
    res = sollya_lib_v_remez(f, n, I, va);
    sollya_lib_printf("Approximation of degree %b is %b\n", n, res);
    va_end(va);
    sollya_lib_clear_obj(n);
    sollya_lib_clear_obj(res);
  }

  return;
}
```

10.18 Warning messages in library mode

10.18.1 Catching warning messages

The philosophy of `Sollya` is "whenever something is not exact, explicitly warn about that". This is a nice feature since this ensures that the user always perfectly knows the degree of confidence they can have in a result (is it exact? or only faithful? or even purely numerical, without any warranty?) However, it is sometimes desirable to hide some (or all) of these messages. This is especially true in library mode where messages coming from `Sollya` are intermingled with the messages of the main program. The library

hence provides a specific mechanism to catch all messages emitted by the `Sollya` core and handle each of them specifically: installation of a callback for messages.

Before describing the principle of the message callback, it seems appropriate to recall that several mechanisms are available in the interactive tool to filter the messages emitted by `Sollya`. These mechanisms are also available in library mode for completeness. When a message is emitted, it has two characteristics: a verbosity level and an id (a number uniquely identifying the message). After it has been emitted, it passes through the following steps where it can be filtered. If it has not been filtered (and only in this case) it is displayed.

1. If the verbosity level of the message is greater than the value of the environment variable `verbosity`, it is filtered.

2. If the environment variable `roundingwarnings` is set to `off` and if the message informs the user that a rounding occurred, it is filtered.

3. If the id of the message has been registered with the `suppressmessage` command, the message is filtered.

4. If a message callback has been installed and if the message has not been previously filtered, it is handled by the callback, which decides to filter it or to permit its displaying.

A message callback is a function of the form `int my_callback(sollya_msg_t msg, void *data)`. It receives as input an object representing the message and a user-defined pointer. It performs whatever treatment seems appropriate and returns an integer interpreted as a boolean. If the returned value is false, the message is not displayed. If, on the contrary, the returned value is true, the message is displayed as usual. By default, no callback is installed and all messages are displayed. To install a callback, use `sollya_lib_install_msg_callback(my_callback, data)`. The `(void *)` pointer `data` is arbitrary (it can be `NULL`) and is simply transmitted as second argument at each call of the callback. It can be used, *e.g.*, to point to a segment of memory where some information should be stored from a call of the callback to another.

Please remember that, if a message is filtered because of one of the three other mechanisms, it will never be transmitted to the callback. Hence, in library mode, if one wants to catch every single message through the callback, one should set the value of `verbosity` to `MAX_INT`, set `roundingwarnings` to `on` (this is the default anyway) and one should not use the `suppressmessage` mechanism.

It is possible to come back to the default behavior, using `sollya_lib_uninstall_msg_callback()`. Please notice that callbacks do not stack over each other: *i.e.*, if some callback `callback1` is installed, and if one installs another one `callback2`, then the effect of `sollya_lib_uninstall_msg_callback()` is to come back to the default behavior, **and not** to come back to callback `callback1`.

Both `sollya_lib_install_msg_callback` and `sollya_lib_uninstall_msg_callback` return an integer interpreted as a boolean: false means failure and true means success.

It is possible to get the current callback using `sollya_lib_get_msg_callback(cb_ptr, data_ptr)`. This stores the current callback at the address referred to by `cb_ptr` (the type of `cb_ptr` is hence `int (**)(sollya_msg_t, void *)`) and stores the current data pointer at the address referred to by `data_ptr` (which has hence `(void **)` type). The arguments `cb_ptr` and `data_ptr` can be `NULL` in which case the corresponding argument is not retrieved (please take care of the difference between `data_ptr` being `NULL` and `data_ptr` pointing to a `(void *)` pointer which value is `NULL`). If no callback is currently installed, the `NULL` value is stored at the addresses referred to by `cb_ptr` and `data_ptr`.

The type `sollya_msg_t` is indeed a pointer and its content is only accessible during the callback call: it does not make sense to keep it for further use after the callback call. Currently the type has only two accessors:

- `int sollya_lib_get_msg_id(sollya_msg_t msg)` returns an integer that identifies the type of the message. The message types are listed in the file `sollya-messages.h`. Please note that this file not only lists the possible identifiers but also defines meaningful names to each possible message number (*e.g.*, `SOLLYA_MSG_UNDEFINED_ERROR` is an alias for the number 2 but is more meaningful to understand what the message is about). It is recommended to use these names instead of numerical values.

- `char *sollya_lib_msg_to_text(sollya_msg_t msg)` returns a generic string briefly summarizing the contents of the message. Please note that this `char *` is dynamically allocated on the heap and should manually be cleared with `sollya_lib_free` when it becomes useless.

In the future, other accessors could be added (to get the verbosity level at which the message has been emitted, to get data associated with the message, etc.) The developers of `Sollya` are open to suggestions and feature requests on this subject.

As an illustration let us give a few examples of possible use of callbacks:

Example 1: A callback that filters everything.

```
int hide_everything(sollya_msg_t msg, void *data) {
  return 0;
}
```

Example 2: filter everything but the messages indicating that a comparison is uncertain.

```
int keep_comparison_warnings(sollya_msg_t msg, void *data) {
  switch(sollya_lib_get_msg_id(msg)) {
    case SOLLYA_MSG_TEST_RELIES_ON_FP_RESULT_THAT_IS_NOT_FAITHFUL:
    case SOLLYA_MSG_TEST_RELIES_ON_FP_RESULT:
    case SOLLYA_MSG_TEST_RELIES_ON_FP_RESULT_FAITHFUL_BUT_UNDECIDED:
    case SOLLYA_MSG_TEST_RELIES_ON_FP_RESULT_FAITHFUL_BUT_NOT_REAL:
      return 1;
    default:
      return 0;
  }
}
```

Example 3: ensuring perfect silence for a particular function call (uses the callback defined in Example 1).

```
...
int (*old_callback)(sollya_msg_t, void *);
void *old_data;
sollya_lib_get_msg_callback(&old_callback, &old_data);
sollya_lib_install_msg_callback(hide_everything, NULL);
  /* Here takes place the function call that must be completely silent */
if (old_callback) sollya_lib_install_msg_callback(old_callback, old_data);
...
```

Example 4: using the `(void *)` data argument to store information from a call to another.

```
int set_flag_on_problem(sollya_msg_t msg, void *data) {
  switch(sollya_lib_get_msg_id(msg)) {
    case SOLLYA_MSG_DOUBLE_ROUNDING_ON_CONVERSION:
      *((int *)(data)) = 1;
  }
  return 1;
}

...

int main() {
  int flag_double_rounding = 0;
  ...
  sollya_lib_init();
  sollya_lib_install_msg_callback(set_flag_on_problem, &flag_double_rounding);
  ...
}
```

More involved examples are possible: for instance, instead of setting a flag, it is possible to keep in some variable what the last message was. One may even implement a stack mechanism and store the messages in a stack, in order to handle them later. (Please remember however that `sollya_msg_t` is a pointer type and that the `sollya_msg_t` object received as argument of a callback call has no more meaning once the callback call returned. If a stack mechanism is implemented it should store information such as the message ID, or the message text, as given by `sollya_lib_get_msg_id` and `sollya_lib_msg_to_text`, but not the `sollya_msg_t` object itself.)

10.18.2 Emitting warning messages

The `Sollya` library offers a way to print a message, as if it were produced by the `Sollya` core. Such a message will go through the entire process described in the previous section, and can eventually provoke a callback call if a callback is installed. The function supporting this feature is

`void sollya_lib_printlibrarymessage(int verb, const char *str)`.

The first argument `verb` is the least verbosity level at which that warning shall be displayed. The second argument `str` is the message to be displayed.

When a message is produced with this function, its message ID (when caught by a callback) is `SOLLYA_MSG_GENERIC_SOLLYA_LIBRARY_MSG`. An important notice is that the character string returned by `sollya_lib_msg_to_text` when such a message is caught by a callback **is currently not** the argument `str` provided to `sollya_lib_printlibrarymessage`, but is instead a generic message. This behavior might change in the future.

10.19 Using `Sollya` in a program that has its own allocation functions

`Sollya` uses its own allocation functions: as a consequence, pointers that have been allocated by `Sollya` functions must be freed using `sollya_lib_free` instead of the usual `free` function. Another consequence is that `Sollya` registers its own allocation functions to the `GMP` library, using the mechanism provided by `GMP`, so that `GMP` also uses `Sollya` allocation functions behind the scene, when the user performs a call to, *e.g.*, `mpz_init`, `mpfr_init2`, etc.

In general, this is completely harmless and the user might even not notice it. However, this is a problem if `Sollya` is used in a program that also uses its own allocation functions and that has already registered these functions to `GMP`. Actually:

- If the main program has already registered allocation functions to `GMP` and if `Sollya` is naively initialized with `sollya_lib_init()`, `Sollya` will register its own allocation functions, thus overriding the previously registered functions.

- If the user initializes first Sollya, and then registers its own allocation functions to GMP, the exact opposite happens: Sollya allocation functions are overridden by those of the user, and this will likely cause Sollya to crash (or worst, silently behave not reliably).

In order to solve this issue, Sollya provides a chaining mechanism that we are now going to describe. The idea is the following: suppose that the main program should use a function custom_malloc. The user should not use mp_set_memory_functions as usual, but should instead initialize Sollya with the initializing function described above. This will cause Sollya to register an allocation function sollya_lib_malloc to GMP. This function overloads custom_malloc: when called, it uses custom_malloc to perform the actual allocation and does nothing else but some internal accounting and verification for that allocation. To repeat, the actual allocation is done by custom_malloc; hence from the point of view of the user, the mechanism is completely transparent and equivalent to directly registering custom_malloc to GMP. The same holds for all other allocation functions: in particular, this is true for free as well: if a function custom_free is given at the initialization of Sollya, then the function sollya_lib_free eventually uses custom_free to free the memory.

The initialization function providing this mechanism is:

```
int sollya_lib_init_with_custom_memory_functions(
          void *(*custom_malloc)(size_t),
          void *(*custom_calloc)(size_t, size_t),
          void *(*custom_realloc)(void *, size_t),
          void (*custom_free)(void *),
          void *(*custom_realloc_with_size)(void *, size_t, size_t),
          void (*custom_free_with_size)(void *, size_t)).
```

None of the arguments is mandatory: if the user does not want to provide an argument, they may use NULL as a placeholder for that argument. In that case, the corresponding Sollya default function will be used. Indeed, the default initializing function sollya_lib_init() is just an alias to sollya_lib_init_with_custom_memory_functions(NULL, NULL, NULL, NULL, NULL, NULL).

Please notice, that if custom_malloc is provided, then the function sollya_lib_malloc will be defined as an overloaded version of custom_malloc. Hence, custom_malloc will eventually be used for all the allocations performed by Sollya (including the allocation of memory for its own purpose). This is true also for custom_calloc, custom_realloc and custom_free. However, this is not the case for custom_realloc_with_size and custom_free_with_size: these functions are only required for the registration to GMP and are not used by Sollya itself (except of course when Sollya allocates function through a call to a GMP, MPFR or MPFI function). Thus, to sum up:

- If the user only wants to register their own functions to GMP through Sollya, they only need to provide custom_malloc, custom_realloc_with_size and custom_free_with_size at the initialization of Sollya (actually an overloaded version will be registered to GMP but this is transparent for the user, as explained above).
- If the user also wants Sollya to use their custom allocation functions for all allocations of memory by Sollya, then they also need to provide custom_calloc, custom_realloc and custom_free.

Of course, even if the user registers custom_malloc, custom_free, etc., at the initialization of Sollya, they stay free to use them for their own allocation needs: only allocations performed by GMP (and consequently MPFR and MPFI) and allocations performed by Sollya have to use the chaining mechanism. However, for the convenience of the user, the library also provides access to the allocation functions of Sollya. They are the following:

- void sollya_lib_free(void *)
- void *sollya_lib_malloc(size_t)
- void *sollya_lib_calloc(size_t, size_t)
- void *sollya_lib_realloc(void *, size_t).

No access to the overloaded version of custom_realloc_with_size and custom_free_with_size is provided, but if the user really wants to retrieve them, they can do it with mp_get_memory_functions since they are registered to GMP.